WRINKLING CRITERIA IN SHEET METAL FORMING FOR SINGLE SIDED CONTACT SITUATION AND ITS APPLICATION

Géza T. Nagy[*], Kasper Valkering[†] and Han Huétink[‡]

[*] Corus RD&T, Packaging Applications, Containers Technology
POB 10000
IJmuiden, 1970CA, The Netherlands
e-mail: geza.nagy@corusgroup.com, web page: http://www.coruspackaging.com

[†] Netherlands Institute for Metals Research
Rotterdamseweg 137
Delft, 2600 GA, The Netherlands

[‡] University of Twente, Faculty of Engineering Technology
Drienerlolaan 5
Enschede, 7500 AE, The Netherlands

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Summary. *The general wrinkling theory based on the rate formulation of the principle of virtual work has been applied in a number of studies on wrinkling during sheet metal forming. No contact with the die was allowed. In this study it is shown that with the right choice of wrinkling modes, the theory can also be used for contact situations. Furthermore, loads normal to the sheet can be taken into account. Therefore, the wrinkling theory has a larger area of application then it was previously considered.*

1   INTRODUCTION

Wrinkling of a sheet during forming can be viewed as both a local and a global phenomenon. When the area prone to wrinkling is large compared to the wrinkle wavelength, local determination of the wrinkling risk gives good quantitative prediction of the possibility for wrinkling. However, when the wrinkle wavelength is short compared to the area, specific boundary conditions must be taken into account. In this study the known *local* wrinkling criteria are further investigated. It is assumed that the area prone to wrinkling can be represented by a doubly curved shell, which can be in contact with one or more dies. The last decades a number of articles have been published which study the wrinkling of sheet metal during (plastic) forming analytically. Most of these articles are based on the wrinkling theory presented by Hutchinson[1] and restricted to a specific geometry. Hutchinson & Neale[2] and Neale & Tugcu[3] used this wrinkling theory to study the wrinkling potential of doubly curved, isotropic, contact free sheet metal during forming. Tugcu[4] used the wrinkling theory in combination with both flow and deformation theory to study the wrinkling behaviour of a flat plate. Wang et. al.[5] studied doubly curved, planar isotropic, contact free sheet metal during

flanging. Finally, Wang & Cao[6] used the theory for prediction of contact free sidewall wrinkling during deep drawing. The studies mentioned are all restricted to a specific contact free geometry. However, the same theory can be used for geometries including single-sided and double-sided contact.

Notation

α ratio between transverse and compressive stress

ε_i strain in principal direction, $i = 1,2$

ε_0 prestrain

ε_{ij} strain components

σ_i stress in principal direction, $i = 1,2$

σ_{ij} stress components

λ_i wave number, $i = 1, 2$

a_i dimensionless amplitude of mode, $i = 1,2,3$

b_{ij} curvature tensor, $i, j = 1,2,3$

C material strength coefficient

C_{ij} compliance moduli, $i, j = 1,2,3$

E_s secant modulus

E_t tangent modulus

E_{ij} stretching strains

F bifurcation functional

K_{ij} bending strains

L wavelength

$\overline{L}_{ijkl}$ instantaneous moduli for plane stress conditions

$\dot{M}_{ij}$ incremental bending moments

$\dot{N}_{ij}$ incremental stress resultants

n strain hardening exponent

p pressure

R anisotropy parameter

R_i radius of principal curvature, $i = 1,2$

S surface

t thickness of sheet

$\dot{u}_i$ incremental buckling displacement in plane, $i = 1,2$

$\dot{\omega}$ incremental buckling displacement normal to sheet

$\overline{\omega}$ external work

V_0 original volume

x_i coordinates, i = 1,2,3

subscripts

$,i$ covariant differentiation with respect to *i*

cr   critical

2 GENERAL WRINKLING THEORY

To study plastic buckling of shells the Donnell-Mushtari-Vlasov (DMV) shell theory was used. The shell coordinates, $x_{i,\ i=1,2}$ are defined on the middle surface of the undeformed body, and are aligned with the directions of the principal curvatures $1/R_1$ and $1/R_2$. Coordinate x_3 is defined normal to the undeformed middle surface (Figure 1).
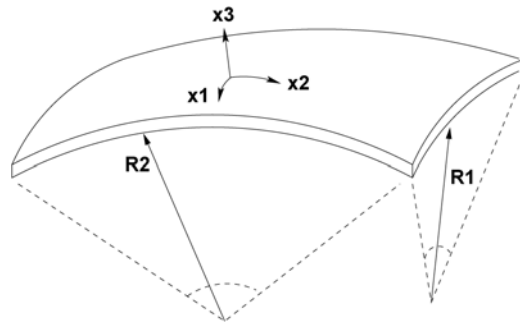

