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A CONSTITUTIVE MODEL FOR METALS OVER A LARGE RANGE OF STRAIN RATES AND TEMPERATURES COUPLED TO AN IMPLICIT CONSISTENT ALGORITHM TO SIMULATE DYNAMIC PROCESSES

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Summary. *This paper presents a hardening relation which takes into account strain, strain rate and temperature is effect to predict precisely the thermoviscoplastic behaviour of material with BCC microstructure. Moreover, an algorithm to integrate the thermoviscoplastic constitutive equations, including the hardening law, is proposed to implement via a subroutine the previous constitutive relation in a commercial finite element code as ABAQUS/Explicit. Finally, this tool is used to simulate the problem of a ring expanding radially in a broad range of strain rates, covering both low and high initial velocities.*

1 CONSTITUTIVE RELATION

To predict the thermoviscoplastic behavior of our material a thermoviscoplastic constitutive relation has been developed taking into account hardening, strain rate and temperature sensitivity. This formulation is based on the process of thermal activation. Thus, in this formulation the stress of plastic flow is presented in an additive form generally used for BCC microstructure:

$$\sigma_Y\left(\bar{\varepsilon}^p,\dot{\bar{\varepsilon}}^p,T\right)=\frac{E(T)}{E_0}\left[\sigma_\mu\left(\bar{\varepsilon}^p,\dot{\bar{\varepsilon}}^p,T\right)+\sigma^*\left(\dot{\bar{\varepsilon}}^p,T\right)\right] \tag{1}$$

where σ_μ and σ^* are respectively the internal and the effective stress component. The first term is directly related to the strain hardening of the material and the second defines the contribution of thermal activation (combination of temperature and strain rate). $E(T)$ is of the Young's modulus as a function of temperature. The explicit form proposed to define the two stress components is inspired by the physical approach via the theory of thermal activation. The components are given by the following expressions:

$$\sigma_\mu = B(\dot{\bar{\varepsilon}}^p,T)\left(\varepsilon_0+\bar{\varepsilon}^p\right)^{n(\dot{\bar{\varepsilon}}^p,T)} \tag{2}$$

$$\sigma^*=\sigma_0^*\left[1-D_1\left(\frac{T}{T_m}\right)\log\left(\frac{\dot{\varepsilon}_{\max}}{\dot{\bar{\varepsilon}}^p}\right)\right]^{m^*} \quad \text{if} \quad \dot{\varepsilon}^p \le \dot{\varepsilon}_c \quad \text{then} \quad \sigma^*=0 \quad \forall T \tag{3}$$

where ε_0 is the strain characterizing the yield stress, $B(\dot{\varepsilon},T)$ and $n(\dot{\varepsilon},T)$ are respectively the modulus of plasticity and the strain hardening exponent, $\dot{\varepsilon}_c$ is the critical strain rate, experimentally obtained and typically very low, m^* is the coefficient that characterizes the temperature and strain rate sensitivity, D_1 is the material constant, σ_0^* is the effective stress at $T = 0\,K$ and $\dot{\varepsilon}^{max}$ is the maximum strain rate limiting the validity of the model.

To take into account the temperature increase for each step of plastic deformation, the heat equation is used. The temperature rise slows the propagation of the two waves, inducing, at large strain rates, a trapping of the plastic wave and a localisation of deformation. Hence the decrease of the Young's Modulus with temperature is considered in the constitutive model and both the hardening exponent and the modulus of plasticity in the internal stress account for thermal (and strain rate) effects throughout the following expressions

$$n\left(\dot{\bar{\varepsilon}}^p, T\right) = n_0\left[1 - D_2\left(\frac{T}{T_m}\right)\log\left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_{\min}}\right)\right] \quad \text{with} \qquad n \geq 0 \quad \forall\; T, \dot{\bar{\varepsilon}}^p \tag{4}$$

$$B\left(\dot{\bar{\varepsilon}}^p, T\right) = B_0\left[\frac{T}{T_m}\log\left(\frac{\dot{\varepsilon}_{\max}}{\dot{\bar{\varepsilon}}^p}\right)\right]^{-\nu} \tag{5}$$

where n_0 is the strain hardening exponent at $T = 0\,K$, D_2 is a constant, $\dot{\varepsilon}_{\min}$ and $\dot{\varepsilon}_{\max}$ are the minimum and maximum strain rates assumed in the model, B_0 is a constant and ν is the temperature sensitivity.

