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EXTENDED FINITE ELEMENTS FOR FRACTURE ANALYSIS OF FUNCTIONALLY GRADED MATERIALS

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Summary. *In this paper we develop a numerical strategy to follow crack propagation in quasi-brittle functionally graded materials (FGMs), endowed with assigned spatially varying elastic and fracture properties. A cohesive model is used within the context of the extended finite element method, formulated to account for continuous elastic and fracture energy gradation.*

1 INTRODUCTION

This work addresses some computational issues concerning failure analysis of FGMs. Gradual variation of the mechanical properties, unlike the abrupt change encountered in two-layer materials, is known to improve failure performance; many research effort is currently devoted to the experimental characterization, analytical interpretation and numerical simulation of fracture in FGMs.

In this paper focus is in particular on quasi-brittle FGMs, endowed with assigned spatially varying elastic and fracture properties. As in[1], a cohesive approach is used to simulate crack propagation and no dissipation is attributed to the bulk material. The traction-displacement discontinuity model is employed within the context of the extended finite element method[2,3]. The method allows to follow crack paths independently of the background finite element mesh; this feature is especially important for FGMs, since gradation of the mechanical properties may lead to complex propagation paths also in simple symmetric tests[4]. Proper implementation of the method is developed to account for continuous grading in elastic and cohesive fracture properties. The extended finite element method together with a graded element formulation, allows for the use of relatively coarse meshes in failure analysis of FGMs.

The proposed methodology is applied to quasi-static crack growth simulation in a functionally graded glass-filled epoxy beam[5].

2 FORMULATION

The numerical strategy for crack propagation analysis here adopted has the following key features. *i)* The continuous graded elastic properties are modeled by the generalized isoparametric element formulation[6]: Young's modulus and Poisson's ratio vary within each element according to its nodal shape functions; *ii)* the inelastic material behavior is captured in a simplified way by a cohesive crack model relating tractions to displacement discontinuities along the crack Γ_d; also strength and fracture energy vary along Γ_d according to the nodal shape functions; *iii)* the extended finite element formulation[3] is used to follow the crack path independently of the underlying space discretization.

In this work a linear softening cohesive crack model, Fig. 1, is assumed. The extension to FGMs is obtained via a phenomenological approach using, as in[7], a volume-fraction based model. Considering, for the sake of brevity, mode I propagation and denoting by V_1 and V_2 the volume fractions of the constituent materials of the FGM at a given location, the cohesive model is expressed as

$$t_i = t_i^{\max}\left(1 - \frac{w}{w_i^{\max}}\right) \qquad G_i = \frac{1}{2} t_i^{\max} w_i^{\max} \qquad i = 1,2 \tag{1}$$

$$t = V_1 t_1 + V_2 t_2 \tag{2}$$

where t is the traction, $t^{\max}$ is its peak value (strength), w is the displacement discontinuity and $w^{\max}$ is the value corresponding to zero tractions, G is the fracture energy. Indices refer to the two phases, symbols without indices refer to the FGM.

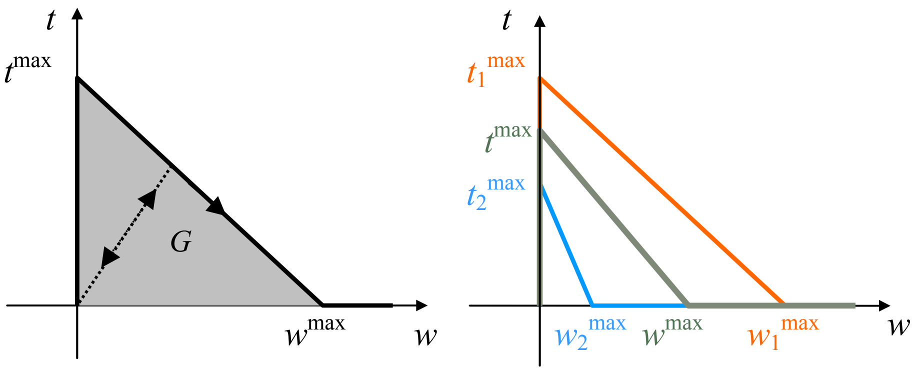

Figure 1: mode I cohesive crack model. (a) nonholonomic behaviour (b) volume-fraction based model

3 NUMERICAL RESULTS

The experimental results of Rousseau and Tippur[5] on functionally graded glass-filled epoxy beams have been considered to check the proposed methodology. The test geometry is shown in Fig. 2. Elastic and fracture properties vary in the vertical direction as a consequence of the variation of the A-glass spheres concentration (volume fraction varying from 0 to 0.5).

Numerical simulations have been carried out considering a gradation with the notch on the less tough side, i.e. on the side with lower t^{max} and G (case 1), and a gradation with the notch on the more tough side (case 2). An unstructured coarse mesh of 4732 constant-strain triangles with 2489 nodes has been adopted. The results are shown in Fig. 3 in terms of length of the crack versus imposed displacement and load versus displacement. For comparison, the curves corresponding to the homogeneous materials with the two extreme values of toughness are also shown.

