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ON THE PERFORMANCE OF 3D ELEMENTS FOR SHELL ANALYSIS WITH NON-LINEAR SOFTENING PLASTICITY MODELS

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Summary. *The present work deals with the comparison of three-dimensional (3d) finite element analysis of thin-walled structures to computations based on reduced, two-dimensional (2d) models which a priori satisfy the plane stress assumption. We apply a plasticity-based non-linear softening material model for concrete[1] to simple four-node 2d plane stress elements and to a 7-parameter 3d shell formulation. Numerical results are compared to experimental data for an L-shaped panel[3].*

1 INTRODUCTION

Along with the rapid development of computer power a trend for general 3d structural and material modeling can be observed. In the context of plates and shells the application of 3d shell finite elements appears to be particularly attractive. But also continuum elements may be used for thick and thin shell analysis if appropriate element technology is applied to avoid shell-typical locking problems, like shear locking and membrane locking.

For the sake of simplicity we study the performance of 3d shell elements considering the special case of a flat problem, allowing to compare its behavior to 2d plane stress elements.

2 NON-LINEAR SOFTENING PLASTICITY MODEL

We are considering the plasticity-based softening material model for concrete described by Menrath[1]. The 3d-multisurface yield criterion (Figure 1) contains two Drucker-Prager (DP) regions (Φ_1 and Φ_2) and a spherical cap (Φ_3). Both Drucker-Prager functions are defined as follows

$$\Phi_i\,(\,\boldsymbol{S}, I_1, \kappa_i\,) \;=\; |\boldsymbol{S}| \,+\, \alpha_i\, I_1 \,-\, \sqrt{\frac{2}{3}}\,\bar{k}_i\,(\kappa_i) \tag{1}$$

where $\boldsymbol{S}$ is the deviatoric stress tensor, I_1 is the first invariant of the stress tensor $\boldsymbol{\sigma}$ and κ_i are internal variables to describe an isotropic hardening/softening behavior. The factors α_i depend on the uniaxial tensile strength f_{ctm} and the uniaxial compressive strength f_{cm}, respectively. A decoupled evolution law is formulated via the damage functions $\bar{k}_i$. Limiting the allowable hydrostatic compressive stresses, the spherical cap is coupled continuously to Φ_2. It is expressed in terms of the midpoint $I_{1,m}$ and the radius R by equation (2).

$$\Phi_3\,(\,\boldsymbol{S}, I_1, \kappa_2\,) \;=\; \sqrt{|\boldsymbol{S}|^2 \,+\, \frac{1}{9}\left[\,I_1 \,-\, I_{1,m}\,(\kappa_2)\,\right]^2} \,-\, R\,(\kappa_2) \tag{2}$$

An exponential and a biparabolic function describe the equivalent stress–equivalent strain relationships in tension and compression respectively. Mesh dependence at localization zones is avoided applying the classical formulation of scaling the fracture energy with an internal length scale[2].

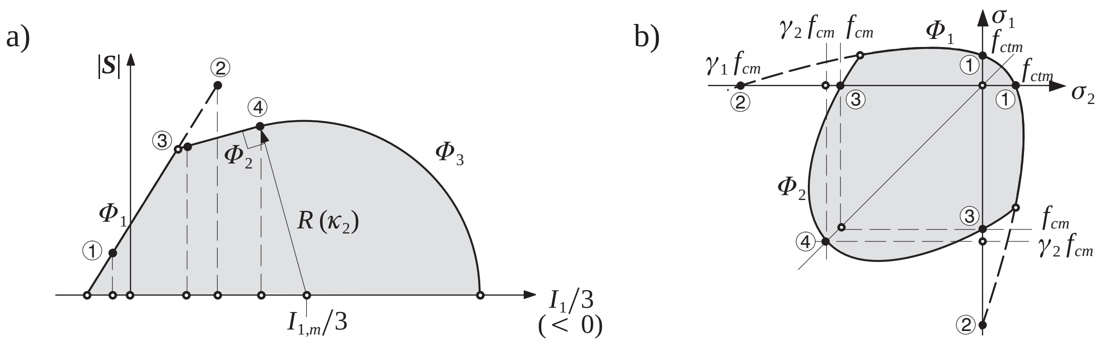


Figure 1: Yield surface in a) ($|\boldsymbol{S}|, I_1$)-space and in b) (σ_1, σ_2)-principal stress space

3 FINITE ELEMENTS

Numerical analyses are done with two different finite element formulations. In the 2d regime for plane structures we use simple four-node plane stress elements whereas for the 3d setting we use a 7-parameter non-linear shell formulation which includes the thickness stretch of the shell[4].

3.1 Three-dimensional shell formulation — the 7–parameter model

Only a very brief overview of the 3d shell formulation will be given. Detailed presentations on this formulation can be found for instance in Büchter and Ramm[4] or Bischoff[5].

