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IMPLEMENTATION OF THE PRE-DISCRETIZATION PENALTY METHOD IN CONTACT PROBLEMS

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Summary. *A verification of the contact algorithm based on the pre-discretization penalty method was provided by means of numerical examples. It was shown that the proposed algorithm passed the convergence numerical tests including stability and contact patch test. An accuracy of the numerical solution was studied on the Hertz contact problem. More accurate results were obtained for quadratic elements whose incorporation into the contact analysis was granted by the pre-discretization procedure.*

In the finite element method the contact constraints can be introduced either before or after the finite element discretization has been performed, leading to the so-called pre-discretization or post-discretization techniques[1]. For example, the standard nodal algorithm belongs to the latter group. In the paper[2] we focused on the pre-discretization approach, showing this technique to lead naturally to the use of surface integration points as contactors. The method was shown to be consistent with the variational formulation of the continuum problem, which enabled easy incorporation of higher-order elements with midside nodes to the analysis. Furthermore, the pre-discretization approach preserved the symmetry of the algorithmic approximation with respect to contact boundaries. As a result, there was nothing like a master or slave definition of contact surface.

Now, let us briefly summarize the main idea of the pre-discretization method. Consider a system of two contacting bodies coming into contact with each other. The initial configuration is described by open domains Ω_1^0, Ω_2^0 with boundaries Γ_1^0 and Γ_2^0; $\Omega^0 = \Omega_1^0 \cup \Omega_2^0$, $\Gamma^0 = \Gamma_1^0 \cup \Gamma_2^0$. A contact is defined to occur if $\Gamma_1 \bigcap \Gamma_2 \neq \emptyset$ in the deformed state. The shared part of deformed boundaries $\Gamma_c = \Gamma_1 \bigcap \Gamma_2$ is referred to as the *contact surface* or the *contact boundary*.

Assuming that the contact surface Γ_c is known, the weak formulation takes the following simple form

$$\int_{\Omega^0} \delta\psi(\mathbf{u})\,\mathrm{d}\Omega = \int_{\Omega^0} \mathbf{f}\cdot\delta\mathbf{u}\,\mathrm{d}\Omega + \int_{\Gamma_S^0} \mathbf{t}\cdot\delta\mathbf{u}\,\mathrm{d}\Gamma + \int_{\Gamma_c} (\delta\mathbf{u}^{(2)} - \delta\mathbf{u}^{(1)}) \cdot \mathbf{p}\,\mathrm{d}\Gamma \qquad (1)$$

where ψ is the strain energy function, $\mathbf{f}$ the body force vector, $\mathbf{t}$ are the surface tractions prescribed on $\Gamma_S^0 \subset \Gamma^0$, and $\mathbf{p}$ is the contact pressure defined on Γ_c.

Applying the standard finite element method to the variational formulation (1), the discrete problem in the form of the governing equilibrium equations is obtained as

$$\mathbf{F}(\mathbf{u}) = \mathbf{R}(\mathbf{u}) + \mathbf{R}_c(\mathbf{u}) \tag{2}$$

where $\mathbf{F}$ is the vector of internal forces, $\mathbf{R}$ the vector of externally applied forces and

$$\mathbf{R}_c = \int_{\Gamma_c} \mathbf{H}^T \mathbf{n}\, p \,\mathrm{d}\Gamma \tag{3}$$

is the equivalent vector of contact forces acting on the nodal points for each body; $\mathbf{H}$ denotes the shape functions, and $\mathbf{n}$ the outward unit normal to the element boundary. Using the Gaussian quadrature, the integral (3) is approximated by

$$\mathbf{R}_c \simeq \sum_{IG=1}^{NIG} \mathbf{H}_{IG}^T \mathbf{n}_{IG}\, p_{IG} w_{IG} \det |J_{IG}^S| \tag{4}$$

with w_{IG} being the weighing coefficients, J^S the surface Jacobian and the subscript IG designates the point of evaluation. The unknown contact pressure p_{IG} can be viewed either as the Lagrange multiplier enforcing contact condition or the penalty traction $p_{IG} = \xi \pi_{IG}$, where ξ denotes the value of the penalty parameter and π_{IG} is the penetration determined at the *Gaussian integration points*. This makes a substantial difference to post-discretization formulations, in which all the quantities are processed at the nodes. Note that the post-discretization formulation imposes no restriction on the element types used in the analysis.

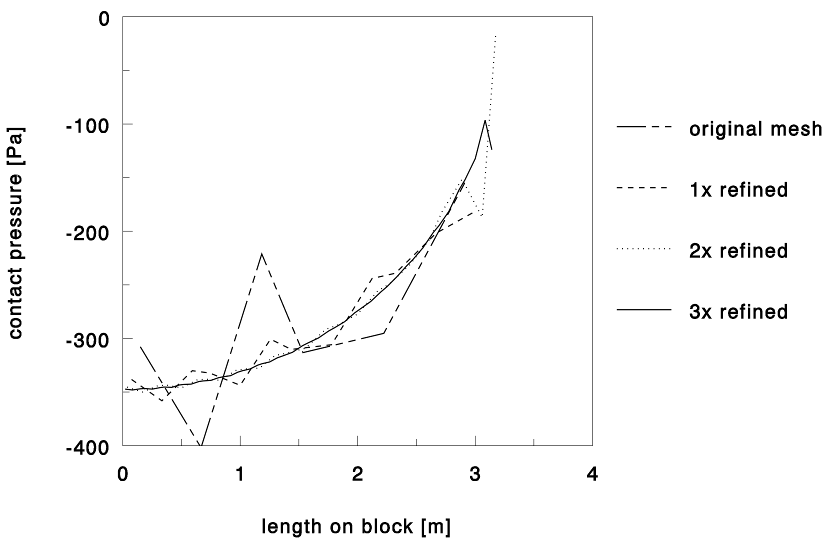

Figure 1: Contact pressure distribution on elastic block.

