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NODE BASED FINITE ELEMENT METHOD WITH HIGH PERFORMANCE ELEMENTS: THE LOCAL ELEMENT METHOD

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Recent advances in computer technology have enabled a number of complicated natural phenomena to be accurately simulated, which were ever only analyzable by experiments. Among various computer simulation techniques, the finite element method has been most widely used due to the capability of analyzing an arbitrary domain. The bottleneck of the finite element analysis, however, is the generation of a huge unstructured mesh, for example exceeding 10 million.

To overcome the above shortcoming of the standard finite element method, so called mesh-free methods have been studied. The element-free Galerkin method [1] is one of them with the use of integration by background-cells instead of elements, based on the moving least square and diffuse element methods. The reproducing kernel particle method [2] is another mesh-free scheme, which is based on a particle method and wavelets. The general feature of these mesh-free methods is that, contrary to the standard finite element method, the connectivity information between nodes and elements is not required explicitly, since the evaluation of the total stiffness matrix is performed generally by the node-wise calculations instead of the element-wise calculations.

Among virtually mesh-free methods is the node-based finite element method named as the free mesh method, which is simple, accurate and still keeps the good features of the standard finite element method [3,4]. In the method, a cluster of elements is generated around each node. Then, the stiffness matrices of each element are calculated and the contributing components to the central node are assembled to the total matrix. Therefore, the solution of the free mesh method is equivalent to the usual finite element method. The features of the free mesh method are summarized as follows,

(1) Easy ,robust and automatic to generate a large-scale mesh
(2) Applicable without being conscious of mesh generation
(3) The result being equivalent to that of the finite element method

In the present paper, a new node based finite element method, called the Local Element Method, is proposed, which is a kind of extension from the free mesh method. Here, the strain field is defined on the same local element cluster as in the free mesh method where each element within the cluster is the usual finite element (see Fig.1). The strain field is assumed as

$$\{\boldsymbol{\varepsilon}(\mathbf{x})\} = [\mathbf{N}]\{\mathbf{a}\} \tag{1}$$

where $\{\boldsymbol{\varepsilon}(\mathbf{x})\}$ is the strain field on the local element cluster. [**N**] is an arbitrary polynomial matrix of the coordinates and {**a**} is a coefficient vector to be determined. In this paper, [**N**] is assumed to be a 1st or 2nd order polynomial. {**a**} is determined approximately by using the points which are called as "strain monitoring points"(see Fig.1). In other words, we consider the least square problem to determine the unknown coefficient {**a**} as follows,

$$J = \sum_{e=1}^{Lelm} \sum_{i=1}^{p} \left[\{\boldsymbol{\varepsilon}(\mathbf{x})\} - \{\boldsymbol{\varepsilon}_i^e\} \right]^2 \tag{2}$$

where J is a functional to be minimized, *Lelm* is the number of local elements, and p is the

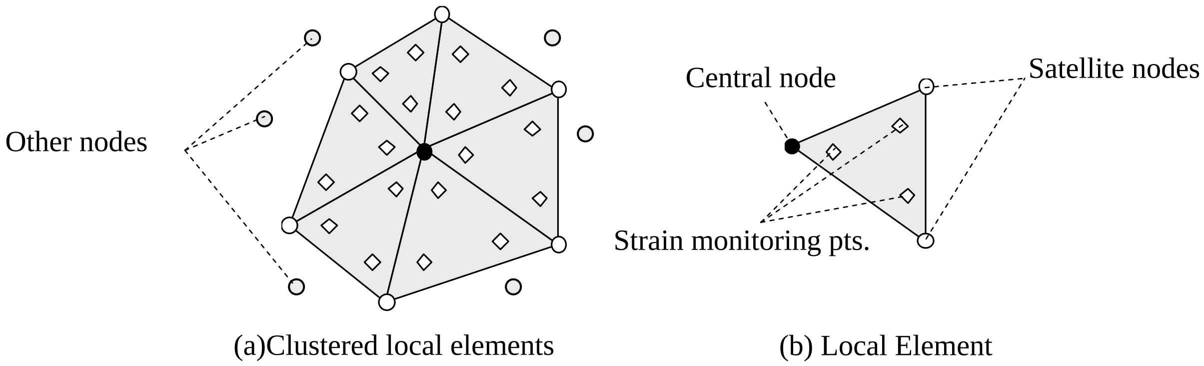

Fig.1 Concept of local elements

number of monitoring points. $\{\boldsymbol{\varepsilon}_i^e\}$ is determined as the values of strains of the local elements, called the mother element, at the strain monitoring points. According to the stationary condition of equation (2), {**a**} is given by

$$\{\mathbf{a}\} = \sum_{e=1}^{Lelm} \sum_{i=1}^{p} \left[\left[[\mathbf{N}_i]^T [\mathbf{N}_i] \right]^{-1} [\mathbf{N}_i]^T \{\boldsymbol{\varepsilon}_i^e\} \right] \tag{3}$$