The geometric description of the initial and the deformed shell body is projected onto a 2d reference surface. Assuming a linear variation of the displacements across the thickness, position vectors of points in the reference configuration (undeformed state) $\boldsymbol{x}$ and current (deformed) configuration $\bar{\boldsymbol{x}}$, as well as the displacement $\boldsymbol{u}$ are expressed as

$$\boldsymbol{x} = \boldsymbol{r} + \theta^3 \, \boldsymbol{a}_3; \quad \bar{\boldsymbol{x}} = \bar{\boldsymbol{r}} + \theta^3 \, \bar{\boldsymbol{a}}_3 \tag{3}$$

$$\boldsymbol{u} = \boldsymbol{v} + \theta^3 \, \boldsymbol{w}; \quad \boldsymbol{v} = \bar{\boldsymbol{r}} - \boldsymbol{r}; \quad \boldsymbol{w} = \bar{\boldsymbol{a}}_3 - \boldsymbol{a}_3 \tag{4}$$

where $\boldsymbol{r}$ ($\bar{\boldsymbol{r}}$) expresses the corresponding point in the mid-surface, $\boldsymbol{a}_3$ ($\bar{\boldsymbol{a}}_3$) is the director and $\boldsymbol{w}$ denotes the relative displacement field between the mid- and the upper-surface (see Figure 2).

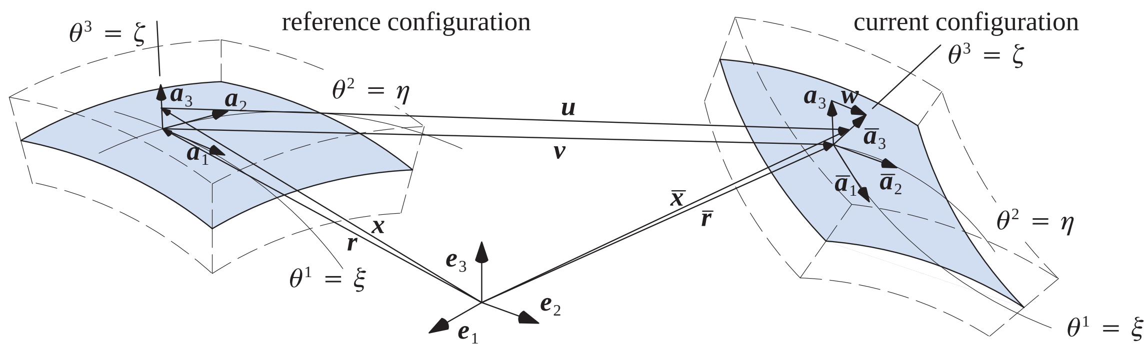


Figure 2: Kinematics of 7-parameter model

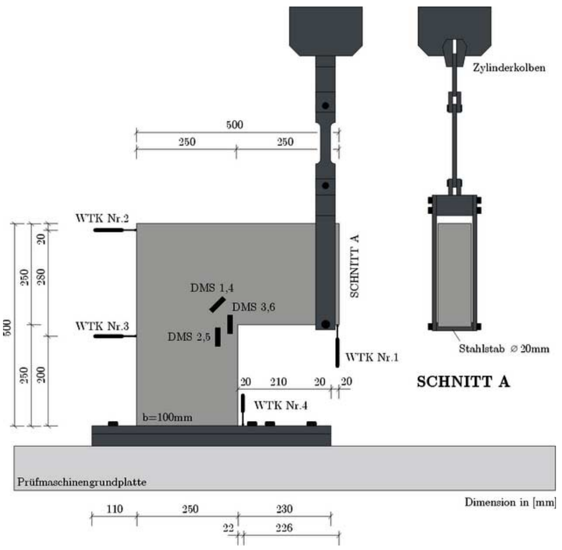
Utilizing the Enhanced Assumed Strain (EAS) Method, the compatible transversal normal strain E_{33}^u is enhanced with an independent degree of freedom $\tilde{\beta}$, leading to a 7-parameter model.

$$E_{ij} = E_{ij}^{u} \quad \text{for} \quad (i,j) \neq (3,3); \quad E_{33} = E_{33}^{u} + \tilde{E}_{33} = \alpha_{33}^{u} + \theta^{3}\, \tilde{\beta} \tag{5}$$

With this extension the strain distribution is completely 3d up to linear terms in the thickness coordinate θ^3, such that fully 3d material laws can be applied without any further modification[5].

4 NUMERICAL RESULTS

As a benchmark test for the validation of material models for the simulation of plain concrete, the L-shaped panel, investigated by Winkler et al.[3], has become a popular test example. Experimental setup, material parameters and finite element meshes are shown in Figure 3.



