VIII International Conference on Computational Plasticity
COMPLAS VIII
E. Oñate and D. R. J. Owen (Eds)

EXTENDED FINITE ELEMENT METHOD IN AN ORTHOTROPIC CRACKED MEDIUM

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Key words: Orthotropic Medium; Extended Finite Element Method (X-FEM).

1 INTRODUCTION

The needs of modeling orthotropic materials have been recently revived with great interest, having enormous applications in various structural systems like aerospace and automobile industries. The main advantages of using these materials can be attributed to their high stiffness and low ratio of weight to strength in comparison to other materials.

Some analytical investigations have been reported on the fracture behaviour of such materials such as the pioneering one by Muskelishvili[1] and Sih et al.[2].

Many numerical methods have been utilized for solving mechanical problems such as the finite difference method, the finite element method and the boundary element method. However, the finite element method is more convenient and applicable because of its ability in modeling complex boundary conditions, loadings, materials and geometries. In order to further improve FEM modeling of discontinuities, Belytschko et al.[3] combined FEM with the concepts of partition of unity developing the eXtended Finite Element Method (XFEM). In the XFEM, the finite element approximation around a crack is enriched with functions derived from fracture analysis of the crack-tip. The main advantage of the XFEM is that the mesh is prepared independent of the existence of any discontinuities. Moës et al[4] have reported successful simulations for 2D isotropic media.

In this study, the method is further extended for modeling one branch of orthotropic materials. The enriching functions are based on the work reported by Viola et al[5]. To verify the robustness of the proposed method, stress intensity factors (SIFs) for a cracked plate are obtained and compared with other numerical or (semi-) analytical methods.

2 MECHANICS OF ORTHOTROPIC MATERIALS

It is assumed that an infinite orthotropic plate consisting of a traction free line crack is subjected to uniform biaxial (T and kT) and shear (S) loads at infinity. Fig. 1 shows the crack geometry, loading and the Cartesian and polar co-ordinates utilized in this study.

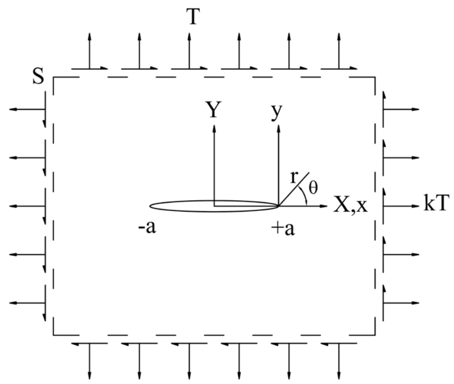

Figure 1: Crack geometry, loading, global and local co-ordinates

In order to derive expressions for an static case from the originally developed formulae for elastodynamic problems by Viola et al.[5], the velocity of the crack propagation is assumed to be zero. The displacement field in X (u) and Y (v) directions can then be written as (Viola et al.[5]):

$$\begin{aligned}u &= -2\beta\left[\left(p_3A_1 - p^4B_1 + p_4B_2\right)Y_1 + \left(p_3B_1 + p_4A_1\right)X_1 + p_3B_2X_2\right] \\ &+ \frac{\beta T}{\mu_{12}D_1}\left\{\left(p_3k_6 + p_4k_5\right)\left[2\left(a + r\cos\theta\right) - \sqrt{2ar}\left(\sqrt{c_1(\theta)}\cos\theta_1/2 + \sqrt{c_2(\theta)}\cos\theta_2/2\right)\right]\right. \\ &\left.- \left(p_3k_5 + p_4k_5\right)\sqrt{2ar}\left(\sqrt{c_1(\theta)}\sin\theta_1/2 - \sqrt{c_2(\theta)}\sin\theta_2/2\right)\right\} \\ &+ \frac{\beta S}{\mu_{12}D_1}\left\{\left(p_3k_3 + p_4k_4\right)\left[X_1 - X_2 + \sqrt{2ar}\left(\sqrt{c_2(\theta)}\cos\theta_2/2 - \sqrt{c_1(\theta)}\cos\theta_1/2\right)\right]\right. \\ &\left.- \left(p_3k_4 + p_4k_3\right)\left[2Y_1 - \sqrt{2ar}\left(\sqrt{c_1(\theta)}\sin\theta_1/2 + \sqrt{c_2(\theta)}\sin\theta_2/2\right)\right]\right\}\end{aligned} \tag{1}$$