Let as consider the simple Constant Strain Triangle as the mother element, in which the displacement field of each local element is defined by

$$\{\mathbf{u}\} = \sum_{i=1}^{3} \{\mathbf{u}_i\}\zeta_i \ , \tag{4}$$

where {**u**} is the displacement field of the local element, $\{\mathbf{u}_i\}$ is the nodal displacement, and ζ_i is the area-coordinate. Therefore, the strain value at the strain monitoring points is given by

$$\{\boldsymbol{\varepsilon}_i^e\} = [\mathbf{B}_i^e]\{\mathbf{u}_i\} \tag{5}$$

where

$$[\mathbf{B}_i^e] = \left[[\mathbf{B}_1] \quad [\mathbf{B}_2] \quad [\mathbf{B}_3]\right]$$

$$[\mathbf{B}_j] = \begin{bmatrix} \partial\zeta_j/\partial x & 0 \\ 0 & \partial\zeta_j/\partial y \\ \partial\zeta_j/\partial y & \partial\zeta_j/\partial x \end{bmatrix} \quad , \quad j = 1,\, 2\,,\, 3 \quad .$$

Finally, the unknown coefficient $\{\mathbf{a}\}$ is determined by

$$\{\mathbf{a}\} = \sum_{e=1}^{Lelm}\sum_{i=1}^{p} \left[\left[[\mathbf{N}_i]^T [\mathbf{N}_i]\right]^{-1} [\mathbf{N}_i]^T \left[\mathbf{B}_i^e\right]\{\mathbf{u}_i\}\right]. \tag{6}$$

Substituting equation (6) into (1), we obtain

$$\begin{aligned}\{\boldsymbol{\varepsilon}(\mathbf{x})\} &= [\mathbf{N}]\sum_{e=1}^{Lelm}\sum_{i=1}^{p} \left[\left[[\mathbf{N}_i]^T [\mathbf{N}_i]\right]^{-1} [\mathbf{N}_i]^T \left[\mathbf{B}_i^e\right]\{\mathbf{u}_i\}\right] \\ &= [\mathbf{A}]\{\mathbf{u}_i\}.\end{aligned} \tag{7}$$

The stiffness matrix of the local element is computed using the new assumed strain as

$$[\mathbf{k}] = \int_\Omega [\mathbf{A}]^T [\mathbf{D}][\mathbf{A}]\mathbf{d}\Omega. \tag{8}$$

where $[\mathbf{D}]$ is the stress-strain matrix, and Ω is the local element. Thus, the stiffness matrix is computed in a node-by-node manner.

It is noted that the present local element method is very closely related to the superconvergent patch recovery proposed by Zienkiewicz and Zhu [5]. In an adaptive finite element method, the Z-Z error estimator has been most widely used to estimate the error. The error estimator requires an exact solution, but generally it is impossible to compute the exact value because the exact solution is not available in the natural phenomena. The Z-Z technique then obtains the recovered solution in a post processing stage. The local element cluster in the present method is equivalent to the superconvergent patch used in the Z-Z technique. The difference lies in that the recovering procedure in the local element method is in a main process stage when computing element stiffness matrices. The use of the assumed strain is therefore, in some sense, equivalent to the "post-process" of the superconvergent patch recovery.

REFERENCES

[1] T.Belytschiko, Y.Y.Lu, and L. Gu, Element-Fee Galerkin Methods, Int. J. Numer. Methods Eng., 37(1994), 229-256.

[2] W.K. Liu, S. Jun, S. Li, J. Adee and T. Belytschiko, Reproducing Kernel Particle Methods for Structural Dynamics, Int. J. Numer. Methods Eng., 38(1995), 1655-1679.

[3] G.Yagawa and T.Yamada , Free mesh method: A new meshless finite element method, Computational mechanics, 18(1996), 383-386.
[4] G.Yagawa, Node-by-node parallel finite elements: a virtually meshless method, Int. J. Numer. Methods Eng., 60(2004), 69-102.
[5] O.C.Zienkiewicz and J.Z.Zhu, The superconvergent patch recovery and *A posteriori* error estimates. Part 1 : The recovery technique, , Int. J. Numer. Methods Eng., 33(1992), 1331-1364.

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