VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D.R.J. Owen (Eds)

DISCONTINUOUS ELEMENT APPROXIMATION FOR DYNAMIC FRACTURE

A. Arranz*, N. Petrinic†, and E. Süli*

*Oxford University Computing Laboratory
Numerical Analysis Group
Oxford University
Oxford, Wolfson Building, Parks Road, OX1 3QD England
e-mail: aurelio.arranz@comlab.ox.ac.uk, endre.suli@comlab.ox.ac.uk

†Department of Engineering Science
Solid Mechanics Group, Impact Team
Oxford University
Oxford, Parks Road, OX1 3PJ England
e-mail: nik.petrinic@end.ox.ac.uk,
Web page: http://www.eng.ox.ac.uk/ftgnp

Key words: Discontinuous Galerkin Methods, Adaptivity, Fracture, Discontinuous Element Approximation

Summary. *A new set of numerical methods for predictive modeling of crack propagation on aircraft structures based on discontinuous interpolation is introduced. These methods solve many shortcomings and limitations of classical FEMs, in particular in terms of accuracy and stability of the numerical approximation. The combination of these methods with appropriate adaptive local re-meshing can circumvent the unreliability and high mesh-dependency of classical approaches, and can effectively model fracture onset and propagation. Some results are presented on the simulation of fracture of ductile and brittle materials.*

1 INTRODUCTION

The main focus of this paper is the development of an accurate and appropriate numerical method for the treatment of crack initiation and propagation. Classical approaches have been shown to render unreliable approximations to the crack propagation path, a fact due to insufficient approximation of the stress field near the crack tip and a crude and inappropriate treatment of the crack growth. When it comes to creating or propagating a fracture, most commercial and customized codes resort to the elimination or removal of the affected elements, which results in a high-mesh dependency of the solution and a completely unphysical behaviour in most cases since the crack ends up following lines given by the mesh in use. Some workarounds have been proposed over the past years and others are still under research[2], but unfortunately most of them rely on classical FEMs, which present many numerical problems and limitations.

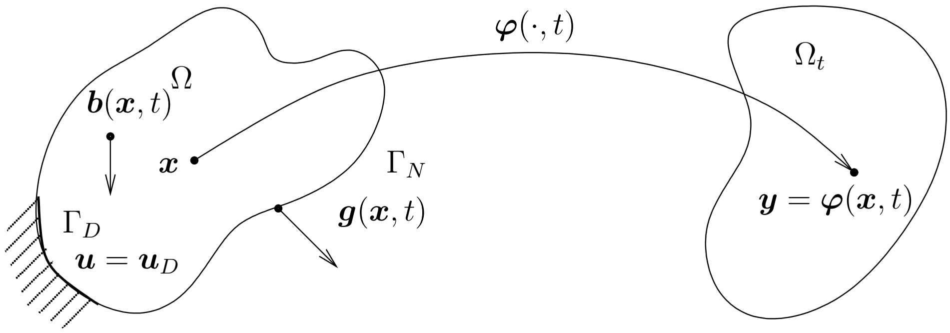

Figure 1: Elastodynamics model problem.

2 DGFEM, the new approach

The alternative proposed makes use of Discontinuous Galerkin Methods[1] (DGM or DGFEM), a wide family of state-of-the-art numerical methods for the solution of PDEs based on discontinuous interpolation and numerical fluxes. These methods lie in between classical Finite Element Methods (FEMs) and Finite Volume Methods (FVMs), and solve many shortcomings and limitations of both methods in the field of computational mechanics.

2.1 DGFEM weak formulation

In order to introduce the ideas employed in these methods, we start by considering a simple elastodynamics problem for a linearly elastic material. It is well known that the elastic response of a domain Ω subjected to both traction and displacement conditions (see Figure 1) is given by the following IVP

$$\left.\begin{aligned} \rho u_{i,tt} - \frac{\partial \sigma_{ij}}{\partial x_j}(\boldsymbol{u}) = b_i, \text{ in } \Omega \times \mathcal{I},\ \forall i = 1, \dots, n \\ \boldsymbol{u} = \boldsymbol{u}_D \text{ on } \Gamma_D \times \mathcal{I}, \\ \sigma_{ij}(\boldsymbol{u}) n_j^N = g_i \text{ on } \Gamma_N \times \mathcal{I},\ \forall i = 1, \dots, n, \\ \boldsymbol{u} = \boldsymbol{u}^0 \text{ and } \boldsymbol{u}_t = \boldsymbol{v}^0 \text{ on } \Omega \times \{0\}. \end{aligned}\right\} \tag{1}$$

