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3D MODELLING OF METAL FLOW IN HOT ROLLING

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1 INTRODUCTION

Many approximation methods allow to analyse the plastic flow in metal forming processes. The upper-bound method (UBM) is one of them. It is based on one of the limit theorems formulated by [1,2,3]. According to this, any assumed velocity field that satisfies the volume constancy and the boundary conditions leads to a power P_I, which is greater than the real one required for processing, P_E. The lowest power P_I is obtained by a velocity field that shows the best approximation to the real one. Therefore, the determination of such a field is an essential step in the application of the theorem to metal-forming problems.

2 DESIGN OF A 3D MODEL

Establishing a velocity field for the roll gap implies a prognosis of dimensions of the deformation zone as a whole. All boundaries of this zone can be subdivided into three parts: free side surface, surfaces being in contact with the rolls, and the surfaces at the entry and exit of the roll gap, which come into contact with the rigid zones of a billet. For the steady state deformation the last two surfaces can be fixed by the circular arc of the rolls and by parallel planes at the roll-gap entrance and exit, respectively. Only the free side surface needs to be modelled.

As soon as the velocity field for the roll gap is established, also the so far unknown though presupposed target width is fixed by the volume constancy; therefore, it can be determined by the variation of a functional. Formulating this, the external power with torque M, speed ω and tensions T and the internal power with the assumed velocities u, v and w together with the flow stress k_f and the strain rate $\dot{\varepsilon}_V$ lead to a functional F, which has to be minimized, equation (1):

$$F = \frac{4T_A}{A_A}\int_{z=0}^{h_A/2}\int_{y=0}^{b/2(x=0,z)} u\Big|_{x=0} dydz - \frac{4T_B}{A_B}\int_{z=0}^{h_B/2}\int_{y=0}^{b/2(x=ld,z)} u\Big|_{x=ld} dydz - 2M\omega + 4m\int_{x=0}^{ld}\int_{y=0}^{b/2(x,z=h/2)} \frac{k_f}{\sqrt{3}}\sqrt{u^{r2}+v^{r2}}\Big|_{z=h/2} dydx +$$
$$4\int_{z=0}^{h_A}\int_{y=0}^{b/2(x=0,z)} \frac{k_f}{\sqrt{3}}\sqrt{v^2+w^2}\Big|_{x=0} dydz + 4\int_{x=0}^{ld}\int_{z=0}^{h/2(x)}\int_{y=0}^{b/2(x,z)} k_f\,\dot{\varepsilon}_v\, dydzdx \;\rightarrow \min. \tag{1}$$

The so called Ritz's coefficients (a_1, a_2, … a_n) are introduced into equation (1) by the formulation of a kinematically admissible velocity field. These coefficients describe the spread function and serve as optimization parameters in equation (1). For this application the following conditions hold $\left[\frac{\partial F}{\partial a_i}\right] = 0$. The Newton method serves for the solution of this equation set. The program is written in C++. Integration and iteration algorithms are derived from [4]. The calculation has to be supervised by a safe control system described in [5].

3 ROLLING TESTS

In a recently published paper [5], the special computer tool is described that allows determining a test schedule and the sequence of the realisation of the experiments. Table 1 shows the basic characteristics used for planning the tests.

Reduction	25	35	45	%
Steel grade	Low alloyed C15	Austenitic 1.4301	Ferritic 1.4016	
b0/h0	6	8	10	
T	1050			°C
Max roll force	530			kN
Max roll torque	9.5			kN*m
Min thickness	4			mm
Rolling speed	0.5			m/s
Roll diameter	188			mm

Table 1 : Test basic characteristics

The most relevant characteristics in this test were: initial thickness *h0*, initial width *b0*, end thickness *h1*, end width *b1* and side surface form. The linear dimension was measured with a caliper, the side surface with a sensing device on a 3D-plane-table. Finding the dependences of the shape of the lateral faces was one of the basic purposes of the side-surface study.

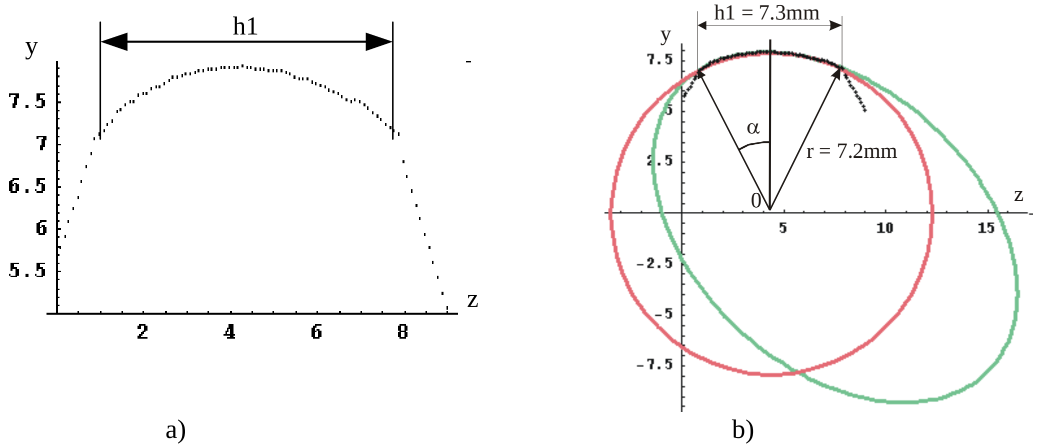


Figure 1: Typical side-form measurement

It is intended to use the obtained results as the static parameters in the UBM. Figure 1 shows the typical result of one measurement a) and an example to evaluate this measurement b). The task is to specify an objective function that connects actuating variables like *eps* as reduction, *ld/hm* as roll-gap aspect ratio and *b0/h0* as width relation with the relative variable sin($\alpha/2$). As approximation function was chosen

$$\sin(\alpha/2) = j_1 \cdot eps^{j_2} \cdot \left(\frac{ld}{hm} - 0.7\right)^{j_3} \cdot \left(\frac{b0}{h0}\right)^{j_4} \tag{2}$$

where j_1, j_2, j_3, j_4 are fit parameters. Figure 2 shows a 3D and a density plot of this