$$\begin{aligned}v &= -\left[\left(\gamma_1A_1 - \gamma_2B_1 - \gamma_2B_2\right)Y_1 + \left(\gamma_2A_1 + \gamma_1B_2\right)X_1 - \gamma_1B_2X_2\right] \\ &+ \frac{T}{2\mu_{12}D_1}\left\{\left(\gamma_1k_6 + \gamma_2k_5\right)\left[\left(X_1 - X_2\right) + \sqrt{2ar}\left(\sqrt{c_1(\theta)}\cos\theta_2/2 - \sqrt{2ar}\cos\theta_2/2\right)\right]\right. \\ &\left.+ \left(\gamma_1k_5 - \gamma_2k_6\right)\left[2Y_1 - \sqrt{2ar}\left(\sqrt{c_1(\theta)}\sin\theta_1/2 + \sqrt{c_2(\theta)}\sin\theta_2/2\right)\right]\right\} \\ &+ \frac{S}{2\mu_{12}D_1}\left\{\left(\gamma_1k_3 - \gamma_2k_4\right)\left[2\left(a + r\cos\theta\right) - \sqrt{2ar}\left(\sqrt{c_1(\theta)}\cos\theta_1/2 + \sqrt{c_2(\theta)}\cos\theta_2/2\right)\right]\right. \\ &\left.+ \left(\gamma_2k_3 + \gamma_1k_4\right)\sqrt{2ar}\left(\sqrt{c_1(\theta)}\sin\theta_1/2 + \sqrt{c_2(\theta)}\sin\theta_2/2\right)\right\}\end{aligned} \tag{2}$$

where p_j (j=1-4), k_j (j=1-6), A_1, B_1, B_2 and D_1 are constant loadings and material property coefficients and

$$X_1 = \left(a + r\cos\theta\right) - \gamma_1 l^2 r\sin\theta, \quad X_2 = \left(a + r\cos\theta\right) + \gamma_1 l^2 r\sin\theta, \quad Y_1 = \gamma_2 l^2 r\sin\theta \tag{3}$$

$$c_j(\theta) = \left(\cos^2\theta + l^2\sin^2\theta + (-1)^j l^2\sin 2\theta\right)^{1/2}, \quad l^2 = \left(\gamma_1^2 + \gamma_2^2\right)^{-1} \tag{4}$$

$$\theta_j = arctg\left(\frac{\gamma_2 l^2\sin\theta}{\cos\theta + (-1)^j\gamma_1 l^2\sin\theta}\right), \quad j = 1,2. \tag{5}$$

$$\gamma_1 = \left[0.5(C_{22}/C_{11})^2 + 0.25(C_{22}/C_{11}) - 0.25(C_{12}^2/[C_{11}C_{33}]) - 0.5(C_{22}/C_{11})\right]^{1/2} \quad (6\text{-}1)$$

$$\gamma_2 = \left[0.5 + (C_{22}/C_{11})^2 - 0.5(C_{22}/C_{33}) + 0.5(C_{12}^2/[C_{11}C_{33}]) + (C_{22}/C_{11})\right]^{1/2} \quad (6\text{-}2)$$

where C_{ij} (i,j=1,2 and 3) are constitutive coefficients.

The displacements are limited to the case that γ_1 and γ_2 have real values (Viola et al.[5]).

3 EXTENDED FINITE ELEMENT METHOD

In XFEM the procedure of preparing the numerical analysis model is divided into two parts. In the first part, the finite element model is created without any considerations about cracks, holes or other discontinuities and then the approximation of displacement is enriched by utilizing asymptotic near-tip functions and the generalized Heaviside function through the framework of partition of unity.