Proceeding in an analogous manner to the one used in classical methods, the weak formulation of this problem can be obtained by multiplying by a test function, integrating by parts element-wise and collecting internal boundary integrals in terms of averages and jumps, thus arriving at the following linear and bilinear forms[3]

$$L(\boldsymbol{v}) \equiv \int_\Omega \boldsymbol{f} \cdot \boldsymbol{v} \, \mathrm{d}\boldsymbol{x} + \int_{\Gamma_N} \boldsymbol{g} \cdot \boldsymbol{v} \, \mathrm{d}s + \sum_{e_a \in \partial \mathcal{E}_h^D} \int_{e_a} \sigma_{ij}(\boldsymbol{v}) n_j^a (\boldsymbol{u}_D)_i \, \mathrm{d}s + \sum_{e_a \in \partial \mathcal{E}_h^D} \frac{\delta_a r^2}{|e_a|^\beta} \int_{e_a} \boldsymbol{u}_D \cdot \boldsymbol{v} \, \mathrm{d}s. \tag{2}$$

and

$$a_{NS}(\boldsymbol{w},\boldsymbol{v}) \equiv \sum_{E\in\mathcal{E}_h}\int_E \sigma_{ij}(\boldsymbol{w})\epsilon_{ij}(\boldsymbol{v})\,\mathrm{d}\boldsymbol{x} + \sum_{e_a\in\partial\mathcal{E}_h^I}\int_{e_a}\{\sigma_{ij}(\boldsymbol{v})n_j^a\}[w_i] - \{\sigma_{ij}(\boldsymbol{w})n_j^a\}[v_i]\,\mathrm{d}s$$

$$-\sum_{e_a\in\partial\mathcal{E}_h^D}\int_{e_a}\sigma_{ij}(\boldsymbol{w})n_j^a v_i\,\mathrm{d}s + \sum_{e_a\in\partial\mathcal{E}_h^D}\int_{e_a}\sigma_{ij}(\boldsymbol{v})n_j^a w_i\,\mathrm{d}s + J_0^{\delta,\beta}(\boldsymbol{u},\boldsymbol{v})$$

defined on broken Sobolev spaces. The term

$$J_0^{\delta,\beta}(\boldsymbol{v},\boldsymbol{w}) = \sum_{e_a\in\partial\mathcal{E}_h^I}\frac{\delta_a r^2}{|e_a|^\beta}\int_{e_a}[\boldsymbol{v}]\cdot[\boldsymbol{w}]\,\mathrm{d}s + \sum_{e_a\in\partial\mathcal{E}_h^D}\frac{\delta_a r^2}{|e_a|^\beta}\int_{e_a}\boldsymbol{v}\cdot\boldsymbol{w}\,\mathrm{d}s, \tag{3}$$

is the penalty term that controls the level of continuity achieved in the solution.

2.2 DGFEM vs. cracks

Bearing in mind the inherent discontinuous nature across element boundaries of the solutions obtained by this method, the increased stability of the approximations and the flexibility and suitability of their use in combination with adaptivity, this type of methods are a good choice for numerical modeling crack initiation and propagation. In this formulation elements are naturally independent from one another, only connected through some integral terms that ensure their proximity. Appropriate use of these terms can relax the degree of mutual bonding of two elements, allowing them to separate, as it is the case in fracture propagation.

3 Numerical results

Two different approaches have been taken in order to simulate fracture with DGFEM. The first consists in calculating the interelemental face with maximum stress and turning it into part of the Neumann boundary. This results in a fast opening crack, appropriate for brittle materials. See Figure 2 for a 2D V-notched specimen clamped on its left end and being pulled on the right. It can be shown that the general crack path is insensitive to mesh changes and topology. In Figure 3 we can see an example of the extension to 3D. A prismatic bar with a pre-existing slit is loaded in an analogous way.