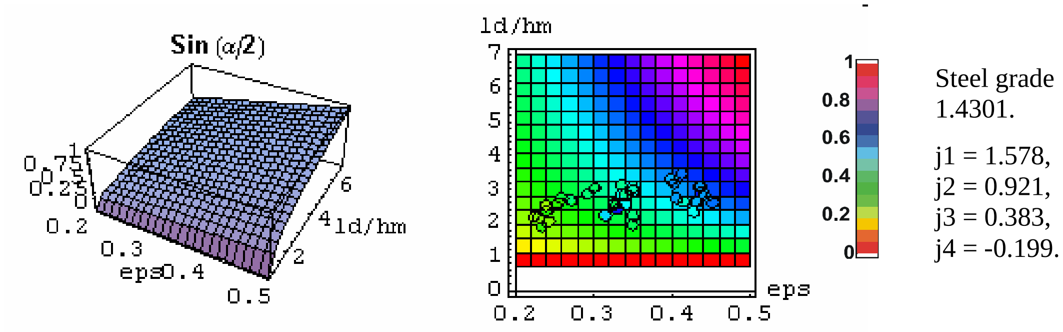


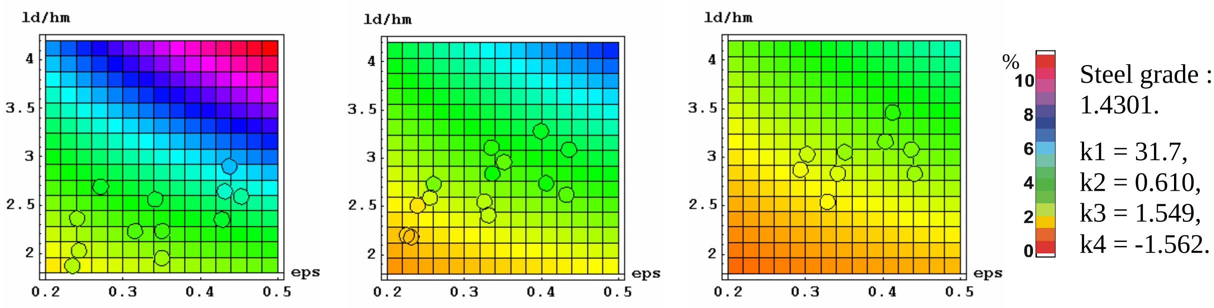
Figure 2: Typical side-form measurement

function for austenitic steel. Points on the graph are measurement points.

Figure 3 shows a comparison of the nominal spread *db/b0* for the austenitic steel 1.4301. Three pictures are presented for each width relation *b0/h0*. The density plots show the approximation function of all measurements for this steel grade. Points on these plots are measuring points. As approximation function for this analysis serves

$$db/b0 = k_1 \cdot eps^{k_2} \cdot \left(\frac{ld}{hm}\right)^{k_3} \cdot \left(\frac{b0}{h0}\right)^{k_4} \tag{3}$$

where k_1, k_2, k_3, k_4 are fit parameters. On these plots it is to be seen how precisely the fit function describes the measuring points. Actually, the better a function approximates the points the closer the points merge with the background.

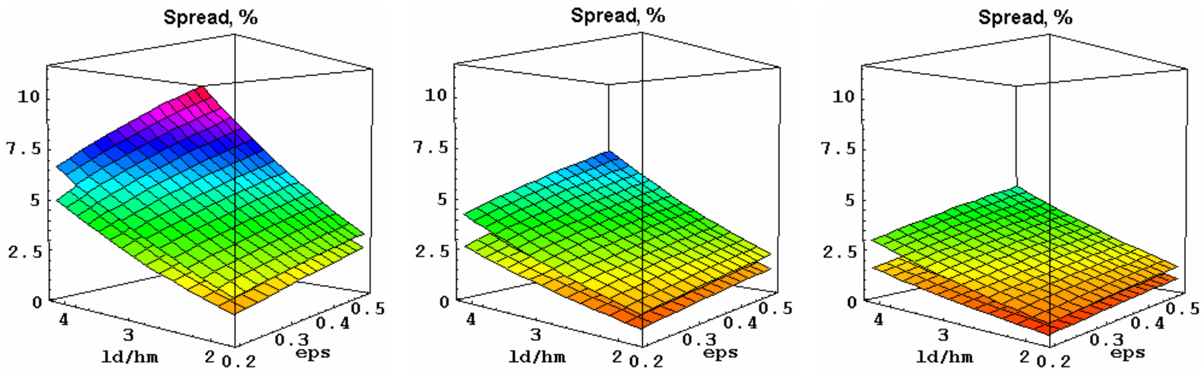


b0/h0 = 6   b0/h0 = 8   b0/h0 = 10

Figure 3: Typical side-form measurement

4 CALCULATION RESULTS AND THEIR VERIFICATION

For conducting the comparison of experimental data with those calculated by UBM the last version of the velocity field described in [5] is chosen. More details to design a velocity field are given in [6]. A comparison of the fit function gained by the measuring points and the fit function evaluated by the calculated values is given in figure 4. The top surface represents the fit function of the measuring points. The surface below denotes the calculated ones.



b0/h0 = 6 b0/h0 = 8 b0/h0 = 10

Figure 4: Nominal spread for austenitic steel 1.4301

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