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CRYSTALLOGRAPHIC HOMOGENIZATION ELASTOPLASTIC FINITE ELEMENT ANALYSES OF POLYCRYSTAL SHEET MATERIAL

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Summary. *In this study, a dynamic explicit finite element (FE) analysis code is newly developed by introducing a crystallographic homogenization method to estimate the polycrystalline sheet metal formability, such as the extreme thinning and "earing". This code can predict the plastic deformation induced texture evolution at the micro scale and the plastic anisotropy at the macro scale, simultaneously. This multi scale analysis can couple the microscopic crystal plasticity inhomogeneous deformation with the macroscopic continuum deformation. We study the plastic anisotropy effects on "earing" in the hemispherical cup deep drawing process of pure ferrite phase sheet metal.*

1 INTRODUCTION

The purpose of the present paper is to introduce the homogenization algorithm into our dynamic explicit FE code [1] to simulate the crystal texture evolution and plastic deformation induced anisotropy (earring) in the polycrystalline sheet metal forming process. The key points for this homogenization procedure are: (1)the specification of boundary condition for the micro crystal structure, which employs the velocity gradient of macro continuum on its boundary, and (2)the derivation of homogenized Cauchy stress at each integration point of macro continuum, which is evaluated in the micro crystal structure. The proposed crystallographic homogenization algorithm is implemented in the updated Lagrangian FE analysis code and the performance of this code is demonstrated by numerical examples of texture evolution and anisotropy (earring) prediction of polycrystal sheet metal.

2 FUNDAMENTAL EQUATIONS FOR THE MICRO AND MACRO STRUCTURES

In the homogenization procedure of the multi scale analysis, a Cartesian coordinates system ($x_i, i = 1,2,3$) and the deformation field $\dot{U}_i(x, y)$ are employed in the macro continuum, as shown in Fig.1(a). Simultaneously, for the pointwise attached micro structure, another Cartesian system (y_i, i = 1,2,3) and the deformation field $\dot{u}_i(x, y)$ of the unit cell are [2]

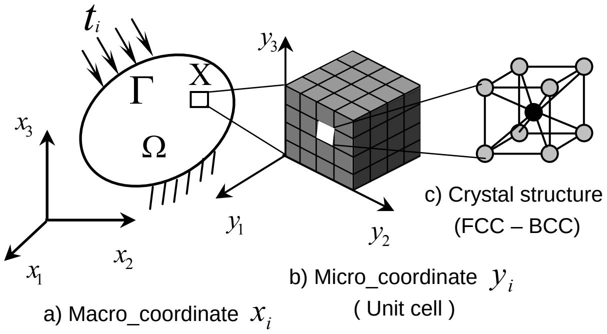

Figure 1 Macro and Micro structures.

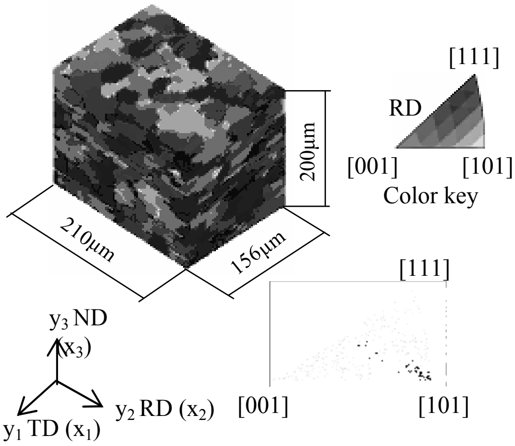

Figure 2 Crystal orientation map of SEM-EBSD measured a rectangular

introduced as shown in Fig.1(b) and Fig. 2. The two coordinate systems are related by the scale ratio λ as $y = x/\lambda$. According to the conventional homogenization procedure, the velocity of macro continuum is defined by the following equation:

$$\dot{U}_i(x, y) = \dot{u}_i^0(x) + \lambda\, \dot{u}_i^1(y) \tag{1}$$

where $\dot{u}_i^0(x)$ means the homogenized macro continuum velocity, and $\dot{u}_i^1(x)$ the perturbed velocity defined in the micro crystal structure, which becomes very small values. $\dot{U}_i(x, y)$ is determined by solving the governing equations and employing the corresponding boundary conditions in the macro continuum. The velocity $\dot{u}_i(x, y)$ in the micro crystal structure can be written in the coordinate system y as:

$$\dot{u}_i(x, y) = \frac{\partial \dot{u}_i^0(x)}{\partial x_j} y_j + \dot{u}_i^1(y) \tag{2}$$

The first term of r.h.s. of Eq. (2) obtained by multiplication of the coordinates y_i and the macroscopic velocity gradient, and $\dot{u}_i^1(y)$ the perturbed velocity in the micro structure.

