VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Onate and D. R. J. Owen (Eds)

VISCOELASTIC TRANSVERSE IMPACT OF A SPHERE ON AN UFLYAND-MINDLIN PLATE

Yuriy A. Rossikhin, Marina V. Shitikova and Alexey A. Loktev

Department of Theoretical Mechanics
Voronezh State University of Architecture and Civil Engineering
st. Moiseeva 47-134, 394055 Voronezh, Russia
e-mail: prtlokt@yandex.ru

Key words: Transverse impact, Uflyand-Mindlin plate, Ray method, Nonstationary wave surface.

Summary. *Mathematical modeling of the impact of a solid body upon a buffer positioned on a thin isotropic plate, whose dynamic behaviour is described by the Uflyand-Mindlin wave equations taking the rotary inertia and shear deformations into account, is investigated. Two types of the isotropic plate are considered in the present paper: elastic and viscoelastic. The buffer represents a steel screw cylindrical spring and a liquid damper connected with the spring consecutively. Wave equations allow to assume that in a plate the transient wave of transverse shear, because of which there is a deformation of a plate material outside of contact area, is generated with final velocity. As a method of the decision the ray method and method of splicing asymptotic expansion received for small times in a contact area and outside of it are used.*

1 INTRODUCTION

The transverse impact of a viscoelastic impactor on an elastic shallow spherical shell was investigated in the paper [1]. Impactor represents a rigid body of mass *m* and viscoelastic Maxwell's element, which one end is connected with the mass and another end impacts upon the shell. The shell of the final sizes was considered, which behaviour was described by the classical system of the equations based on Kirchhoff-Love hypotheses.

The problem with the similar calculating scheme is considered in the present paper, a non-classical plate of the unlimited sizes is used as a target, whose dynamic behaviour is described by the Uflyand-Mindlin equations, taking the rotary inertia and transverse shear deformations into account, the behaviour of the viscoelastic buffer is described by the Maxwell model. The wave approach based on splicing on the border of the solution contact area for required function inside a contact disk and outside of it, is used for the problem solution [2,3].

2 PROBLEM FORMULATION AND GOVERNING EQUATIONS

A rigid body with the mass *m* comes closer with the velocity V_0 to the free end of a buffer, whose another end is clamped in the center of a circular isotropic plate. It is assumed that the sphere moves along the buffer's axis, which is perpendicular to the plate (Fig.1). The viscoelastic buffer does not lose its stability during deformation, its stiffness is expressed in terms of the operator, and thus the relationship for contact force takes on the integral form with a function of the relaxation for Maxwell's model.

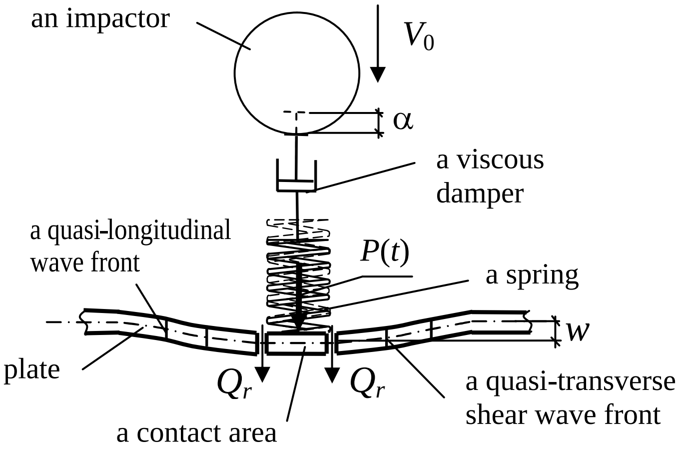

Fig. 1 Scheme of the shock interaction of a body and a viscoelastic buffer embedded into a plate

The dynamic behaviour of the elastic isotropic Uflyand-Mindlin plate behind the nonstationary elastic wave's fronts is described in the polar coordinates by the following equations [2]:

$$\frac{1}{r}\left(M_r - M_\varphi\right) + \frac{\partial M_r}{\partial r} + Q_r = \frac{\rho h^3}{12}\dot{B}_r\ ,\quad \frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} = \rho h \dot{W}\ , \tag{1}$$

$$\dot{M}_r = D\left(\frac{\partial B_r}{\partial r} + \sigma\frac{B_r}{r}\right),\ \dot{M}_\varphi = D\left(\frac{B_r}{r} + \sigma\frac{\partial B_r}{\partial r}\right),\ \dot{Q}_r = K\mu h\left(\frac{\partial W}{\partial r} - B_r\right), \tag{2}$$

where r and φ are the polar radius and angle, respectively, Ì$_r$ and Ì$_\varphi$ are the bending moments, Q_r is the transverse force, B_r is the angular speed of the normal to the plate's middle surface in direction r, W is the lateral displacement velocity, ρ is the density, h is the plate thickness, $D = Eh^3/12(1-\sigma^2)$, E is the modulus of elasticity, σ is Poisson's ratio, μ is the shear modulus, $\hat{E} = \pi^2/12$, and an overdot denotes a derivative with respect to time t.