Tangent modulus of elasticity	E_{cm}	25850	[N/mm^2]
Cylindrical compressive strength	f_{ck}	31.00	[N/mm^2]
Axial tensile strength	f_{ctm}	2.70	[N/mm^2]
Tensile fracture energy	G_t	0.065 − 0.09	[N/mm]

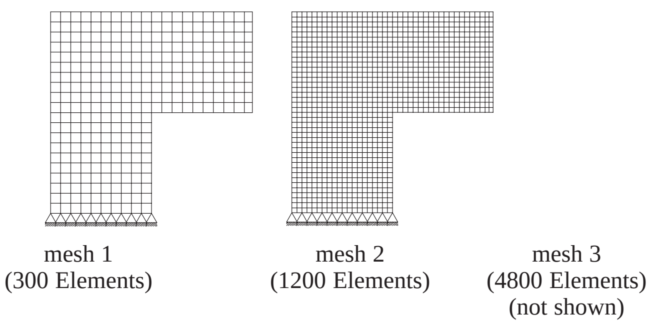


Figure 3: Test setup, material properties and finite element meshes (from Winkler et al.[3])

Geometry and experimental setup suggest that the plane stress assumption is justified. Numerical results are shown in Figure 4. Besides the fact that for both element types the failure load increases with a finer mesh, the performance of 3d shell elements is significantly worse compared to simple 2d plane stress elements. Using 3d shell elements, the failure load is severely overestimated and an unrealistic structural response in the softening branch can be observed.

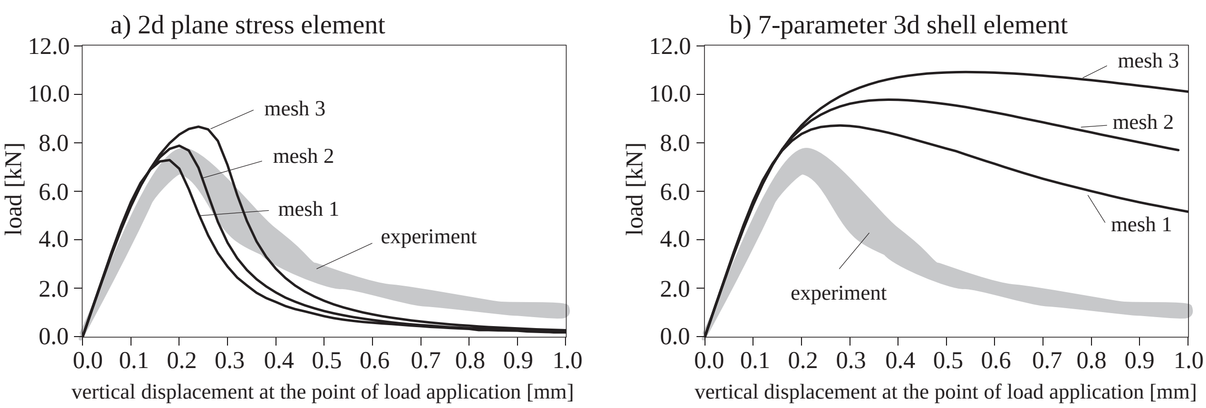


Figure 4: Load-displacement curves for a) four-node 2d plane stress element and b) four-node 3d shell element

An in-depth investigation of the phenomenon revealed that the reason for the inferior behavior of the 3d elements has its origin in parasitic transverse normal stresses at the reentrant corner. They originate from Poisson's effect along with in-plane normal stress concentrations. Due to the softening behavior this initially local phenomenon influences global structural response.

The underlying numerical effect has some similarities to the phenomenon of volumetric locking, known from finite element analysis of nearly incompressible materials. The idea to use finite element formulations, like the EAS method, which avoid this locking phenomenon is thus self-suggesting. The resulting load-displacement curve (Figure 5) matches experimental results quite nicely.

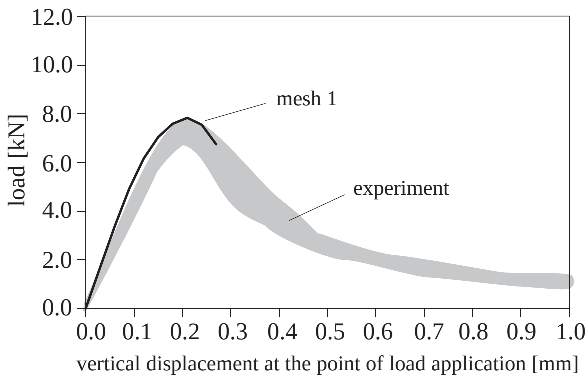


Figure 5: Load-displacement curve for four-node 3d shell element with EAS

5 CONCLUSIONS

We studied the performance of 3d shell elements in a special geometrical setting, to compare its performance to 2d plane stress elements. A numerical effect, occurring in 3d analysis of thin-walled structures using a plasticity based non-linear softening material law, as well as a strategy to avoid it have been described. We can conclude that a fully 3d simulation does not necessarily lead to more accurate results than a 2d one. It is essential to realize that 3d analysis of shells, combining complex material models, demands sophisticated knowledge of the interaction between mechanical model, material model and finite element technology.

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