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INTER-ELEMENT FORCES IN THE FEM FORMULATION AND IMPLICATION FOR SMOOTHING PROCEDURES

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Summary. *This paper describes a method for the "a posterior" determination of the inter-element forces and stresses in the standard FEM-displacement formulation. These stresses can be used to predict the crack initialization along mesh lines, or simply as values for graphic post-processing ("smoothing procedure").*

1 INTRODUCTION

The simulation of crack propagation within a FE domain represents one of the key aspects in the area of numerical modelling of damage and fracture. A recent approach[1], 2 consists in considering each line in the mesh as a potential crack, evaluating inter-element forces and stresses, and allowing crack opening/sliding as appropriate strength criteria are exceeded. Evaluation of stress tractions transmitted across mesh lines, however, in general requires to insert interface elements with double nodes. Otherwise, stress evaluation is only trivial at mid-side nodes of quadratic elements[1].

2 FORMULATION

At an "interior corner node" of a 2D FE mesh (node not at the domain boundary, which sits at the corner of surrounding elements), equilibrium equations alone are not enough to determine the inter-element forces $\mathbf{r}^{(k)}$ ($k = 1, n$ of elements converging at node) or stress tractions $\mathbf{t}^{(k)} = \frac{1}{\Omega^{(k)}}\mathbf{Q}^{(k)}\mathbf{r}^{(k)}$ ($\mathbf{Q}^{(k)}$ =rotation matrix, $\Omega^{(k)}$ =contributing area), remaining one undetermined force vector $\bar{\mathbf{r}}$. The missing condition can be obtained using an objective error function Φ, defined as

$$\Phi = \sum_{k=1}^{N}(\boldsymbol{n}^{(k)}\mathbf{T}\boldsymbol{n}^{(k)} - \sigma^{(k)})^2 + (\boldsymbol{t}^{(k)}\mathbf{T}\boldsymbol{n}^{(k)} - \tau^{(k)})^2 \tag{1}$$

being $\mathbf{T}$ the stress tensor at the point, $(\sigma^{(k)}, \tau^{(k)})$ the components of $\mathbf{t}^{(k)}$, and $\mathbf{n}^{(k)}$ the corresponding normal vectors. A double minimization of the objective function Φ, first with respect to the components of $\mathbf{T}$ and then to the components of $\mathbf{r}$, provides the required vectorial equation that solved together with the equilibrium conditions leads to the values of $\mathbf{r}$, $\mathbf{T}$ and $\mathbf{t}^{(k)}$. In the case of nodes at the domain boundary, the problem is simpler since equilibrium alone leads to the problem solution. More details of the formulation can be found in literature[3].

3 APPLICATION EXAMPLES

3.1 Non-uniform stress state in square specimen

A square specimen under gravity load is considered in fig. 1. With linear elements, progressively refined meshes lead to progressively more accurate tractions and stress tensor. With quadratic elements, exact solution is obtained from the simplest mesh (a).

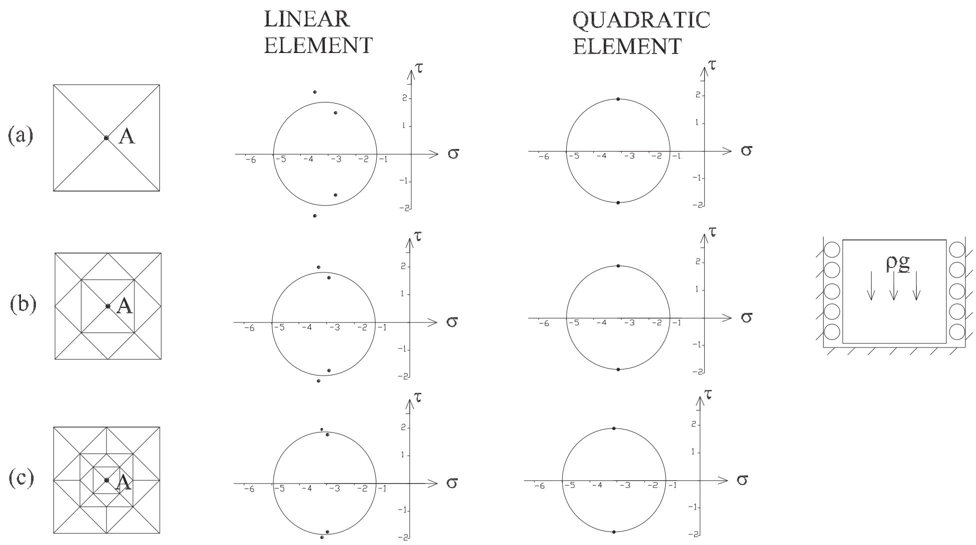

Figure 1: Non-uniform stress state in square specimen

3.2 Uniform stress state in square specimen

A uniform stress load is applied on the linear-triangular mesh as shown in fig. 2; on the right side, the stress state obtained at the interior node A is presented in terms of Mohr's circle representing $\mathbf{T}$, and a set of dots representing the stress tractions along the various inter-element planes. The error Φ is equal to zero in this case (exact solution).