The second approach takes advantage of the fact that the penalization parameter can be modified locally, thus being able to control the speed at which the discontinuity, i.e. the crack, grows. This technique, combined with *hp*-adaptivity and plastic models will enable us to obtain good approximations for fracture surfaces in ductile materials.

4 CONCLUSIONS

DGFEM represents a brand-new and promising approach for the treatment of fracture problems. Its increased stability and accuracy offers a numerical approximation more reli-

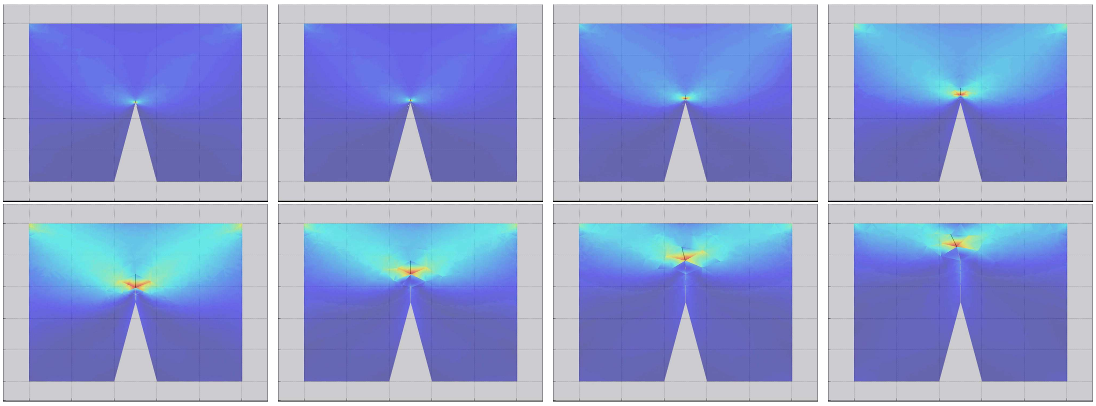

Figure 2: Fracture onset and propagation in a 2D V-notched specimen with quadratic triangles using NIPG and backward Euler time-stepping on a mesh with 404 elements.

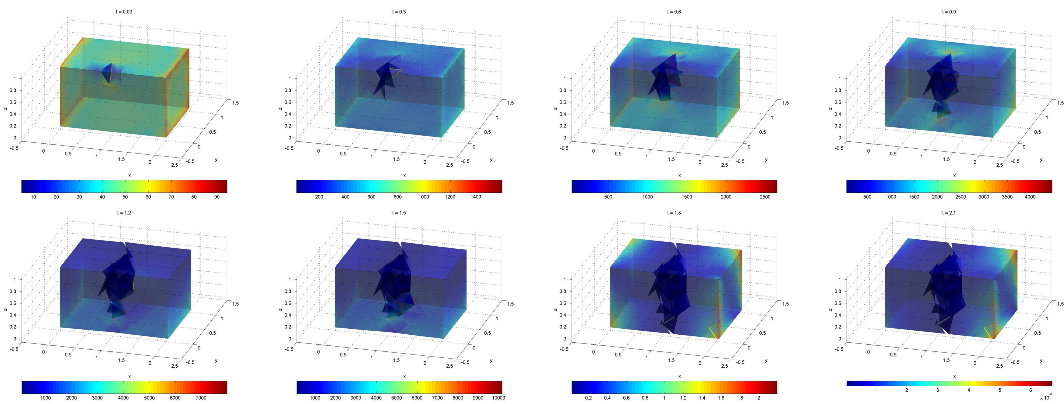

Figure 3: Fracture onset and propagation in a 3D V-notched specimen with quadratic tetrahedra using NIPG and backward Euler time-stepping on a mesh with 648 elements.

able than classical methods. Their inherent discontinuous approximation offers a natural and easy to handle framework for the treatment of fracture problems.

REFERENCES

[1] D.N. Arnold, F. Brezzi, B. Cockburn, and L.D. Marini. Unified analysis of discontinuous Galerkin methods for elliptic problems. *SIAM J. Numer. Anal.*, **39**(5):1749–1779, 2002.

[2] N. Petrinic and P. Johnson. Element splitting. Work in progress, 2004.

[3] B. Rivière and M.F. Wheeler. Optimal error estimates for DGM applied to linear elasticity problems. Tech. Rep. 30, TICAM, 2000.