General Framework for Analytic Sensitivity Analysis for Inverse Identification of Constitutive Parameters

Benny Endelt* **and Karl Brian Nielsen****

The Sheet Metal Forming Group
Department of Production, Aalborg University, Denmark
*e-mail: endelt@production.aau.dk
**e-mail: kbn@production.aau.dk

ABSTRACT

The dominating material test for sheet metal is the uni-axial tensile test, the test does however, cover a very limited deformation-range. The demand for alternative identification schemes for identification of constitutive parameters is getting more pronounced as the complexity of the constitutive equations is increasing i.e. the number of parameters subject to the identification is increasing. Furthermore, the constitutive parameters can is some cases not be determined solely from the uni-axial tensile test e.g. for an exact identification of the coefficients for Barlat's Yld2000-2d and Yld96, data for the biaxial yield stress is needed.

1 INTRODUCTION

A general framework for inverse identification of constitutive parameters associated with sheet metal forming is proposed in this paper. Through minimization of the least square error between a experimental punch force, F^{em}, sampled from a deep drawing operation and modeled punch force, F^{fit}, produced from a coherent finite element model.

The framework for analytic sensitivity analysis is presented and some implementation issues are covered as well. The sensitivity analysis is based on the assumption that the total punch force can be separated into a plastic contribution and a contribution from friction, thus $F^{total} = F^{plastic} + F^{fric}$, where an expression for the plastic force can be derived from incremental plastic work, dw. The analytic sensitivity scheme is implemented in a user-defined material routine in LS-Dyna. Further, a simple strategy for analytic sensitivity analysis of the friction coefficient is presented.

Finally, the proposed inverse modeling scheme was tested on data produced from a square deep-drawn cup, where the punch-force was sampled during the drawing, the tool dimensions correspond to the Benchmark tool for the NumiSheet'93 conference. The constitutive parameters were identified through iterative minimization of the least square error between the experimental punch force and the coherent modeled punch (Finite element). Barlat's 2D criterion [1] was applied for modeling the anisotropic behavior and an exponential hardening behavior is assumed. The solution of the proposed inverse problem is highly sensitive to the capability of the applied inverse solver or optimization scheme, e.g. appropriated bounds on the solution space, the strategy for steep size regulation etc.

A least square formulation of the object function was applied utilizing the structure of the object function, for further introduction to the inverse solver see [2, 5, 4, 3].

2 ANALYTIC SENSITIVITY

The punch force as a function of punch displacement reflects the energy input to the system. The basic idea is that two identical punch forces can only be obtained with the same set of constitutive parameters. The constitutive parameters were identified, by iteratively minimizing the objective function, $f(\boldsymbol{x}) = \boldsymbol{r}^T\boldsymbol{r}$, where the residual vector was defined as $r_j = (F_j^{fit} - F_j^{em})$, where F^{em} represents the empirical punch force, and F^{fit} represents the fitted data and was produced by LS-Dyna .

The objective was to minimize $f(\boldsymbol{x})$. The minimum was identified through gradient based optimization techniques, thus an analytical expression, representing the Jacobian matrix which holds the derivatives $\frac{\partial r(\boldsymbol{x})}{\partial \boldsymbol{x}}$ was needed. Barlat's 2D yield criterion $\Phi(a, h, p, M)$ describes the relation between the equivalent stress $\bar{\sigma}$ and Lankford's coefficients which can be expressed through the parameters a, h, p and M. Further, a similar relation exists between the equivalent stress and the exponential hardening law, described by the strength coefficient K and hardening coefficient n.

The following scheme was applied to establish the relation between $\bar{\sigma}$ and F^{fit}. The total punch force from the model F^{fit} is defined as:

$$F^{fit} = F^{plast} + F^{fric} \tag{1}$$

where F^{plast} denotes the plastic force, i.e. the contribution to the total punch force from plastic deformation of the blank. The contribution from friction is denoted F^{fric}. Independency between F^{plast} and F^{fric} was assumed. The incremental plastic work dw for one element in the finite element model can be expressed as:

$$dw = \bar{\sigma} d\bar{\epsilon} \tag{2}$$

The total plastic work increment was calculated by summation of the contribution for each element. The plastic force F^{plast} can now be stated as:

$$F^{plast} = \sum_{i=1}^{n_{el}} \frac{V_i \bar{\sigma}_i d\bar{\epsilon}_i}{\Delta s} \tag{3}$$

where n_{el} represents the number of elements for the blank, V_i blank and the number of integration points, respectively. V_i represents the element volume and Δs is the increment for the punch displacement Δs.

Under the assumption that F^{plast} is independent of the friction coefficient, the Jacobian matrix can now be defined as:

$$J_{ij} = \left[\frac{1}{\Delta s_i} \sum_{k=1}^{n_{el}} V_k d\bar{\epsilon}_k \frac{\partial \bar{\sigma}_k}{\partial x_j} \right] \quad \begin{aligned} i &= 1, 2, \ldots, m \\ j &= 1, 2, \ldots, n \end{aligned} \tag{4}$$

The Jacobian is a $(m \times n)$ matrix, where n is the number of constitutive parameters, m is the number of points in the residual vector. The derivatives of the residual vector ($r_i(\boldsymbol{x}) = F_i^{fit} - F_i^{em}$) with respect to $\boldsymbol{x} = [K, n, a, h, p, M]$ were assumed to be independent of the friction coefficients.

The friction was modeled using Coulomb's friction law ($F^{fric} = \mu F^N$), and the sensitivity for the friction coefficient was evaluated analytically using a very simple strategy. Where the normal force is approximated using the following relation:

$$\tilde{F}^N \simeq \frac{F^{fit} - F^{plast}}{\mu} \tag{5}$$

using the above approximation of the normal force, the sensitivity for the friction coefficient can be defined as:

$$\frac{\partial F^{fric}}{\partial \mu} = \tilde{F}^N \tag{6}$$

assuming equal friction coefficients for all the contact interfaces between the tool parts and the sheet.

