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CONTACT SMOOTHING IN SENSITIVITY ANALYSIS AND OPTIMIZATION OF MULTI-BODY CONTACT PROBLEMS

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Summary. *Sensitivity analysis (SA) is developed for 2D and 3D multi-body frictional contact problems. The direct differentiation method (DDM) is applied to obtain response sensitivities with respect to arbitrary design parameters (parameter and shape SA). The FE formulation of contact employs smoothing of the master surface, and the augmented Lagrangian technique is used to enforce the contact and friction conditions. Numerical examples, including application for optimization, illustrate the approach.*

1 INTRODUCTION

Sensitivity analysis (SA) is a technique that allows efficient computation of derivatives of the solution to the problem at hand, the *direct problem*, with respect to parameters defining the problem, the *design parameters*. These derivatives are implicit, as the dependence of the solution $\mathbf{u}$ on the design parameters ϕ_i is defined through the governing equation,

$$\mathbf{R}(\mathbf{u}(\phi_i), \phi_i) = 0, \qquad \frac{\partial \mathbf{R}}{\partial \mathbf{u}} \frac{\mathrm{D}\mathbf{u}}{\mathrm{D}\phi_i} = -\frac{\partial \mathbf{R}}{\partial \phi_i}, \tag{1}$$

here provided for the simplest case of the steady-state nonlinear system, cf. [1]. The general framework of SA for path-dependent problems (transient coupled nonlinear systems), such as those of incremental elasto-plasticity, can be found in [1].

Several applications for frictional contact problems have recently been published [2, 3, 4, 5], however, restricted classes of problems have only been considered. In this work, DDM-based sensitivity analysis is developed for a general class of 2D and 3D multi-body frictional contact problems. Moreover, SA is applied for smooth contact formulations which are beneficial for numerical efficiency and for accuracy.

2 CONTACT SMOOTHING

Boundaries of discretized bodies are typically C^0-continuous. In implicit finite element formulations of contact problems, special treatment is thus necessary in order to avoid severe convergence problems due to sudden changes of surface normal. Contact smoothing is a possible approach to overcome the related problems. Additionally, it provides more accurate approximation of the real shape in case of a coarse mesh.

Within the master-slave approach, adopted in this work, a node of one surface, the *slave node*, contacts with the nearest segment (or facet) of the other surface, the *master segment*. Accordingly, contact smoothing amounts to constructing a smooth representation of the master surface. As contact smoothing in 3D is much more complex than that in 2D, below we focus on the 3D case only.

Several approaches to contact smoothing can be found in the literature. Bézier and B-spline interpolations involving 16 master nodes are used in [6], however, this approach is applicable only for structured meshes. Smoothing of unstructured meshes is presented in [7] based on Gregory patches which provide tangent plane (G^1) continuity. This method is more general, but the computational cost is much higher. Bézier surface interpolation passing through the centroids of four adjacent quadrilateral facets has been proposed in [8]. The latter approach is also adopted in this work, and it is extended for unstructured meshes.

3 SENSITIVITY ANALYSIS OF CONTACT PROBLEMS

Several developments of sensitivity analysis for frictional contact problems have recently been reported employing either the penalty [2, 4] or the augmented Lagrangian [3, 5] formulation. Generally, the sensitivity analysis framework for path-dependent problems, such as those in elasto-plasticity [1], is directly applicable also for frictional contact problems.

The common approach to the augmented Lagrangian method, which employs the Uzawa algorithm, implies that the exact sensitivity analysis is not a single linear problem, but requires iterations corresponding to the iterative update scheme for Lagrange multipliers. This is avoided in [3] by solving an approximate non-iterative sensitivity problem with oversized penalties. In an alternative approach [6] to the augmented Lagrangian formulation, the necessary conditions of the saddle-point problem are solved using a generalized Newton method for the primal (displacements) and dual (Lagrange multipliers) variables at the same time. Sensitivity analysis for contact problems employing the latter approach has been developed in [5]. This formulation is particularly suitable for sensitivity analysis, because the direct differentiation method leads to a non-iterative linear sensitivity problem at each time increment. Moreover, as desired, the operator of the sensitivity problem is exactly the tangent operator of the iterative Newton scheme of the direct problem.

In this work, the approach of [5] is extended to the case of 3D multi-body contact problems including contact smoothing. The problem is formulated as the non-linear transient coupled system, cf. [1]. The kinematic contact variables (normal gap and tangential slip increment) are defined by the orthogonal projection of the slave node onto the master surface [6]. The corresponding local contact search procedure constitutes the dependent problem in the iteration-subiteration scheme.

Application of smooth contact formulations results in severe complexity of the characteristic expressions (residual vector, tangent matrix, sensitivity pseudo-load vector). These are efficiently derived using *AceGen*, the symbolic system for automatic differentiation, optimization and code generation, cf. [9].

4 NUMERICAL EXAMPLE

A 3D problem of two hyperelastic pipes in frictional contact is used to check the accuracy of the sensitivity analysis, cf. Fig. 1. The analytical DDM-based sensitivities have been compared to the numerical finite difference (FD) sensitivities and an excellent agreement has been obtained with the relative error below 10^{-4}, cf. Table 1.

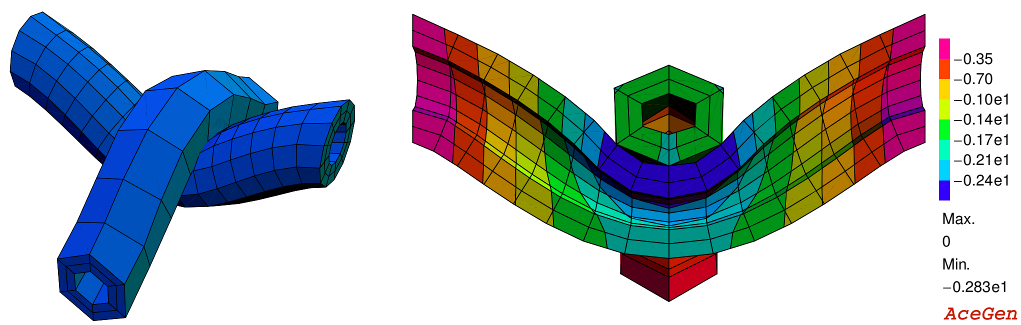

Figure 1: Hyperelastic pipes: deformed mesh and (shape) sensitivity of the z-displacement with respect to the outer radius of the upper pipe, $\mathrm{D}u_z/\mathrm{D}R_o$.

Table 1: Comparison of DDM and FD sensitivities (FD step $\Delta\mu = 10^{-5}$, $\Delta R_o = 10^{-5}$).

Node	DDM	FD	Error	DDM	FD	Error
	$\mathrm{D}u_z/\mathrm{D}\mu$			$\mathrm{D}u_z/\mathrm{D}R_o$		
32	-0.01126618	-0.01126612	5.3E-6	-0.64451519	-0.64450526	1.5E-5
64	-0.03031119	-0.03031102	5.6E-6	-1.49473608	-1.49470542	2.1E-5
393	-0.00535480	-0.00535474	1.1E-5	-0.27672344	-0.27672016	1.2E-5
667	0.00811398	0.00811385	1.6E-5	-0.34567623	-0.34565159	7.1E-5

Other applications of the present framework include sensitivity analysis of the complete workpiece-tooling-press system in cold forging (3D example) and preform optimization of a two-stage forming process (2D example). In the latter case, response sensitivities allow application of an efficient gradient-based optimization algorithm.

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