For a point **x** of a domain, in XFEM, the displacement approximation for modeling an arbitrary crack can be written as (Moës et al[4])

$$\mathbf{u}^h(\mathbf{x}) = \sum_{\substack{I \\ n_I \in \mathbf{N}}} \phi_I(\mathbf{x})\mathbf{u}_I + \sum_{\substack{J \\ n_J \in \mathbf{N}^g}} \mathbf{b}_J \phi_J(\mathbf{x}) H(\mathbf{x}) + \sum_{k \in \mathbf{K}^1} \phi_k(\mathbf{x}) \left(\sum_l \mathbf{c}_k^{l1} \mathrm{F}_l^1(\mathbf{x}) \right) + \sum_{k \in \mathbf{K}^2} \phi_k(\mathbf{x}) \left(\sum_l \mathbf{c}_k^{l2} \mathrm{F}_l^2(\mathbf{x}) \right) \quad (7)$$

where $\mathbf{N}$ is the set of all nodes in the domain, $\mathbf{K}^1$ and $\mathbf{K}^2$ are the sets of nodes that crack tips 1 and 2 are in their support domain, respectively, $\mathbf{N}^g$ is a set of nodes that the crack is in their support domain except for $\mathbf{K}^1$ and $\mathbf{K}^2$, $\mathbf{b}_J$ and $\mathbf{c}_k$ are the set of additional degrees of freedom related to the discontinuities, ϕ_I is the finite element shape function and $H(\mathbf{x})$ is the generalized Heaviside function defined by

$$H(\mathbf{x}) = \begin{cases} +1 & ;if\ (\mathbf{x} - \mathbf{x}^*).\mathbf{e_n} > 0 \\ -1 & ;otherwise \end{cases} \quad (8)$$

where $\mathbf{x}^*$ is the nearest point on the crack to **x** and $\mathbf{e_n}$ is the unit vector normal to the crack alignment.

In Eq. 7, $\mathrm{F}_l^1(\mathbf{x})$ and $\mathrm{F}_l^2(\mathbf{x})$ are near-tip enrichment functions; noting that these functions must span the displacement fields in Eqs. (1) and (2); therefore one can write:

$$\{F_l(r,\theta)\}_{l=1}^4 = \left\{ \sqrt{r}\cos\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\cos\frac{\theta_2}{2}\sqrt{g_2(\theta)}, \sqrt{r}\sin\frac{\theta_1}{2}\sqrt{g_1(\theta)}, \sqrt{r}\sin\frac{\theta_2}{2}\sqrt{g_2(\theta)} \right\} \quad (10)$$

4 EXAMPLE

In this example the proposed method is applied to a slanted crack of length 2a located in a finite two-dimensional orthotropic plate under constant applied tension (Fig. 2) where $2a = 2\sqrt{2}$.

Stress intensity factors are evaluated by the method reported by Kim and Paulino[6] and compared with results reported by Sih et al[2], Alturi et al[7], Wang et al[8] and Kim and Paulino[13]. According to Table 1, the results are different 2.6% for K_I and 3.6% for K_{II} in comparison to Sih et al[2].

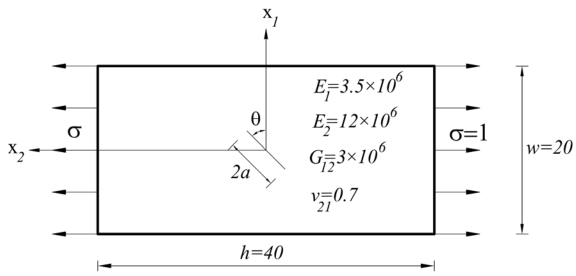

Figure 2 : Geometry of a plate with a slanted crack under remote tension

Method		K_I	K_{II}
Sih et al[2]		1.0539	1.0539
Atluri et al.[7]		1.0195	1.0795
Wang et al.[8]		1.023	1.049
Kim and Paulino[6]	MCC	1.067	1.044
	DCT	1.077	1.035
Proposed method		1.081	1.092

Table 1: SIFs in an orthotropic plate with a slanted crack under uniform remote tension loading

5 CONCLUSIONS

In this paper, an extended finite element method is proposed for analyzing cracked orthotropic materials. A set of partition of unity based enriching functions are added to the finite element approximation so the crack geometry can be taken into account without any special meshing. Analytical displacement field around a crack-tip in orthotropic media is used to extract near-tip enrichment functions. The robustness of suggested method was tested with evaluating stress intensity; all in good agreement with other numerical and semi-analytical methods.

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