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FURTHER STUDIES ON ASSESSING DUCTILE FRACTURE USING CONTINUOUS DAMAGE COUPLED TO AN ELASTO-PLASTIC MATERIAL MODEL

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Summary. *In recent years, failure assessment of metal forming operations using numerical simulation has been gaining momentum due to the rapid development of computational techniques for elasto-plastic problems. Furthermore, the recent proposals for numerical evaluation of ductile fracture has also potential for further improvement in tool design by means of reduction of forming defects. The present work aims at discussing possible improvements of the damage equation to better accommodate the transition from tensile to compressive stress states. Aspects of ductile failure are also presented.*

1 INTRODUCTION

In recent years, the rapid development of computational techniques for elasto-plastic problems at finite strains has favoured further research on predicting failure onset in general metal forming operations. The literature shows a wide range of strategies used to model metal fracture, amongst which the most referred are the Critical Open Displacement (COD), J-Integral and Continuum Damage Mechanics (CDM). The present work is based on the latter in association with the formulation proposed by Lemaitre[1]. This model correlates material degradation to void growth and has experienced great success to describe ductile failure in tensile-dominant problems[2]. However, in most metal forming operations compressive stress states prevail, which bring difficulties to the original damage formulation. The damage law is established so that the damage parameter increases with increasing plastic deformation and void growth, leading, eventually, to material failure. Compressive states cause, however, voids to close, yet leaving a material discontinuity. Such behaviour requires a distinctive description of the relationship between damage evolution and progressive plastic deformation. This work summarises a discussion on possible improvements of the damage equation to better accommodate the transition from tensile to compressive stress states.

2 DAMAGE MODELLING

Lemaitre's theory[1] assumes that a process is governed by a damage variable, D, which, physically, represents the net area of an unit surface cut by a given plane corrected for the presence of existing cracks and cavities. By assuming isotropic damage, the *effective stress tensor*, $\tilde{\boldsymbol{s}}$, and the corresponding yield function can be represented as

$$\tilde{\boldsymbol{s}} = \frac{\boldsymbol{s}}{1-D} \qquad \text{and} \qquad \Phi(\boldsymbol{s}, a, D) = \frac{s_{eq}}{(1-D)} - \left[s_{Y0} + q(a)\right] \tag{1}$$

where $\boldsymbol{s}$ is the stress tensor for the undamaged material, s_{Y0} is the initial yield stress, s_{eq} is the *von Mises* equivalent stress and a is the isotropic hardening parameter. The plastic flow equation and the evolution laws for internal variables are

$$\overline{\boldsymbol{D}}^{p} = \frac{3}{2}\frac{\operatorname{dev}[\boldsymbol{s}]}{(1-D)\, s_{eq}}, \qquad \dot{a} = \dot{g} \qquad \text{and} \qquad \dot{D} = \dot{g}\frac{1}{1-D}\left(\frac{-Y}{r}\right)^{s} \tag{2}$$

in which $\overline{\boldsymbol{D}}^{p}$ is the plastic deformation rate, $\dot{g}$ is the plastic consistency parameter, r and s are material damage parameters and Y is the *damage strain energy release rate*. The fully coupling between the damage evolution law and the elasto-plastic equations requires a simultaneous solution of equations (1) and (2), which, in the present work, is accomplished based on the algorithm proposed by De Souza Neto[3] for isotropic materials.

2.1 Damage strain energy release rate and void closure

The void closure effect has been introduced in the original model based on the principle that tensile and compressive principal stresses impose different effects on material degradation. The former leads to obvious void growth (and fracture) whereas the latter causes smaller material damage. One of the first attempts to utilise the concept in the context of metal forming simulation was presented by Andrade Pires *et al.*[4] based on a rigid-plastic material model and Andrade Pires *et al.*[5] in association with an explicit time integration scheme. Both solutions redefine the *effective stresses*, $\tilde{\boldsymbol{s}}$, and split principal stresses and ($-Y$) into tensile and compressive components,

$$\tilde{\boldsymbol{s}}^{-} = \frac{\boldsymbol{s}^{-}}{1-hD} \quad , \quad \boldsymbol{s} = \boldsymbol{s}^{+} + \boldsymbol{s}^{-} \quad \text{and} \quad (-Y)_F = (-Y)^{+} + h(-Y)^{-} \tag{3}$$

in which $(-Y)^{+}$ and $(-Y)^{-}$ represents the *damage strain energy release rate* computed using tensile, $\boldsymbol{s}^{+}$, and compressive, $\boldsymbol{s}^{-}$, principal stresses respectively and h is the void closure parameter $(0 \le h \le 1)$, so that

$$(-Y)_F = \frac{1}{2E(1-D)^2}\left[(1+n)\boldsymbol{s}^{+} : \boldsymbol{s}^{+} - n\,\mathrm{tr}[\boldsymbol{s}^{+}]^2\right] + \frac{h}{2E(1-hD)^2}\left[(1+n)\boldsymbol{s}^{-} : \boldsymbol{s}^{-} - n\,\mathrm{tr}[\boldsymbol{s}^{-}]^2\right]. \tag{4}$$

