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ALE FORMULATION FOR THE NUMERICAL SIMULATION OF FRICTION STIR WELDING.

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Summary. *An Arbitrary Lagrangian Eulerian approach is used to simulate the Friction Stir Process. This formulation, which has been implemented into the Forge3® FE software, uses a splitting method: 1) calculation of the material velocity/pressure fields, 2) calculation of the mesh velocity, and 3) treatment of advection terms. This latter stage consists in remapping the variables necessary to the next computation step. Transport of nodal variables is carried out relatively simply by using an upwind technique. However, remapping variables stored at integration points, as the stress field, require more complex operations. Different techniques, based on patch recovery approach, have been investigated.*

1 INTRODUCTION

The stationary welding phase of the Friction Stir Welding Process (FSWP), is described on fig. 1. The simulation of the plunging phase, and the formation of eventual defects (voids) during the welding phase, require taking into account the free surfaces' movements whereas high deformations are involved under the tool. And the only approach, which proved to be suitable for removing this problem, is the Arbitrary Lagrangian Eulerian one [1]. So, an ALE formulation has been implemented, into the FORGE3® FE software, using a splitting method: a Lagrangian phase is followed by a convection (transport) phase. Treatment of convective terms has always been one of the main difficulties of this method [2]. Transport of variables, which can be discontinuous (P0) per element, has to be sufficiently accurate and consistent. Furthermore, remapping the stress tensor (necessary in elasto-viscoplastic computation) should verify mechanical equilibrium. This work reports on investigations concerning different transport techniques.

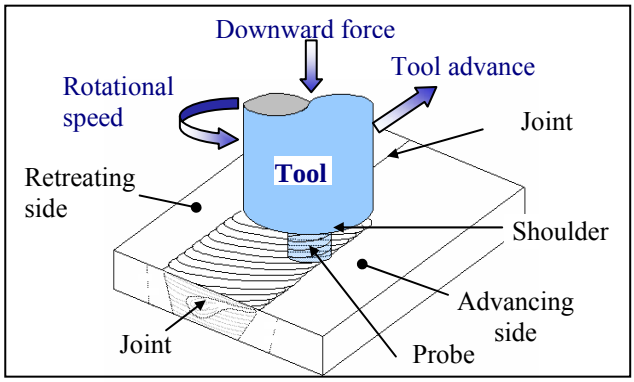

Figure 1 : Welding phase of FSWP

2 LAGRANGIAN PHASE

Forge3® is extensively used to simulate hot, warm and cold forging of 3D geometry parts, using a pure Lagrangian formulation. At each time step, this formulation is used to solve the

thermal and mechanical equilibrium equations. This is the first step of the implemented ALE method.

2.1 Mechanical Formulation

The finite element discretization of the part is based on an enhanced (P1+/P1) 4-node tetrahedron element. A mixed velocity-pressure formulation is used. The weak form of the momentum equation and weak form of incompressibility of the plastic deformation are solved by using a Newton-Raphson algoritm with a preconditioned gradient solver.
Viscoplastic, Elasto-plastic and elasto-viscoplastic constitutive models are available to simulate both elastic and plastic deformation, with elastic spring back and residual stress calculations.
Contact and friction conditions at the interface between part and tools are modeled by Tresca, Coulomb or Norton friction laws. Furthermore, different laws are available to describe some rheological and tribological parameters as functions of the temperature.

2.2 Thermal Coupling

The thermal computation is coupled to the mechanical computation. So the discretized form of the heat equation is solved at each time step. Heat generated by material deformation and by friction with the tool, thermal exchanges by conduction, convection and radiation, are all taken into account.