Figure 1: The undeformed sheet

2.1 Incremental strains, bending moments, and stress resultants

The incremental stretch strains, $\dot{E}_{ij}$ and incremental bending strains, $\dot{K}_{ij}$ due to shallow wrinkling modes are defined by:

$$\dot{E}_{ij} = \frac{1}{2}\left(\dot{U}_{i,j} + \dot{U}_{j,i}\right) + b_{ij}\dot{W} + \frac{1}{2}\dot{W}_{,i}\dot{W}_{,j} \tag{1}$$

$$\dot{K}_{ij} = -\dot{W}_{,ij} \tag{2}$$

Here $\dot{U}_i$ are the incremental displacements in the x_1, x_2 -directions respectively, $\dot{W}$ is the incremental displacement normal to the middle surface of the sheet, b_{ij} is the curvature tensor of the middle surface of the pre-buckling state and a comma denotes covariant differentiation with respect to a surface coordinate. The first term and third term of the incremental stretching strain represent the strain in plane and out of plane respectively, and the second

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term is due to a radius increase in case the sheet has a curvature.

The stress and strain increments can be related by $\dot{\sigma}^{ij} = \overline{L}^{ijkl}\dot{\varepsilon}_{kl}$, where $\overline{L}_{ijkl}$ are the plane-stress incremental moduli[5]. Then, the incremental bending moments, $\dot{M}_{ij}$ and stress resultants, $\dot{N}_{ij}$ are given by Hutchinson & Neale[2]

$$\dot{M}_{ij} = \int_{-t/2}^{t/2} \dot{\sigma}^{ij} x_3 \ dx_3 = \int_{-t/2}^{t/2} \overline{L}^{ijkl}\dot{\varepsilon}_{kl} x_3 \ dx_3 \tag{3}$$

$$\dot{N}_{ij} = \int_{-t/2}^{t/2} \dot{\sigma}^{ij} \ dx_3 = \int_{-t/2}^{t/2} \overline{L}^{ijkl}\dot{\varepsilon}_{kl} \ dx_3 \tag{4}$$

where t is the current sheet thickness. Elastic unloading at one surface and plastic compression at the other is not taken into account. The incremental Langrangian strain tensor for points in the shell equals

$$\dot{\varepsilon}_{ij} = \dot{E}_{ij} + x_3 \dot{K}_{ij} \tag{5}$$

2.2 Rate formulation of the principle of virtual work

In shallow shell theory the principle of virtual work states that for all admissible increments δU_i and δW it holds [7]

$$\int_S \left(M^{ij}\delta K_{ij} + N^{ij}\delta E_{ij} \right) dS = \left[external\_work \right] \tag{6}$$

$$\delta E_{ij} = \frac{1}{2}\left(\delta U_{i,j} + \delta U_{j,i} \right) + b_{ij}\delta W + \frac{1}{2}\left(W_{,i}\delta W_{,j} + W_{,j}\delta W_{,i} \right) \tag{7}$$

In rate notation, denoted with a dot, the principle of virtual work (6) becomes

$$\int_S \left(\dot{M}^{ij}\delta K_{ij} + \dot{N}^{ij}\delta E_{ij} + N_{ij}\dot{W}_{,i}\delta W_{,j} \right) dS = 0 \tag{8}$$

where the dependence of δE on the displacement W and the chain rule is used.

Wrinkling occurs when the stable shell with only in-plane deformations looses its stability to a wrinkle mode with a rate, $\dot{u}_i$, $\dot{\omega}$, not equal to zero. Substitution of this solution rate in the rate formulation of the principle of virtual work and choosing the increments conveniently it must hold[1]

$$F \equiv \int_S \left(\dot{M}^{ij}\delta \dot{K}_{ij} + \dot{N}^{ij}\delta \dot{E}_{ij} + N^{ij}\dot{\omega}_{,i}\dot{\omega}_{,j} \right) dS = 0 \tag{9}$$

where substituting (5) in (3) and (4), the bending moments and resulting stress are given by

$$\dot{M}_{ij} = \int_{-t/2}^{t/2} \overline{L}^{ijkl} x_3 dx_3 \dot{E}_{kl} + \int_{-t/2}^{t/2} \overline{L}^{ijkl} x_3^2 dx_3 \dot{K}_{kl} \tag{10}$$

$$\dot{N}_{ij} = \int_{-t/2}^{t/2} \overline{L}^{ijkl} dx_3 \dot{E}_{kl} + \int_{-t/2}^{t/2} \overline{L}^{ijkl} x_3 dx_3 \dot{K}_{kl} \tag{11}$$

The bifurcation functional, F represents the total energy for wrinkling occurrence. The first term represents the bending ($i = j$) and twisting ($i \neq j$) energy of a wrinkle, the second term is the strain energy due to the membrane stresses, and the third term may be interpreted as the work done by the applied in-plane stresses in the middle surface[5]. If the work produced by membrane forces is smaller than the internal energy of any possible wrinkle, the sheet will be under stable conditions. However, if for some mode $\dot{u}_i$, $\dot{\omega}$, the functional equals zero, bifurcation becomes possible.