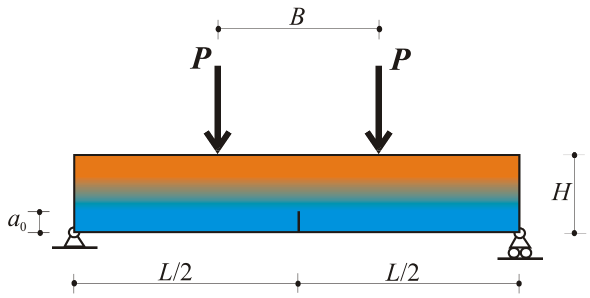

Figure 2: symmetric four-point bending test. Specimen geometry and sketch of the material gradation

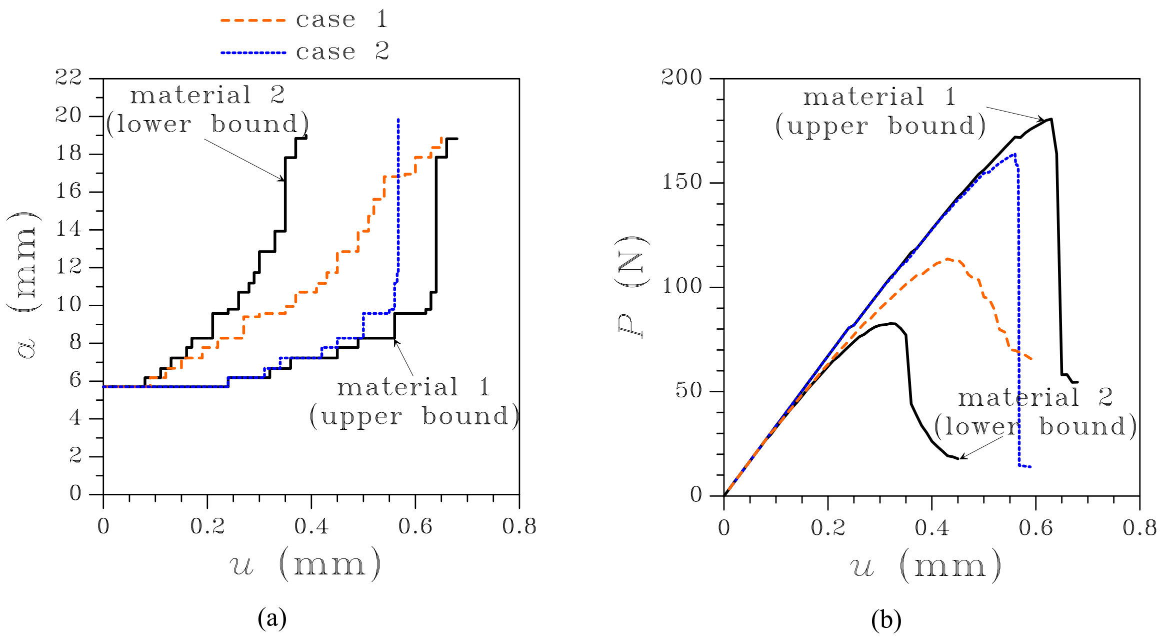

Figure 3: effect of fracture energy gradation on crack growth and specimen response. (a) length of the crack versus imposed displacement; (b) load versus displacement

As expected, when the crack propagates in material layers with increasing toughness (case 1), crack propagation is stabilized and an overall ductile behavior is obtained (see Fig. 3b). On the contrary, in case 2 due to high toughness at the initial notch tip, the onset of propagation is postponed, but crack propagation is unstable and an overall brittle behavior is observed.

Figure 4 shows the contour plot of the in-plane maximum principal stress and the crack path on a close-up of the ligament region (for u=0.35 mm and u=1.2 mm, case 1): the extended finite element formulation allows to follow an almost perfectly straight crack propagation, independently of the background mesh.

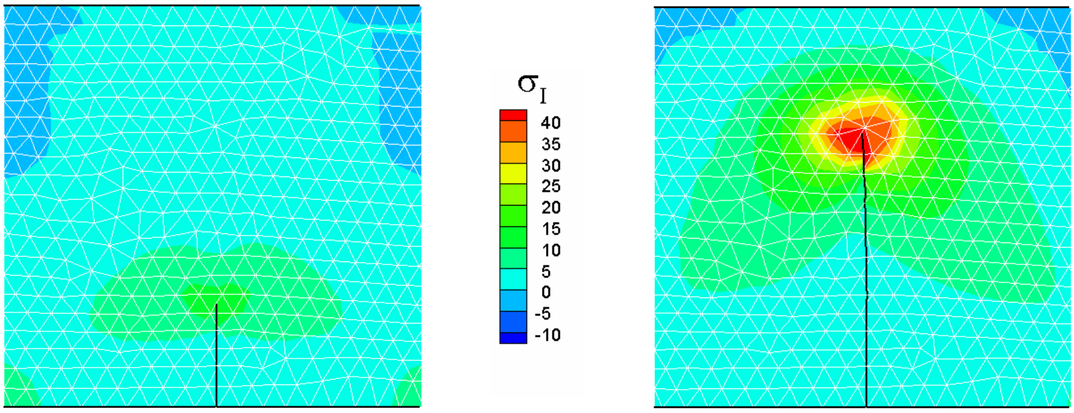

Figure 4: crack path and level sets of in-plane maximum principal stress, case 1. (a) onset of crack propagation, u=0.35 mm; (b) propagation at u=1.2 mm

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