In the present paper the viscoelastic plate is also considered. Viscoelastic properties of the plate's material under shear deformations are described by the representation of the shear modulus and Young's modulus in terms of the operator, and the Hooke's law takes on the integral form with an arbitrary kernel of relaxation, in so doing Poisson's ratio does not depend on viscoelastic properties of the material. For viscoelastic plate equations (2) will be represented in form

$$\dot{M}_r = -D_\infty\ \left[\left(\frac{\partial\dot{\beta}_r}{\partial r} + \sigma\frac{\dot{\beta}_r}{r}\right) - \int_0^t g\left(t-t'\right)\ \left(\frac{\partial\ddot{\beta}_r}{\partial r} + \sigma\frac{\ddot{\beta}_r}{r}\right)dt'\right], \tag{3}$$

$$\dot{M}_\varphi = -D_\infty\ \left[\left(\frac{\dot{\beta}_r}{r} + \sigma\frac{\partial\dot{\beta}_r}{\partial r}\right) - \int_0^t g\left(t-t'\right)\ \left(\frac{\ddot{\beta}_r}{r} + \sigma\frac{\partial\ddot{\beta}_r}{\partial r}\right)dt'\right], \tag{4}$$

$$\dot{Q}_r = K\mu_\infty h\ \left[\left(\frac{\partial W}{\partial r} - \dot{\beta}_r\right) - \int_0^t g\left(t-t'\right)\ \left(\frac{\partial\dot{W}}{\partial r} - \ddot{\beta}_r\right)dt'\right], \tag{5}$$

where $D_\infty = E_\infty\left(1-\sigma^2\right)^{-1} h^3/12$, $\mu_\infty = E_\infty \big/ 2(1+\sigma)$, $g(t-t') = 1-e^{-(t-t')/\tau}$, $g(t)$ is the relaxation function for Maxwell's model, τ is the time of relaxation of the plate, t' - variable of the integration, $Å_\infty$ è μ_∞ – nonrelaxational magnitude of the Young's modulus and Poisson's ratio, accordingly.

3 RECURRENT RELATIONS OF THE RAY METHOD

Assume that as a result of the dynamic action on a plate, a cylindrical wave Σ of a strong or weak discontinuity propagates in the plate, in the form cylindrical surfaces-strips are circumferences extending with the normal velocities $G^{(\alpha)}$ (indices α take on the values 1 and 2). The solution for the desired function Z (r,t) behind the front of the wave surface Σ is constructed in terms of the ray series [2,3]

$$Z(r,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[Z_{,(k)}\right]_{t=r/G}\left(t-\frac{r-r_0}{G}\right)^k H\left(t-\frac{r-r_0}{G}\right), \tag{6}$$

where $[Z_{,(k)}]=Z^{+}_{,(k)} - Z^{-}_{,(k)}=[\partial^k Z/\partial t^k]$ are the jumps of the kth derivatives of the function Z with respect to time on the wave surface Σ, i.e. at $t=(r-r_0)/G^{(\alpha)}$, r is the polar radius, r_0 is the initial radius, and $H(t)$ is the unit Heaviside function.

To determine coefficients of the ray series (6) for the desired function Z, it is necessary to differentiate Egs.(1-2) for the elastic plate or Eqs (1) and (3-5) for the viscoelastic plate k times with respect to time, take their difference on the different sides of the wave surface Σ, and apply the condition of compatibility for the physical components of the value [2]

$$G\left[\frac{\partial Z_{,(k)}}{\partial r}\right] = -\left[Z_{,(k+1)}\right] + \frac{\delta\left[Z_{,(k)}\right]}{\delta t}, \tag{7}$$

where $\delta/\delta t$ is the δ - derivative with respect to time.

