VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

DETERMINATION OF MATERIAL PARAMETERS FOR DIFFERENT HYPERELASTIC MODELS

Pedro A.L.S. Martins[⋆‡], **Renato M. Natal Jorge**[⋆§], **and Antonio J.M. Ferreira**[†]

[⋆]IDMEC-Polo FEUP
Faculdade de Engenharia da Universidade do Porto
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
e-mail: [‡]palsm@fe.up.pt, [§]rnatal@fe.up.pt

[†]DEMEGI-Departamento de Engenharia e Gestao Industrial
Faculdade de Engenharia da Universidade do Porto
Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
e-mail: ferreira@fe.up.pt

Key words: Biomechanics, Soft Tissues, Hyperelastic models, Optimization

Summary. *In this paper the authors propose a experimental/theoretical process to study the behavior of biological soft tissues with a uniaxial tension test. An optimization process is performed using the Levenberg-Marquardt (LM) algorithm.*

1 INTRODUCTION

In the analysis of the mechanical behavior of solids, the correct modeling of the constitutive laws that more accurately represent such behavior is of great importance[1,2]. On those constitutive laws (material laws), there are sets of parameters assumed to have a particular value for each material. Such parameters must be determined by experimental procedures. Nowadays, to study the mechanical behavior of soft tissues, the usage of the most adequate constitutive law, and the determination of the material parameters is object of intense research, especially on the subject of biological soft tissues[3,4]. On this study, the material parameters for different hyperelastic constitutive laws are determined with an inverse technique. The authors search the optimal set of material parameters, for each material model considered, that allow to describe in an optimal way the nonlinear behavior shown by soft tissues subjected to a uniaxial tension test. The optimization of the material parameters is performed with an implementation of the Levenberg-Marquardt algorithm[5] in FORTRAN90.

2 THEORETICAL SURVEY

For the purpose of illustrating the method, we consider Yeoh's material model, whose strain-energy function[2] is

$$\Psi = \sum_{i=1}^{3} c_i (I_1 - 3)^i \tag{1}$$

For the particular case of an uniaxial tension test, the Cauchy stress is related with strain invariants according to

$$\sigma = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(\frac{\partial \Psi}{\partial I_1} + \frac{1}{\lambda}\frac{\partial \Psi}{\partial I_2}\right) \tag{2}$$

Combining equations 1 and 2 the relation,

$$\sigma_{Yeoh} = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(c_1 + 2c_2(I_1 - 3) + 3c_3(I_1 - 3)^2\right) \tag{3}$$

appears as the Cauchy tension in the traction direction.

The set of unknowns $\{c_i\}$ is then found with a gradient method (LM algorithm[4]) which is based on the optimization of a cost function of the type

$$\begin{array}{rcl} O(\overrightarrow{C}) & = & \sum_{i=1}^{m} {\omega_i}^2 [f_i^{an}(\overrightarrow{C}) - f_i^{exp}]^2 \\ \omega_i & \longrightarrow & \textit{weight coefficient} \\ f_i^{exp} & \longrightarrow & \textit{experimental measurements} \\ f_i^{an} & \longrightarrow & \textit{model approximation} \end{array} \tag{4}$$

3 EXPERIMENTAL RESULTS

The following results where obtained with rectangular strips of pig meat. All the mechanical testing was made with the apparatus shown in figure 1

To evaluate the quality of the fitting between hyperelastic model results and experimental results, the calculation of the correlation coefficient (CC)[6]

$$C.C._{model} = \frac{\sum_{i=1}^{m}\left(f_i - \overline{f}\right)_t \left(f_i - \overline{f}\right)_e}{\sqrt{\sum_{i=1}^{m}\left(f_i - \overline{f}\right)_t}\sqrt{\sum_{i=1}^{m}\left(f_i - \overline{f}\right)_e}} \tag{5}$$

is shown on table 1

Money-Rivlin	0,99964
Yeoh	0,99959
Humphrey	0,99907
Ogden (n=3)	0,99974
Ogden (n=4)	0,99973
Neo-Hookean	0,98849
Martins	0,99976
Veronda-Wetmann	0,99948

Table 1: Correlation Coefficient for each material model

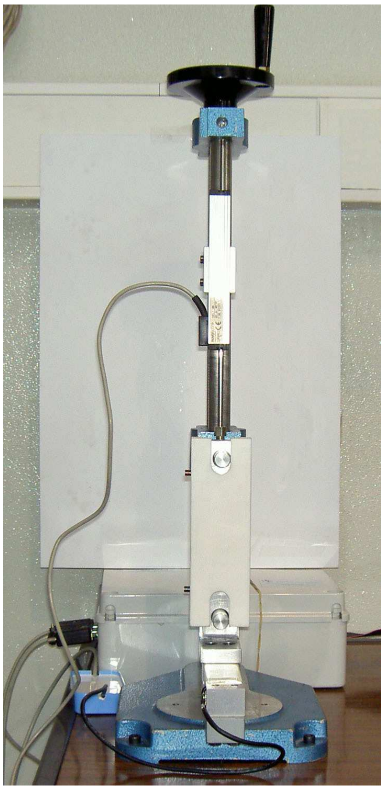

Figure 1: Manually operated mechanical testing machine

4 CONCLUSIONS

With most of the hyperelastic models used a good correlation (< 0.99) between experimental and theoretical data is observed. Although, the experimental process as it is need further improvements, specially the automation of the tension machine that is manually controlled. This issue compromises the stability of the optimal parameters set (obtained via optimization process), since the experimental load displacement curve lacks the smoothness that is possible to achieve with an automated process.

4.1 Future Work

There are several improvements to the present work that might (should in fact) be considered in order to develop a mechanical characterization process for biological soft tissues:

- Automation of the testing machine
- Preconditioning of the soft tissues in order to avoid hysteresis effects
- Use a viscoelastic approach, which is known to explain in a more accurate way[7] the mechanical behavior of soft tissues

- Use alternative optimization processes (like genetic algorithms)
- Upgrade the experimental apparatus in order to perform biaxial essays

REFERENCES

[1] R. W. Ogden. *Non-Linear Elastic Deformations*, Dover Publications inc., 1984.

[2] G. A. Holzapfel. *Nonlinear Solid Mechanics*, John Wiley and Sons, 2000

[3] H.M. Yin, L.Z. Sun, G. Wang, M.W. Vannier. Modeling of elastic modulus evolution of cirrhotic human liver. *IEEE Transactions on Biomedical Engineering*, **51(10)**, 1854-1856, 2004.

[4] D.F. Meaney. Relationship between structural modeling and hyperelastic material behavior: application to CNS white matter. *Biomechan. Model. Mechanobiol.*, **1(4)**, 279-293, 2003.

[5] T. Karvonen. *Soil and Groundwater Hydrology*, http://www.water.hut.fi/∼tkarvone/sgh.htm, 10 May 2003.

[6] J.D. Humphrey. *Cardiovascular Biomechanics-Cells, Tissues and Organs*, Springer-Verlag, 2002.

[7] H.W. Haslach Jr. Nonlinear viscoelastic, thermodynamically consistent, models for biological soft tissues. *Biomechan. Model. Mechanobiol.*, **3**, 172-189, 2005.