3 ALE FORMULATION: CONVECTIVE PHASE

3.1 Mesh velocity

The second step of the ALE method consists in computing the mesh velocity $\mathbf{v}_{msh}$. This velocity can be different of the material one. Respecting a consistency condition at boundaries (see equ. 1), $\mathbf{v}_{msh}$ is computed to optimize element shapes independently from the material deformation. The relationship between the material time derivate and the grid time derivate of a variable φ is given by equ. 2. The convective terms account for the transport of material trough the grid and are treated in this method by a remapping step described above.

$$\left(\mathbf{v}_{\text{msh}} - \mathbf{v}_{\text{mat}}\right)\cdot\mathbf{n} = 0 \qquad (1)$$

$$\frac{\mathrm{d}_{\mathrm{g}}\varphi}{\mathrm{d}t} = \dot{\varphi} + \left(\mathbf{v}_{\text{msh}} - \mathbf{v}_{\text{mat}}\right)\cdot\nabla\varphi \qquad (2)$$

3.2 Transport Techniques

A first order linearization of equation 2, which leads to equation 3, provides a quick and simple method to transport nodal variables. The gradient has to be computed on the upwind element which is determined as shown on fig. 2.

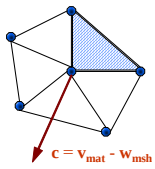

Figure 2: Upwind element

$$\varphi_{ALE}^{t+\Delta t} = \varphi_{LAG}^{t+\Delta t} - \left(v_{mat} - v_{msh}\right).\nabla\varphi^{t+\Delta t} \qquad (3)$$

However, this method can not be applied to transport the variables stored at integration points: the stress field σ or the equivalent strain $\bar{\varepsilon}$ are often discontinuous per elements.

The first and simplest idea is to apply a P0 transport: value at the ALE integration point is approximated by the value of the nearest updated Lagrangian integration point. But higher are the gradients in regard to the elements' size, higher is the resulting error. Therefore, defining a local continuous solution is preferred.

Simple least square method, or more elaborated Superconvergent Patch Recovery method [3] (equ. (4), (5) or (7) introduced above), can be used to firstly compute a continuous solution at Lagrangian nodes from their topological neighbor (see fig. 4). Then, this solution is interpolated at ALE integration points.

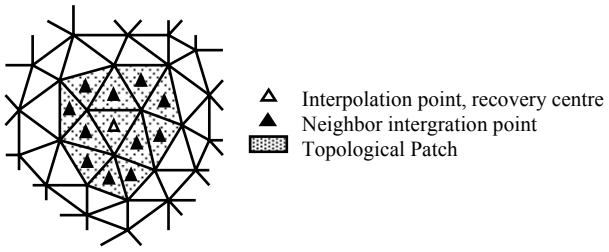

Figure 3: Patch centred on an element in 2D

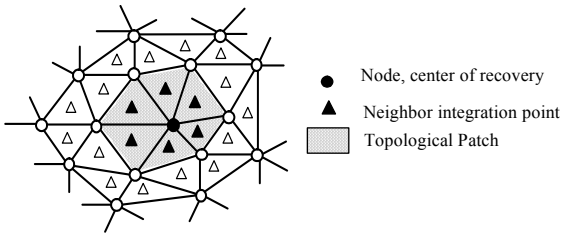

Figure 4: Patch centred on a node in 2D

A recovered continuous solution $\tilde{\sigma}_h^k$ can be also directly constructed at each integration points. Equ. (4) gives the second order polynomial expression of $\tilde{\sigma}_h^k$. Coefficients a^k, which represent the variable gradient, are determined on the Lagrangian configuration by minimizing the least square expression (5) on the integration point centered patch (fig. 3).

$$\tilde{\sigma}_h^k(\Delta x,\Delta y,\Delta z) = P.a^k \qquad \begin{cases} P = (1,\Delta x,\Delta y,\Delta z,\Delta x^2,\Delta y^2,\Delta z^2,\Delta x\Delta y,\Delta x\Delta z,\Delta y\Delta z) \\ a^k = (a_1^k,a_2^k,a_3^k,a_4^k,a_5^k,a_6^k,a_7^k,a_8^k,a_9^k,a_{10}^k)^t \\ \Delta x = x - x_h \quad \Delta y = y - y_h \quad \Delta z = z - z_h \end{cases} \tag{4}$$

At the Lagrangian center of recovery (x_h, y_h, z_h): $\tilde{\sigma}_h^k = \sigma_h^k \Leftrightarrow a_1^k = \sigma_h^k$.

$$\Pi(a^k) = \sum_{i=1}^{NG}\left(\sigma_h(i) - \tilde{\sigma}_h^k(i)\right)^2 = \sum_{i=1}^{NG}\left(\sigma_h(i) - P(x_i,y_i,z_i).a^k\right)^2 \tag{5}$$

Value of the variable at any integration point of the ALE configuration is next extrapolated from the nearest updated Lagrangian integration point using expression (4).