As a result from the equations of motion (1) for the viscoelastic plate, we obtain

$$\left(1-\frac{\rho h^3 G^2}{12D}\right)\omega_{(k+1)} = 2\frac{d\omega_{(k)}}{dt} + Gr^{-1}\omega_{(k)} + \omega_{(k)}\Gamma(0) + bGX_{(k)} + F_{1(k-1)}, \tag{8}$$

$$\left(1-\frac{\rho G^2}{K\mu}\right)X_{(k+1)} = 2\frac{dX_{(k)}}{dt} + Gr^{-1}X_{(k)} + X_{(k)}\Gamma(0) - G\omega_{(k)} + F_{2(k-1)}, \tag{9}$$

where $\omega_{(k)}=[\beta_{\alpha\ ,(k)}]\ \nu_\alpha$, $X_{(k)}=[W_{,(k)}]$, $b=K\mu hD^{-1}$, $\Gamma(t)=\dot{g}(t)$ is the kernel of relaxation, $F_{1(k-1)}$ and $F_{2(k-1)}$ is the values dependent from discontinuities k-1th of the order.

When deducing Eqs. (8-9), it should be considered the axially symmetric character of the problem and, in consequence, wave characteristic independence from the angle φ. Then from Eqs. (8-9) at k = -1, 0, 1…3 one can find the discontinuity for the first and second waves, allowing us to write the expressions for the desired functions W and Q_r in terms of the truncated ray series with an accuracy to constant integration.

$$W \cong \sum_{\alpha=1}^{2}\sum_{k=0}^{4}\frac{1}{k!}X_{(k)}^{(\alpha)}\left(y_\alpha\right)^k H\left(y_\alpha\right), \tag{10}$$

$$Q_r \cong K\mu h\sum_{\alpha=1}^{2}\sum_{k=0}^{4}\frac{1}{k!}\left[\left(-\frac{X_{(k)}^{(\alpha)}}{G^{(\alpha)}}+\frac{\delta X_{(k-1)}^{(\alpha)}}{\delta t}\frac{1}{G^{(\alpha)}}-\omega_{(k-1)}^{(\alpha)}\right)-\sum_{i=0}^{k-1}\left(-\frac{X_{(i)}^{(\alpha)}}{G^{(\alpha)}}+\frac{\delta X_{(i-1)}^{(\alpha)}}{\delta t}\frac{1}{G^{(\alpha)}}-\omega_{(i-1)}^{(\alpha)}\right)\Gamma(0)_{(k-i-1)}\right]\left(y_\alpha\right)^k H\left(y_\alpha\right), \tag{11}$$

where $y_\alpha = t - (r - r_0)G^{(\alpha)-1}$, and the values $X_{(k)}^{(\alpha)}$ and $\omega_{(k)}^{(\alpha)}$ are calculated at $y_\alpha=0$.

At $\Gamma(0)=0$, from Eqs.(8-11) we obtain expressions for the elastic isotropic plate.

4 GOVERNING SYSTEM OF THE EQUATIONS

The process of interaction of the rigid body with the buffer and the plate can be described by the following equations [3]:

$$m\ddot{y} = -P(t), \quad \rho h\pi r_0^2 \ddot{w} = 2\pi r_0 Q_r + P(t), \tag{12}$$

where $ó=\alpha+w$ is the total displacement of the impactor which is the sum of the displacements of the spring's upper end, α, and the lower end, w, Q_r is the transverse force in the contact region and $P(t)$ is the contact force for Maxwell's model accepts the following kind

$$P(t) = E_1(\alpha - w) - E_1\tau_1^{-1}\int_0^t\left[\alpha(t') - w(t')\right]\exp\left(-\frac{t-t'}{\tau_1}\right)dt', \tag{13}$$

here E_1 is the buffer's elastic modulus, and τ_1 is the time of relaxation of the buffer.

Substituting the values y and $P(t)$ into Eqs. (12) and taking into account the condition that the tangent to the median surface of the plate should be horizontal in the contact region, we are led to the set of equations defining the process of the plate, buffer and impactor interaction. Substituting the relationships (10), (11) written down on the boundary of the contact region $r = r_0$ and the degree series on time t for the function α into Eqs. (12), and equating in the net equations the coefficients at equal powers of t, we obtain on each step three algebraic equations for determining three unknown constants $c_i^{(1)}$, $c_i^{(2)}$ (i = 0,1,2) and α_i (i = 0…4). After definition of constant integration it is possible to write down the dynamic displacement and contact force as truncated degree series with known coefficients at t.

REFERENCES

[1] J. Hammel, "Aircraft Impact on a Spherical Shell", *Nuclear Engineering and Design*, **37**, 205-223 (1976).

[2] Yu.A. Rossikhin, and M.V. Shitikova, "A Ray Method of Solving Problems Connected with a Shock Interaction", *Acta Mechanica*, **102**, 103-121 (1994).

[3] Yu.A. Rossikhin,M.V. Shitikova and A.A. Loktev, "Impact of a Sphere upon a Nonlinear Elastic Buffer Embedded into an Overlaping Plate" (in Russian), *Izvestija Vuzov. Stroitelstvo*, **11**, 16-22 (2004).