Equation (4) does not approximate the original (–*Y*) definition when *h* approaches unity. This study explores the possibility of different definitions of (–*Y*), which may accommodate a better transition between tensile to compressive states. Therefore, an alternative definition to equation (3a), $\tilde{\boldsymbol{s}}^{-} = h^{1/2}\, \boldsymbol{s}^{-}/(1-h\,D)$, is proposed, which represents exactly the original description when *h* = 1, and yet contains the additive split of (–*Y*),

$$\left(-Y\right)=\left(-Y\right)_F - h^{1/2}\,\frac{n}{E}\,\frac{\mathrm{tr}[\boldsymbol{s}^{+}]}{\left(1-D\right)}\,\frac{\mathrm{tr}[\boldsymbol{s}^{-}]}{\left(1-hD\right)}. \tag{5}$$

The direct comparison of equations (4) and (5) is presented in Figure 1, which pictures the evolution of the damage variable and *von Mises* equivalent stresses. The example represents a tensile stress state with positive and negative components of the principal stresses (*h* = 1).

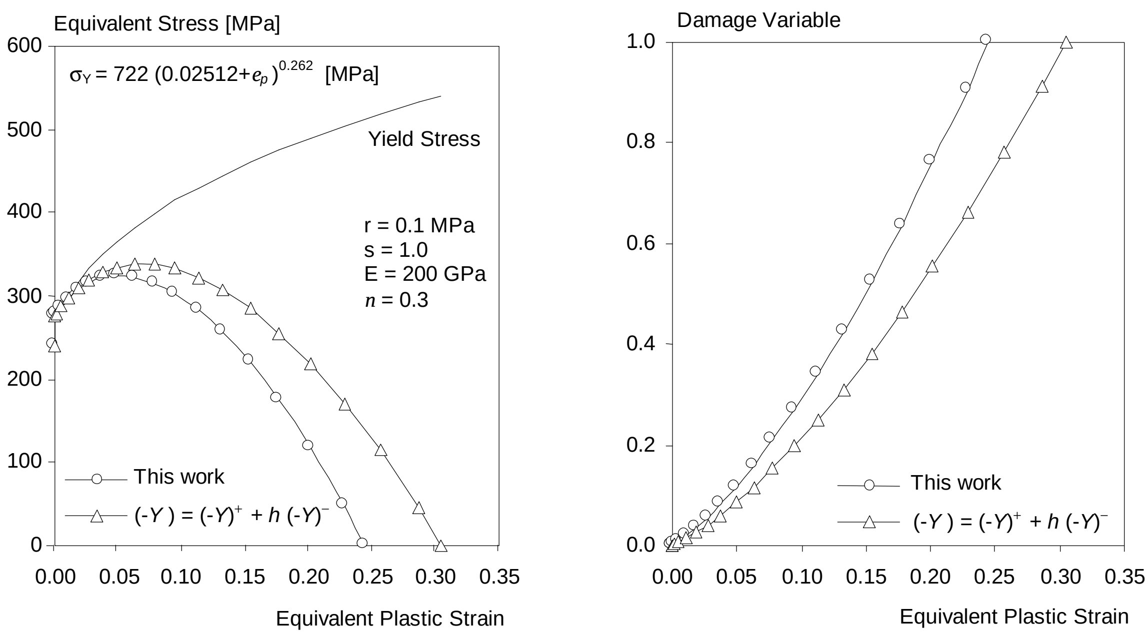

Figure 1: Equivalent stress and damage evolution

2.1 Ductile fracture

A ductile fracture criterion for damaged materials based on the *total damage work* was proposed by Vaz Jr. and Owen[7] aiming at applications which require high gradients at regions close to fracture onset,

$$I_{w_D} = \int_0^{D_c} (-Y)\, dD\,. \tag{6}$$

3 NUMERICAL EXAMPLES

The simulation of upsetting of a cylinder using elasto-plastic materials, although apparently simple, presents most characteristics of a typical forging operation. The cylinder height and radius used in the simulations are H = 30 *mm* and R_0 = 10 *mm* respectively. A total displacement U = 6.5 *mm* is applied to the upper die. Due to symmetry, only quarter of the billet is modelled, in which a quadrilateral structured mesh has been used. Material data for carbon steel AISI 1015 is inserted Figure 1 (h = 0.001). Rigid contact between workpiece and die is assumed with a Coulomb friction coefficient m = 0.2.