In case of an Elasto-viscoplastic computation, a different expression can be minimized to better satisfy mechanical equilibrium. The recovered solution should satisfy equation (6) on the patch domain for any virtual velocity u_h^*. So the least square expression (5) can be replaced by equ. (7). Its minimization provides an "equilibrated remap".

$$\forall\, u_h^* \in U_0^{ca},\ \int_{\Omega_k}\left(\tilde{\sigma}_h^k - \sigma_h\right) : \varepsilon_h(u_h^*)\, d\omega = 0 \ \rightarrow\ \Pi(a^k) = \left(\int_{\Omega_k}\left(\sum_{m=1}^{4} P_m.a_m^k - \sigma_h\right) : \varepsilon_h(u_h^*)\ d\omega\right)^2 \quad (6)\rightarrow(7)$$

3.3 Test and first results

To evaluate the efficiency of the different transport techniques, error estimation is performed. Two norms are used to compute the error η induced by remapping: the first one is a classical L2 norm (8) and the second one is an "equilibrium norm" (9), traducing the disequilibrium at the beginning of the new increment. $\tilde{\sigma}_h^{ALE}$ is the stress remapped on mesh after an arbitrary distortion and σ_h^{LAG} is the Lagrangian value, which would have been directly computed on this distorted mesh with the same boundary conditions.

$$\eta = \left(\int_\Omega \left(\tilde{\sigma}_h^{ALE} - \sigma_h^{LAG} \right)^2 d\omega \right)^{1/2} \qquad (8)$$

$$\eta = \left(\int_\Omega \left(\tilde{\sigma}_h^{ALE} - \sigma_h^{LAG} \right) : \varepsilon_h \left(u_h^* \right) d\omega \right)^{1/2} \qquad (9)$$

Another more classical test to visualize the conservativity of the method consists in remapping variables without any mechanical or thermal computation: a constant velocity is imposed to material, which moves across an Eulerian mesh. Fig. 5 shows the difference observed between a stress spot remapped using a two different techniques.

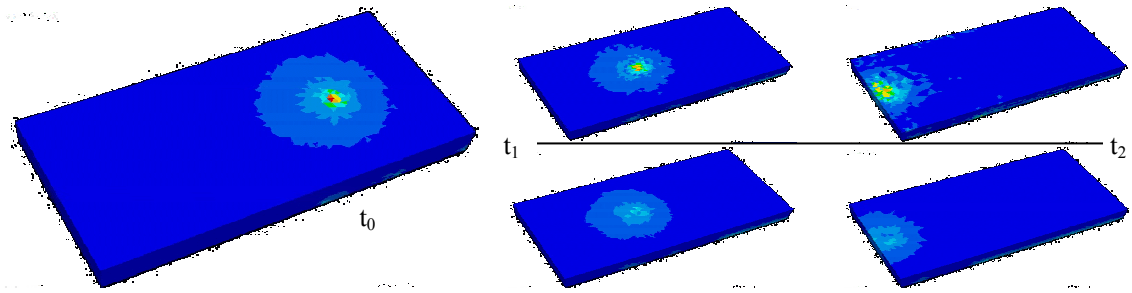

Figure 5: Transport of a stress spot across a fixed mesh, using :
i) a nodal smoothing technique at the top, ii) a SPR technique centered on elements at the bottom.

4 CONCLUSIONS

A new ALE formulation has been implemented in the FORGE3® FE software to simulate all steps of the FSWP. To correctly remap the variables stored at integration points, such as stress components, different transport techniques have been implemented.

These techniques have been enhanced to better satisfy mechanical equilibrium after stress remapping. Benchmark tests have been developed to evaluate the conservativity of the transport algorithm. Error estimation has been computed in order to compare the efficiency of each technique in function of the value of the convective velocity.

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