When the incremental moduli are independent of x_3 the functional F simplifies to

$$F(\dot{u},\dot{\omega}) = \int_S \left[\frac{t^3}{12} \overline{L}^{ijkl} \dot{K}_{kl} \dot{K}_{ij} + t\overline{L}^{ijkl} \dot{E}_{kl} \dot{E}_{ij} + N^{ij} \dot{\omega}_{,i} \dot{\omega}_{,j} \right] dS \tag{12}$$

where S is the region of the sheet middle surface over which the wrinkles occur.

2.3 Wrinkling criterion

Let the axes of principal stress coincide with the axes of principal curvature. Then the initial post-buckling displacements can be described with

$$\begin{bmatrix} \dot{\omega} \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = t \begin{bmatrix} a_1 \cos\left(\frac{\lambda_1}{l} x_1\right)\cos\left(\frac{\lambda_2}{l} x_2\right) \\ a_2 \sin\left(\frac{\lambda_1}{l} x_1\right)\cos\left(\frac{\lambda_2}{l} x_2\right) \\ a_3 \cos\left(\frac{\lambda_1}{l} x_1\right)\sin\left(\frac{\lambda_2}{l} x_2\right) \end{bmatrix} \tag{13}$$

In choosing these post-buckling modes we assume that the wrinkling spans an area S consisting of several wavelengths. Then boundary conditions along the edges of S can be neglected. The variables $a_{i, \ i=1,2,3}$ denote the dimensionless mode amplitude. When the dimensionless mode amplitudes, $a_i << 1$, $i=1,2,3$, the bifurcation functional (12) reduces to[1]

$$F = \beta St \left(\frac{t}{l}\right)^2 a^t M a \tag{14}$$

with $\beta = \frac{1}{2}$, if either λ_1 or λ_2 is zero, and otherwise $\beta = \frac{1}{4}$. The components of the matrix M are given by

$$M_{11} = \tfrac{1}{12}\left(\tfrac{t}{l}\right)^2\left[\overline{L}^{1111}\lambda_1^4 + \overline{L}^{2222}\lambda_2^4 + 2\left(\overline{L}^{1122} + 2\overline{L}^{1212}\right)\lambda_1^2\lambda_2^2\right] +$$
$$+ \left[\overline{L}^{1111}\left(\tfrac{l}{R_1}\right)^2 + \overline{L}^{2222}\left(\tfrac{l}{R_2}\right)^2 + 2\overline{L}^{1122}\left(\tfrac{l}{R_1}\right)\left(\tfrac{l}{R_2}\right)\right] - \left[\sigma_1\lambda_1^2 + \sigma_2\lambda_2^2\right]$$

$$M_{12} = M_{21} = \overline{L}^{1111}\lambda_1\left(\tfrac{l}{R_1}\right) + \overline{L}^{1122}\lambda_1\left(\tfrac{l}{R_2}\right)$$

$$M_{13} = M_{31} = \overline{L}^{2222}\lambda_2\left(\tfrac{l}{R_2}\right) + \overline{L}^{1122}\lambda_2\left(\tfrac{l}{R_1}\right)$$

$$M_{22} = \overline{L}^{1111}\lambda_1^2 + \overline{L}^{1212}\lambda_2^2$$

$$M_{23} = M_{32} = \left(\overline{L}^{1122} + \overline{L}^{1212}\right)\lambda_1\lambda_2$$

$$M_{33} = \overline{L}^{2222}\lambda_2^2 + \overline{L}^{1212}\lambda_1^2$$

A bifurcation point is found whenever the functional *F* equals zero. Then the determinant of the matrix *M* vanishes, or

$$det\ M = 0\ \textit{for wrinkling to occur.} \tag{15}$$

The critical buckling stress state is given by the minimum stress for which (15) holds. Therefore the critical stress, σ_{cr} along the principal axis, and the critical wave number, λ_{cr} are found by solving

$$\frac{\partial \det M}{\partial \lambda_i} = 0, \qquad i = 1,2 \tag{16}$$

The plane stress incremental L_{ijkl} can be found in several forms in a number of articles[2, 4, 5]. They depend on the stresses, and therefore generally an implicit expression for the critical stress is derived.

3   CURVED SURFACES WITHOUT CONTACT

For curved sheets with an expected wavelength, which is much smaller than the dimensions of the sheet, and with the principal axes of stress coinciding with the principal axes of curvature, the modes are given by (13). Here, $l = \sqrt{R_i t}$ and the radius of curvature R_1 or R_2 is taken as appropriate.