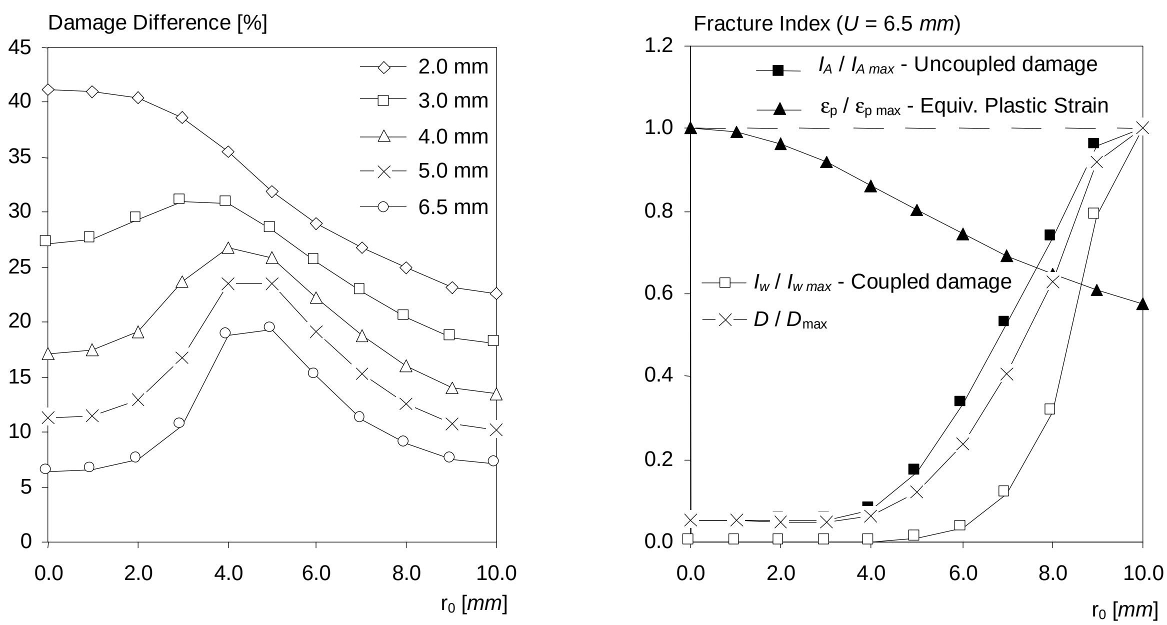

Figure 2: Evolution of the damage differences with U, and fracture index along the *R-R′* symmetry line

Evolution of the damage differences (equations 4 and 5) along the *R-R′* symmetry line is presented in Figure 2(a). The simulations show that differences decreases with increasing compression, i.e., smaller differences are associated to larger damage states.

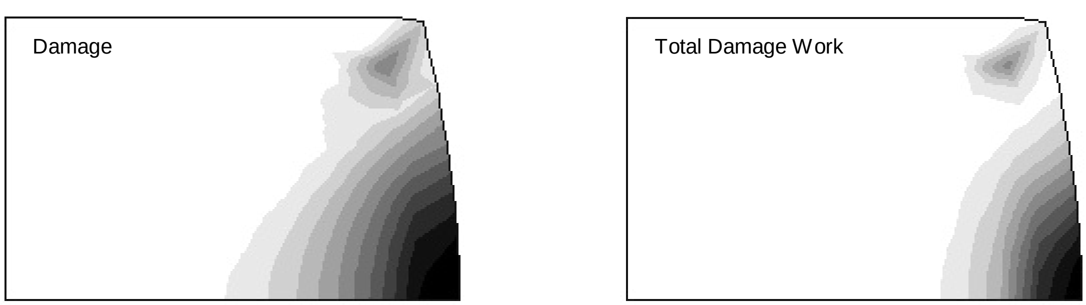

Figure 3: Damage variable (D_{max} = 0.1578) and Total Damage Work for U = 6.5 *mm*

The failure indicator computed by Equation (6) provides higher gradients then the damage variable itself (see Figure 3). It is also possible to define a failure indicator for classical von Mises materials based on the uncoupled integration of the damage law (equation 2c),

$$I_C = \int_0^{e_f} \left(\frac{1}{2\,E\,r} \right)^s \Big\{ \big[(1+n)\boldsymbol{s}^+ : \boldsymbol{s}^+ - n \,\mathrm{tr}[\boldsymbol{s}^+]^2 \big] + h \big[(1+n)\boldsymbol{s}^- : \boldsymbol{s}^- - n \,\mathrm{tr}[\boldsymbol{s}^-]^2 \big] - h^{1/2} n \,\mathrm{tr}[\boldsymbol{s}^+]\mathrm{tr}[\boldsymbol{s}^-]^2 \Big\}^s d\bar{e}_p \qquad (7)$$

Figure 2(b) exhibits the failure index, I / I_{max}, along the symmetry line for failure criteria defined according to equations (6), for fully coupled damage, and (7) for von Mises materials.

4 CONCLUDING REMARKS

The differences found in the simulations (Figure 2a) can be credited to stress states which present two compressive principal stresses on the plane *R-Z* and a tensile principal stress s_{qq}. Such characteristics is captured by the last term of equation (5). Despite the fact that classical fracture indicators fail when used in compressive problems, equations (6) and (7) have been successful to predict the correct location of fracture onset for coupled and uncoupled damage material models respectively.

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