Several experimental comparisons have been reported in [1] in term of temperature and strain rate sensitivity. A comparison has been also performed between several others constitutive relations [1]. The advantages of this model is the low number of constants which is equal to eight and the possibility to obtain the analytical expression of the first derivative necessary to define the yield surface evolution as it will be discussed in the following part.

2 CONSISTENCY ALGORITHM

The proposed algorithm follows the *consistency* approach to viscolpasticity proposed by Wang, Sluys and de Borst [2]. Contrary to the *overstress* models, these authors included rate effects in the yield function, so that the consistency condition and the Kuhn-Tucker complementary conditions could be satisfied. The aim of the algorithm herein described [3] is to include thermal effects in the consistency viscoplasticity model and to develop a robust algorithms to integrate it, keeping in mind that in dynamic applications the spatial configuration of the solid commonly diverges from the material one and a large deformation frame has to be considered. To integrate the constitutive equations, incremental objectivity is achieved by rewriting them in a rotated configuration. Within this configuration, all rate equations are form-identical as they were in the spatial one, and the classical return mapping algorithm for small deformation is proposed to solve them. Once the updated stress has been obtained, it is pushed to the spatial configuration. Next, the small deformation part of the algorithm is described. Correction to the trial stress is performed at time n+1

$$\sigma_{n+1} = \sigma_{n+1}^{trial} + \Delta\sigma^T + \Delta\sigma^{ret} \tag{6}$$

$\Delta\sigma^{ret}$ and $\Delta\sigma^T$ are given by:

$$\Delta\sigma^{ret} = -3G\Delta\lambda \ \ s_{n+1} / \bar{\sigma}_{n+1} \qquad\qquad \Delta\sigma^{T} = -3K\alpha\Delta T I \tag{7}$$

where G and K are the shear modulus and the bulk mudulus, $\bar{\sigma}_{n+1}$ the updated equivalent stress, $\Delta\lambda$ the plastic multiplier increment and s_{n+1} is the updated deviatoric stress tensor given by:

$$s_{n+1} = s_{n+1}^{trial} - 3G\Delta\lambda \ \ s_{n+1} / \bar{\sigma}_{n+1} \tag{8}$$

An implicit rule is also used to approximate the temperature increase

$$\Delta T = \frac{3}{2}\beta\Delta\lambda\sigma_{n+1} : s_{n+1} \Big/ \left(\rho_{n+1} C_p \bar{\sigma}_{n+1}\right) = \beta\bar{\sigma}_{n+1}\Delta\lambda \Big/ \left(\rho_{n+1} C_p\right) \tag{9}$$

According to the consistency model, the yield condition is forced to be satisfied

$$f_{n+1} = f\left(\bar{\sigma}_{n+1}^{trial} - 3G\Delta\lambda, \lambda_n + \Delta\lambda, \Delta\lambda/\Delta t, T_n + \beta\left(\bar{\sigma}_{n+1}^{trial}\Delta\lambda - 3G\Delta\lambda^2\right) \Big/ \left(\rho_{n+1} C_p\right)\right) = 0 \tag{10}$$

If using a Newton-Raphson to solve this equation, the iterative increment of $\delta\lambda$ could be calculated

$$\delta\lambda^{(k)} = f^{(k)}\left[3G + H^{(k)} + S^{(k)}/\Delta t + Z^{(k)}\,\beta\left(\bar{\sigma}_{n+1}^{trial} - 6G\Delta\lambda^{(k)}\right) \Big/ \rho_{n+1} C_p\right]^{-1} \tag{11}$$

being H the plastic modulus, S the strain rate sensitivity and T the thermal sensitivity

$$H = \partial\sigma_Y / \partial\bar{\varepsilon}^p \qquad\qquad S = \partial\sigma_Y / \partial\dot{\bar{\varepsilon}}^p \qquad\qquad Z = \partial\sigma_Y / \partial T \tag{12}$$