When the principal compressive stress is given by σ_1, the wrinkle wave front is perpendicular to the X_1 direction. Therefore we can set $\lambda_2 = 0$. The relation between stresses σ_i and wave numbers λ_i follows from (15). From (16), the critical stress is then implicitly determined by

$$\sigma_1 = \frac{1}{\sqrt{3}} \frac{t}{R_2} \sqrt{L_{11}L_{22} - L_{12}^2} \tag{17}$$

where the abbreviated notation $L_{11} = \overline{L}^{1111}$, $L_{12} = \overline{L}^{1122}$, $L_{22} = \overline{L}^{2222}$ and $L_{44} = \overline{L}^{1212}$ is used. Then, with $l = \sqrt{R_2 t}$ and using the critical stress found, the critical wave number equals

$$\lambda_{cr} = \left(2\sqrt{3} \frac{\sqrt{L_{11}L_{22} - L_{12}^2}}{L_{11}} \right)^{\frac{1}{2}} \tag{18}$$

In case a Nadai material model is assumed a closed form solution for the critical stress and wave number can be found[2].

4 CURVED SURFACES WITH SINGLE SIDED CONTACT

For a sheet of which the lower side is in contact with a die, the displacement normal to the middle surface is restricted to $\dot{\omega} \geq 0$. This condition is satisfied when the modes are chosen as

$$\begin{bmatrix} \dot{\omega} \\ \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = t \left(\begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 \cos\left(\frac{\lambda_1}{l} x_1\right) \cos\left(\frac{\lambda_2}{l} x_2\right) \\ a_2 \sin\left(\frac{\lambda_1}{l} x_1\right) \cos\left(\frac{\lambda_2}{l} x_2\right) \\ a_3 \cos\left(\frac{\lambda_1}{l} x_1\right) \sin\left(\frac{\lambda_2}{l} x_2\right) \end{bmatrix} \right) \tag{19}$$

with $l = \sqrt{R_i t}$, using R_1 or R_2 as appropriate. The bifurcation functional can be as in (14)

$$F = St \left(\frac{t}{l} \right)^2 a^t \widetilde{M} a \tag{20}$$

where, in case of compression in the first principal direction, the matrix $\widetilde{M}$ is given by

$$\widetilde{M} = M\big|_{\lambda_1 = \lambda_2 = 0} + \frac{1}{2} M\big|_{\lambda_2 = 0} \tag{21}$$

Choosing $l = \sqrt{R_2 t}$, the critical buckling stress follows from

$$\sigma_1 = \frac{1}{\sqrt{3}} \frac{t}{R_2} \sqrt{2L_{11}^2 \left(\frac{R_2}{R_1} \right)^2 + 3L_{11}L_{22} + 4L_{11}L_{12} \left(\frac{R_2}{R_1} \right) - L_{12}^2} \tag{22}$$

and the critical wave number is subsequently given by

$$\lambda_{cr} = \left(2\sqrt{3} \; \frac{\sqrt{2L_{11}^2 \left(\frac{R_2}{R_1}\right)^2 + 3L_{11}L_{22} + 4L_{11}L_{12}\left(\frac{R_2}{R_1}\right) - L_{12}^2}}{L_{11}} \right)^{\frac{1}{2}} \qquad (23)$$

The tensor $M\big|_{\lambda_1=\lambda_2=0}$ represents the stretch energy due to increase of the average radius of the shell. The only non-zero term is

$$M_{11} = \left[L_{11}\left(\frac{l}{R_1}\right)^2 + L_{22}\left(\frac{l}{R_2}\right)^2 + 2L_{12}\left(\frac{l}{R_1}\right)\left(\frac{l}{R_2}\right) \right] \qquad (24)$$

Again using the Nadai material model a closed form solution can be found. When the shell is flat $M\big|_{\lambda_1=\lambda_2=0} = 0$ since no radius increase occurs.

5 CONCLUSIONS

The wrinkling theory[1] based on the rate formulation of the principle of virtual work was validated by a number of authors for various geometries. The field of application of the wrinkling theory is however larger than the geometries studied so far. Depending on the choice of the possible modes, the theory can be also applied for wrinkling prediction with contact. When the external work done is added to the formulation, wrinkling with double-sided contact under axial loading can also be dealt with. In combination with FEM-calculations, the local wrinkling theory as presented here therefore offers a valuable tool for qualitative prediction of wrinkling in sheet metal forming. For quantitative wrinkling predictions the condition that the wavelength is short compared to the area prone to wrinkling must be satisfied. By using the presented theory on die necking of Beer&Beverage cans it was shown that the method is suitable to assess and evaluate the risk of pleat forming at certain industrial configurations.

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