3.3 Stress at boundary node in four-point ending test

A beam subject to four-point bending test is discretised in fig. 3 with different meshes from b) to f). The analysis is carried out using linear and quadratic elements. The

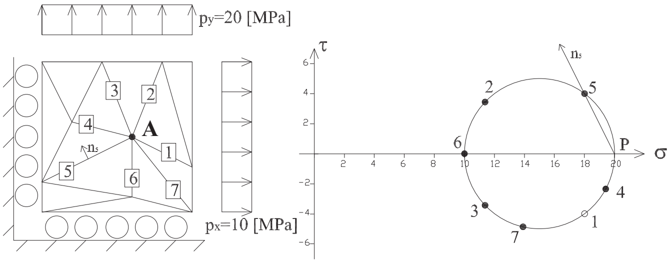

Figure 2: Uniform stress state in square specimen

stress state around boundary point A is computed using the proposed method, as well as using a standard smoothing procedure based on the average of the stresses at the Gauss points of the surrounding elements. The results are plotted in the bottom graphic. For quadratic elements, the proposed procedure yields exact results from the coarsest mesh, while clear convergence is obtained for linear elements. In both cases, the proposed procedure provides better accuracy than standard smoothing.

4 CONCLUDING REMARKS

The proposed post-processing technique makes it possible to evaluate inter-element tractions and stress state at the "corner nodes" of a standard FE-displacement calculation. The example results obtained show good convergence, or even exact solution for simple stress distribution, and always more accurate than traditional smoothing. More details of the procedure and additional cases and examples can be found in literature[3]. Current work aims to a 3D extension of this technique, as well as to its application in the contest of crack nucleation and propagation along mesh lines without the use of a double-node interface elements.

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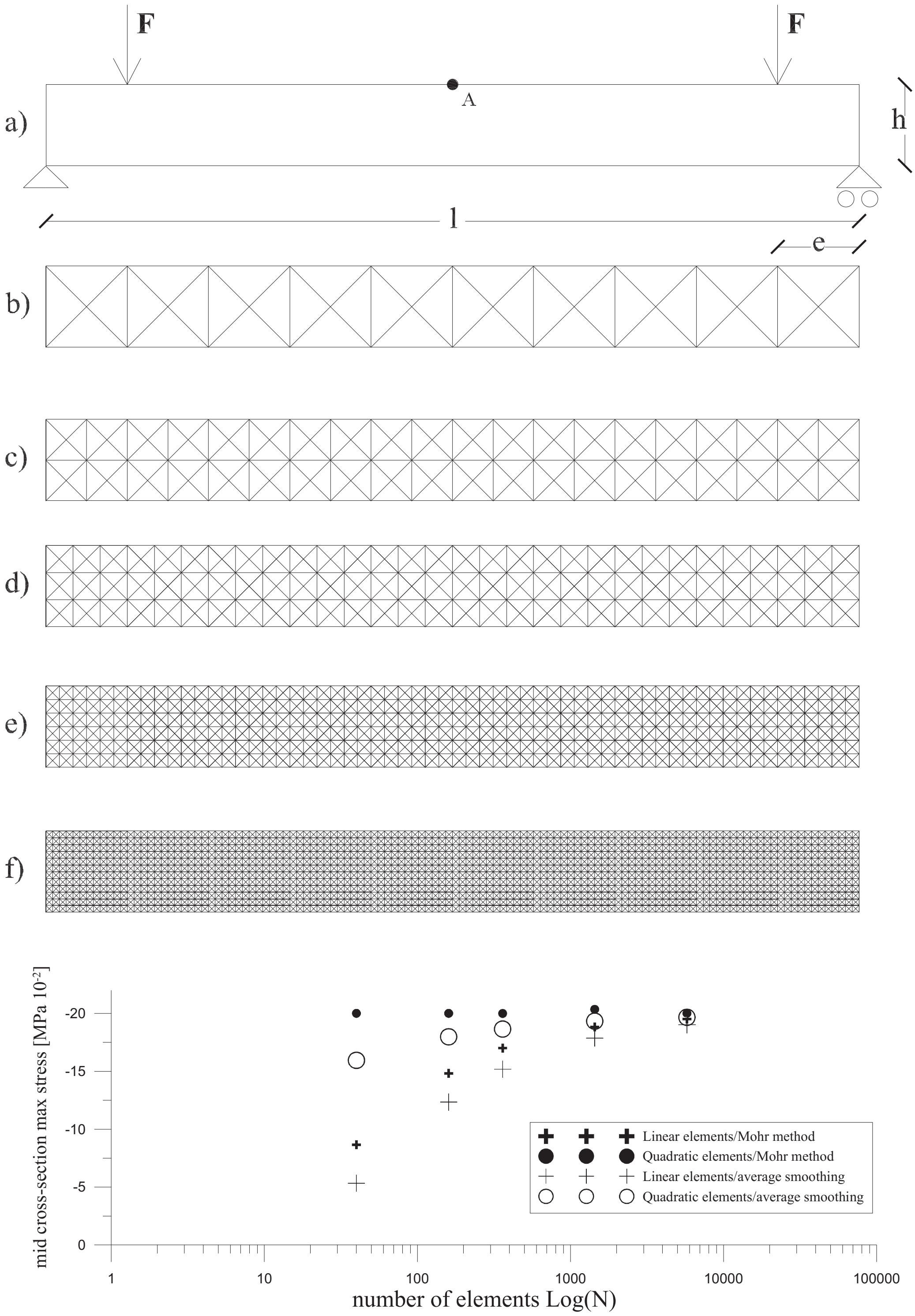

Figure 3: Stress at boundary node in four-point ending test