In this work we focused on the numerical verification of the proposed algorithm. First, the convergence tests including stability and contact patch test were performed. The contact patch test presented in Reference[3]was used to test the ability of the proposed contact formulation to exactly transmit constant normal pressure between two contacting surfaces regardless of their discretization. The different non-matching meshes[3] involving combination of linear and quadratic elements were used. The proposed algorithm passed the contact patch test for both linear and quadratic elements and their combination.

The stability of algorithm was tested by means of a rigid punch problem proposed in References[4,5] where a rigid cylindrical punch was indented into a rectangular elastic block. The geometry, finite element model with eight-node quadratic elements and material parameters were taken from Reference[4].

The contact pressure distribution on the surface of elastic block is plotted in Fig. 1 when the original mesh was thrice regularly refined. In contrast to the predicted instability for 3-point Gaussian quadrature rule reported by Oden[4] a smooth approximation of the contact pressure was obtained. The deformed configuration of the model for the finest mesh is illustrated in Fig. 2 showing the non-matching grids in the contact interface.

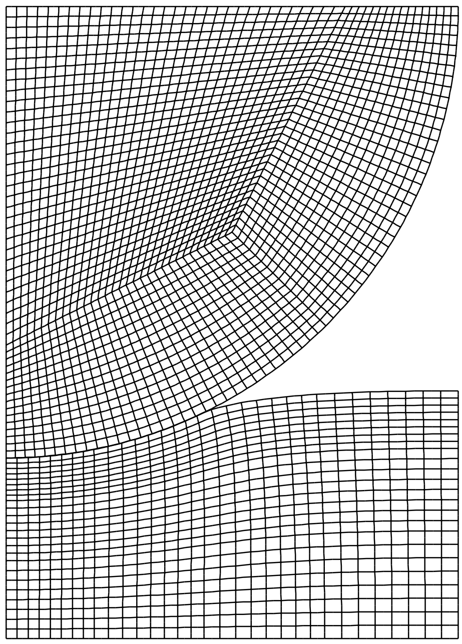

Figure 2: The deformed configuration of the model for the finest mesh.

It was concluded that stability test was passed. No stress oscillations of the type reported by Oden[4] were encountered. A possible explanation may be that in the linearized formulation[4] the normal vector is defined at the initial configuration (or at the beginning of an increment) whereas in the present formulation the normal is implicitly computed at the converged state.

Next, accuracy of the numerical solution was studied on the Hertz contact problem of two infinitely long parallel cylinders. The influence of various values of the penalty parameter and different numbers of elements in contact was investigated. The performance of linear and quadratic elements is compared in Fig. 3 where the contact area is modelled with two quadratic and four bilinear elements, respectively. It should be pointed out that the mesh with two quadratic elements per contact area yielded more accurate results than the one composed of bilinear elements, using even less number of degrees of freedom.

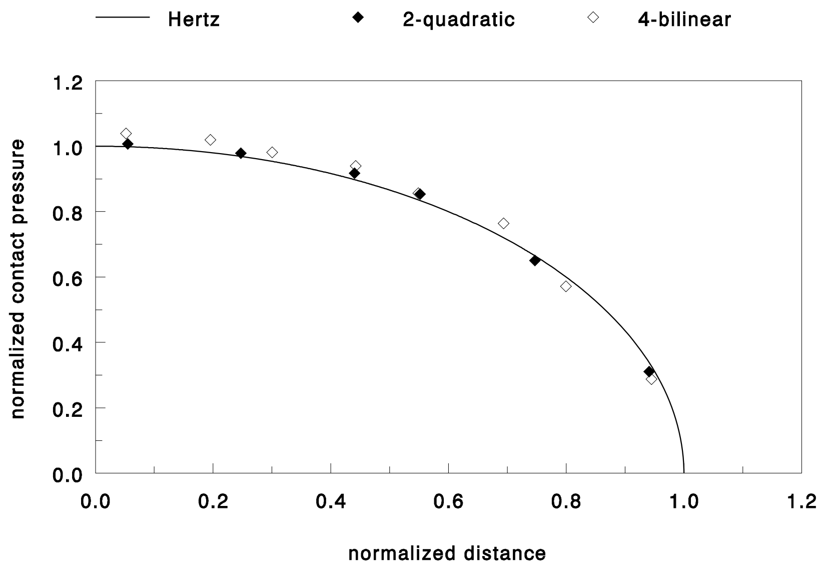

Figure 3: Comparison of solution accuracy for quadratic and bilinear elements.

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