(1) Constitutive equation for micro structure

The conventional elastic/ crystalline viscoplastic constitutive equation is employed to predict deformation of the micro crystal structure.

$$\dot{\gamma}^{(\alpha)} = \dot{\gamma}_0^{(\alpha)} \left[\frac{\tau^{(\alpha)}}{g^{(\alpha)}}\right] \left[\left|\frac{\tau^{(\alpha)}}{g^{(\alpha)}}\right|\right]^{(1/m)-1} \qquad g^{(\alpha)} = g^{(\alpha)}(\gamma), \quad \gamma = \sum_{(\alpha)} \left|\gamma^{(\alpha)}\right| \qquad \dot{g}^{(\alpha)} = \sum_{(\beta)} h_{\alpha\beta} \left|\dot{\gamma}^{(\beta)}\right| ,$$

$$h_{\alpha\beta} = q h(\gamma) + (1-q) h(\gamma) \delta_{\alpha\beta} \qquad h(\gamma) = h_0 \mathrm{sech}^2\left[h_0 \gamma / (\tau_s - \tau_0)\right] \tag{3}$$

Here, $\dot{\gamma}^{(\alpha)}$ denotes the shear strain rate at the slip system(α), $\tau^{(\alpha)}$ the resolved shear stress(RSS), $g^{(\alpha)}$ the reference shear stress, $\dot{\gamma}_0^{(\alpha)}$ the reference shear strain rate, m the material rate sensitivity, $h_{\alpha\beta}$ the hardening coefficients, q_{ab} the self and latent hardening matrices, τ_0, the critical resolved shear stress (CRSS) , τ_s the saturated RSS, h_0 the initial hardening ratio, n the strain hardening exponent.

(2) Derivation of homogenized stresses σ_{ij}^{H}

To evaluate the macroscopic stress tensor $\boldsymbol{\sigma}^{H} = < \boldsymbol{\sigma} >$, which means the homogenized stress tensor obtained by using the stresse $\boldsymbol{\sigma}^{e}$ of each element e of micro crystal structure – the unit cell -, which is obtained by averaging the Cauchy stresses $\bar{\boldsymbol{\sigma}}^{e}$ at the integration points as follows:

$$\boldsymbol{\sigma}^{e} = \sum_{i=1}^{N_g} | J_i \; | \bar{\boldsymbol{\sigma}}_i^{e} \tag{4}$$

where N_g is the number of integration points, and $| J_i |$ the jacobian. The macroscopic homogenized stresses are calculated over the whole finite elements in the unit cell. Hence,

$$\sigma^{H} = < \sigma > = \sum_{e=1}^{N_e} \sigma^{e} \Big/ \sum_{e=1}^{N_e} | J_e | \tag{5}$$

where N_e is the total number of the finite elements in the unit cell of micro crystal structure..

3 HEMISHERICAL CUP DEEP DRAWING

Table. 1 Material properties for crystal mode	
C_{11} (GPa)	237.0
C_{12} (GPa)	141.0
C_{44} (GPa)	116.0
τ_o (MPa)	75.0
h_o (MPa)	125.0
τ_s (MPa)	133.0

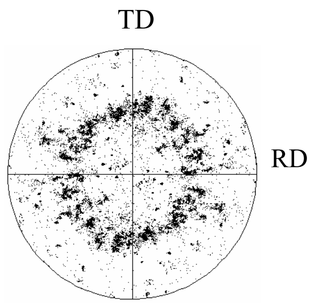

Figure 3 {100} pole figures by SEM · EBSD

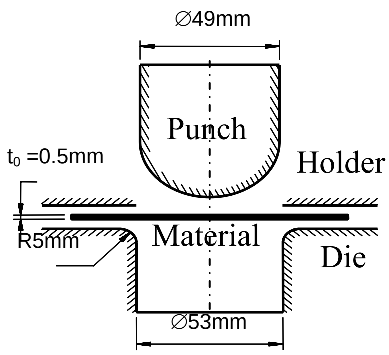

Figure 4 Tool set-up geometry and the circular disk.

