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# MODELLING MASONRY USING THE PARTITION OF UNITY METHOD

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**Key words:** Cohesive zone models, masonry, displacement discontinuities, partition of unity.

**Summary.** *A methodology to model masonry on a mesoscopic scale using the partition of unity concept of finite element shape function is presented. Cracked joints are modelled as displacement discontinuities and these discontinuities are only activated when the stress state in the joint has reached a critical level. The joint behaviour is governed by a plasticity-based cohesive law.*

## 1 INTRODUCTION

Since a few years, the modelling of displacement discontinuities within the framework of the partition of unity property of finite element shape functions has become popular. Using this property, enhanced nodal degrees of freedom can be added to the regular degrees of freedom during the computational process. The behaviour inside the discontinuity is governed by enhanced degrees of freedom. The method is widely used for crack path prediction and failure prediction of quasi brittle materials[1,2,3].

Contrary to general structural problems, where the crack path is not known in advance, in masonry structures cracks often follow the joints. Consequently, in these structures the potential crack paths are known. Until now, interface elements, representing the joints, are placed on the brick interface for a so-called mesoscopic modeling of masonry[4,5]. However, this method shows several disadvantages. Firstly, a large number of additional degrees of freedom must be generated from the beginning of the computation. Secondly, the interfaces are present from the beginning, demanding a very high 'dummy' stiffness to correctly describe the 'elastic' behavior of the joint. The high stiffness may lead to numerical problems such as oscillations and ill-conditioned problems.

In this work, the joints are modelled using the partition of unity method. Basically, all joints are defined in the beginning of the computation, but they are not active. This means that there is no need for a 'dummy' stiffness, and the extra number of degrees of freedom related to the joint are not present at the beginning of the computation. After each loading step, the stress in the joint is evaluated. If necessary, the additional degrees of freedom are enhanced when the opening criterion for the discontinuity is met and the joint is activated. Consequently, only those joints that fracture are enhanced.

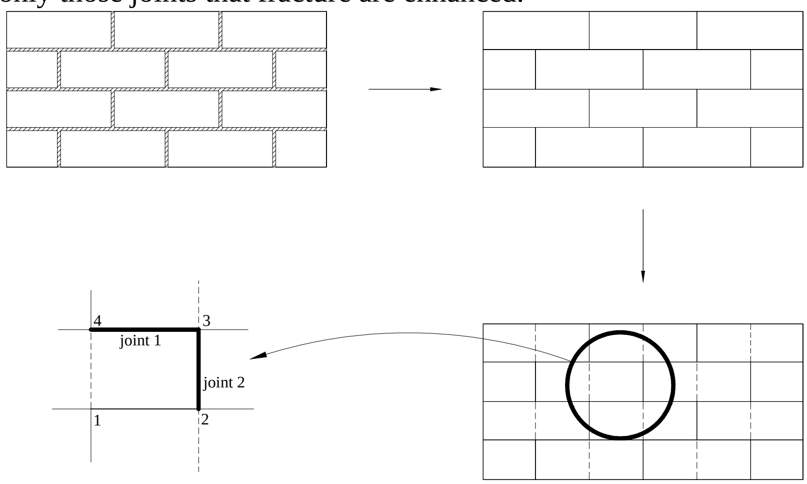

Figure 1: Geometrical model for the masonry element.

## 2 PARTITION OF UNITY CONCEPT FOR DISPLACEMENT DISCONTINUITIES.

### 2.1 Modeling methodology

Cracks in the masonry joints are modeled as displacement discontinuities and are implemented within the finite element context using the partition of unity property of the finite element shape functions. When a crack is activated within a finite element, nodes are locally enhanced by additional degrees of freedom with the Heaviside step function as an enhancement basis[1,2,3]. The displacement field for a body crossed by a discontinuity is obtained by:

$$\mathbf{u} = \mathbf{N}\mathbf{a} + H_{\Gamma}\mathbf{N}\mathbf{b} \tag{1}$$

In which **N** contains the finite element shape functions, $H_{\Gamma}$ is the Heaviside step function centred on the discontinuity, **a** are the regular degrees of freedom and **b** are the enhanced degrees of freedom.

### 2.2 Element technology

A quadrilateral four node element is used as underlying basis. Each element represents a half brick and has two possible positions for a joint: the upper and the right boundary. When a complete brick is formed, the joint which is present in the middle is not activated (dashed lines in figure 1). In a later stage, this displacement discontinuity can be used to model cracking of the brick. For joint 1, node 3 and node 4 are enhanced and for joint 2 node 2 and node 3 are enhanced.

## 3 MATERIAL MODEL

The material model used in the joints is a simplified version of the joint model proposed by Lourenco[4]. The yield surface exists of a Mohr-Coulomb criterion, cut-off by a tensile and a compressive cap, as shown in figure 2. The elastic region is bound by:

$$\begin{aligned} F_1 &= \sigma - f_t \\ F_2 &= f_c + \sigma \\ F_3 &= |\tau| + \sigma \tan\phi - c \end{aligned} \tag{2}$$

in which $f_t$ and $f_c$ are respectively the tensile and the compressive strength, $c$ is the cohesion and $\phi$ is the internal friction angle.

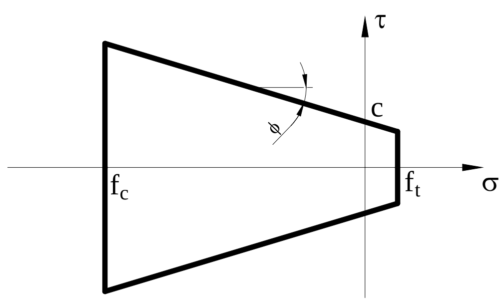


Figure 2: Yield surface for joint behaviour.

When the stress state in a joint is situated outside the area defined by the three surfaces, the enhanced degrees of freedom of the joint are activated. Then, dependent on which surface is violated, the joint behaves non-linear. For the tensile cap, the updated normal tractions are:

$$T_n = f_t \exp\left[-\frac{f_t}{G_{fI}}\Delta_n\right] \tag{3}$$

in which $G_{fI}$ is the mode I fracture energy. For the compression we have:

$$T_n = f_c\left[(1+\alpha_1)\exp(-\alpha_2\Delta_n) - \alpha_1\exp(-2\alpha_2\Delta_n)\right] \tag{4}$$

in which $\alpha_1$ and $\alpha_2$ are two model parameters to control the non-linear behaviour. The shear tractions are assumed to remain elastic. When the Mohr-Coulomb surface is violated, the shear tractions are computed as:

$$T_t = c\exp\left[-\frac{c}{G_{fII}}\Delta_t\right] \tag{5}$$

in which $G_{fII}$ is the mode II fracture energy. In this case, the normal tractions remain elastic. When the stress point is situated in the region where two surfaces are violated, the combined traction update is adopted.

## 4 CONCLUSIONS

In this contribution, masonry is modeled using the partition of unity property of finite element shape functions. When the stress state in a joint reaches a critical value, a displacement discontinuity is introduced by enhancing the correct degrees of freedom. Using this methodology cancels out the use of a 'dummy' elastic stiffness, since the behavior in a discontinuity is completely inelastic.

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