3 APPLICATION TO RING EXPANSION

The behaviour of materials at high strain rates could be determined by conventional dynamic tests such as tension, compression or shear, although only the last one allows to reach large deformations, close to. Among other non-conventional tests, the impulsive expansion of a thin ring allows to reach also large plastic strains. In this test, the inertia effect provides a force resisting the localization of strain as a necking plastic flow instability tries to form. Moreover, the solid exhibits a dynamic uniaxial stress state along the circumference without the wave propagation problems arising in tensile tests [4]. To show the performance of the thermoviscoplastic approach, a numerical analysis of the expansion of a mild steel ring with of 50 mm diameter, 1 mm thickness and a cross section of 1 mm^2, submitted to a broad range of imposed radial velocities $1 \le V_0 \le 500\ \mathrm{m/s}$ and room initial temperature is presented. The numerical analysis was performed using the Finite Element commercial code ABAQUS/Explicit [5]. A mesh with 300 8-node trilinear reduced integration brick elements including hourglass control was used.

The loss of homogeneous deformation triggers the neck development. Using the Considère instability criterion ($\partial\bar{\sigma}/\partial\bar{\varepsilon}^p = \bar{\sigma}$) and Equation (1) in adiabatic conditions, the plastic strain corresponding the onset of instability is $\varepsilon_{neck} = 0.1$ for a mild steel (hardening exponent $n_o = 0.28$). For the same material, the strain at failure ($\partial\bar{\sigma}/\partial\bar{\varepsilon}^p = 0$) is ten times greater and close to $\varepsilon_{failure} = 1.2$. For brittle metals with lower values of the hardening exponent, the strain at failure is much lower. To arrive at a division of the ring through the development of fragments in the simulation, a failure criterion was considered consisting in a critical value of the equivalent plastic strain $\bar{\varepsilon}^{p} = \varepsilon_{failure}$. In this work he folowing relation between hardening exponent and failure strain was proposed

$$\varepsilon_{failure} = \lambda n_{o} \tag{13}$$

with $\lambda = 4$, which agrees with the value proposed by Triantafyllidis et al. [6]. This approach served to analyse the effect of the hardening exponent on the number of fragments.

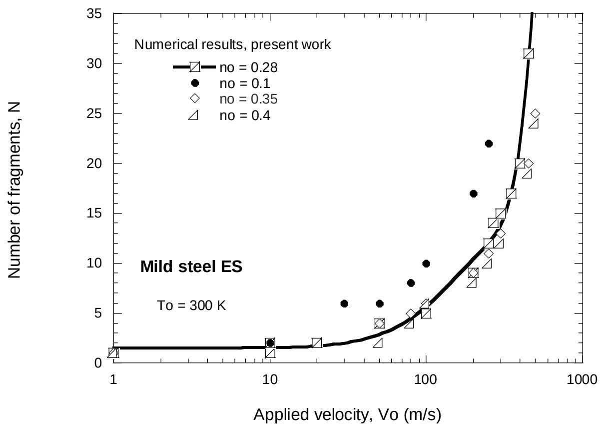

Fig. 1. Effect of hardening value and applied velocity on the number of fragments

Thus, if the hardening parameter n_0 decrease, Eq. 4, the number of fragment N increase inducing apparition of ductile-brittle transition, Fig. 1, as observed during experiments.

CONCLUSIONS

The combination of an original thermoviscoplastic model and an integration scheme was presented. It allows to simulate and study a variety of processes of dynamic loading and impact. Special attention was paid to the proposal of the constitutive relation which will allow to identify precisely the physical processes. As an example, the problem of ring expansion was analyzed, in which the hardening and velocity strongly affects the number of fragments.

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