This crystallographic homogenization FE analysis code has been applied to analyze the hemispherical cup deep drawing process, and predict the earing in the micro scale, due to the macroscopic plastic strain induced anisotropy, simultaneously caused by the crystal texture evolution at the micro crystal structure.

The initial texture of ferrite phase is presented in Fig.3. The material properties of BCC ferrite crystal of sheet metal are shown in Table.1. The tool set-up and geometry description are shown in Fig.4 with the disk of 100.0 mm diameter and 0.5mm thickness. The circular disk was divided into 224 macro finite elements, and that of micro finite elements attached to each macro finite element integration point is 27 (3x3x3). It means that at each macroscopic FE integration point an aggregate of 216 single crystal grains is allocated and the evolution of their orientation constitutes the representation of texture evolution.

Figs. 5 and 6 show comparisons of thickness strain distributions and deformed shapes, and the earring, obtained by both FE analyses and experimental observation. In general there is the presence of four-fold ears aligned with the rolling direction (0 degree), 90 degree, 180

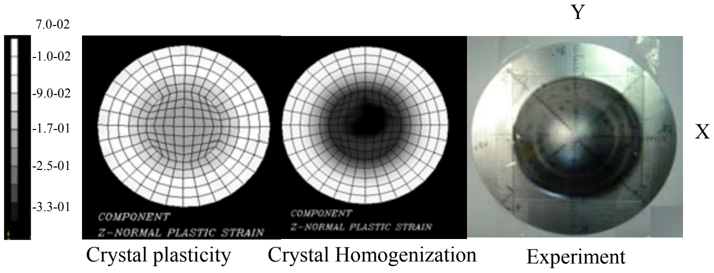

Figure 5 Distributions of thickness strain and experimental

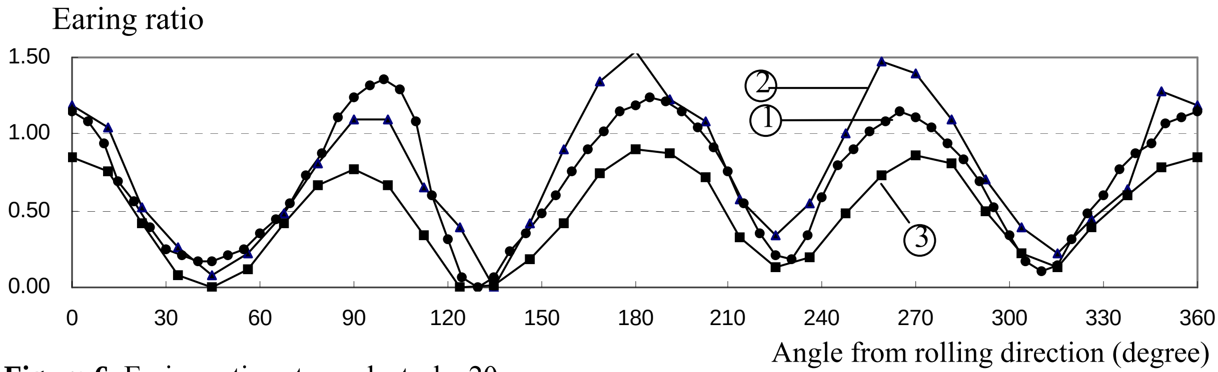

Figure 6 Earing ratios at punch stroke 20mm ;
(1) : experiment (2) : conventional crystal plasticity FE (3) : the crystallographic homogenization FE.

degree and 270 degree directions. Three results show good agreement of four-fold earing occurrence, but there are the discrepancy of earing ratio between two FE models and experiment. An abrupt change of earing ratio is observed for the conventional FE one due to the stiff constraints imposed on crystal grains, while for crystal homogenization model smooth transition of earing is obtained.

4. CONCLUSION

A crystallographic homogenization method has implemented in our dynamic explicit elastic/crystal viscoplastic FEM for the multi scale analysis of the polycrystal material to assess the sheet formability. It is confirmed that this homogenization FE can predict the crystal texture evolution induced by the plastic straining in the micsro scale and further “earing” of the circular disk in the macro scale.

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