	Finite difference	Analytic
$\frac{\partial F^{fric}}{\partial \mu}$	-595.1	-618.4

Table 1: Comparison between analytically defined friction sensitivity and friction sensitivity approximated by finite difference, using a finite difference increment $\delta\mu = 0.001$.

For comparison between the proposed analytical friction sensitivity scheme and the finite difference approximation, see table 1, where an increment $\delta\mu = 0.001$ was used for the finite difference approximation. In conclusion, the difference between the two sensitivity schemes was insignificant, thus, the simple analytical definition of the friction sensitivity seemed valid.

3 IMPLEMENTATION

The analytic sensitivity scheme was implemented as an user defined material routine in LS-Dyna version 970 double precision, the additional calculation steps compared to a normal material routine are summarized in table 2. The material routine is based on the implementation by K.B. Nielsen [6].

Initialize the element thickness e_{th}, number of integration points h_{ipt}, material parameters and the Jacobian dump interval t_{dump}. And define the variables Jacobian $\boldsymbol{J}$, volume $\boldsymbol{V}$, plastic work $\boldsymbol{dw^p}$ and the counter $k = 1$.
if ($t = 0.0$ and j=1) **then**
    Calculate the volume of the i'th element and store the result in V_i.
end if
Update the stress and incremental plastic strain $d\bar{\epsilon}$ for the i'th element and j'th integration point, see e.g. [6].
if ($t > t_{count}$) **then**
    Write the Jacobian $\boldsymbol{J}$ and incremental plastic work $\boldsymbol{dw^p}$.
    $k = k + 1$
    $t_{count} = t_{count} + t_{dump}$
end if
Update the incremental plastic work array $dw_k^p = dw_k^p + \frac{V_i}{h_{ipt}} \bar{\sigma} d\bar{\epsilon}$
Update the Jacobian $\boldsymbol{J}$
$J_{k1} = J_{k1} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial K}$.
$J_{k2} = J_{k2} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial n}$.
$J_{k3} = J_{k3} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial a}$.
$J_{k4} = J_{k4} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial h}$.
$J_{k5} = J_{k5} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial p}$.
$J_{k6} = J_{k6} + \frac{V_i}{h_{ipt}} d\bar{\epsilon} \frac{\partial \bar{\sigma}}{\partial M}$.
End.

Table 2: Illustration of the additional calculation compared to a normal material routine. The number of rows in the Jacobian matrix was controlled by t_{dump} and the termination time. The Jacobian and plastic work array is updated for each time step. Finally, the volume of the element is assumed equally distributed over the integration points.

4 RESULTS

The process was simulated with the explicit code LS-Dyna version 970. The process time was scaled to 20 milliseconds and a forced time step $\delta t = 2.0\ 10^{-7}$ was applied, utilizing mass scaling to improve computational efficiency [6]. The blank was modeled using 1368 Belytschko-Tsay shell elements with 7 intergrarion points through the thickness.Due to symmetry only a quarter of the cup was modeled.

De04 was used for the sheet material with an initial thickness of 0.75[mm] and a diameter of 160[mm] . A square deep-drawing tool, corresponding to Benchmark tool for the NumiSheet'93 conference, was applied and the produced cup was 38[mm] high and a blank-holder force of 20[kN] was applied.

Two initial sets of parameters were tested, the initial parameters and the corresponding identified solution are represented in table 3.

Lankford's coefficients

	Uni-axial	Initial 1	Inverse 1	Initial 2	Inverse 2
μ	-	0.1	0.1129	0.1	0.1139
K	544.68	550	584.29	550	586.57
n	0.239	0.25	0.2284	0.24	0.2286
R_{00}	1.758	1.8	1.9106	1.9	1.9619
R_{45}	1.287	1.4	1.1952	1.4	1.2137
R_{90}	2.028	2.3	1.8196	2.2	1.7869
M	-	6.0	6.9437	6.0	6.9649

Table 3: Inverse identified parameters, where equal friction coefficients are assumed between the tool parts and the blank.

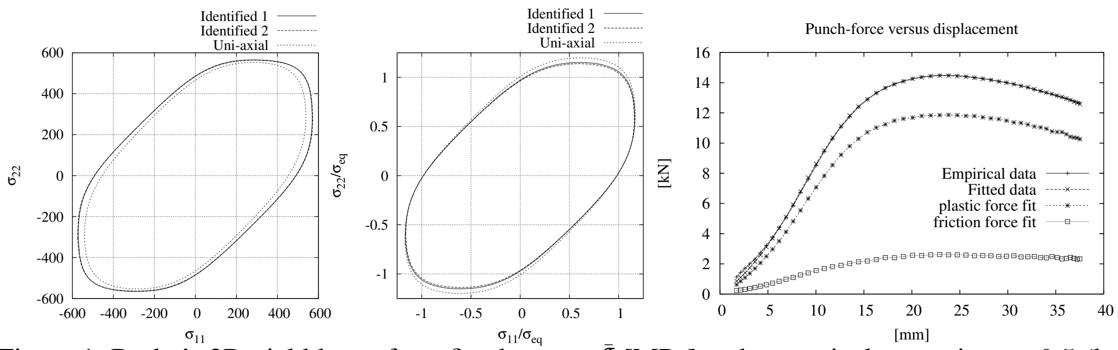

Figure 1: Barlat's 2D yield locus for a fixed $\sigma_{12} = \frac{\bar{\sigma}}{10}$[MPa] and an equivalent strain $\bar{\epsilon} = 0.5$ (left) and the normalized locus (center). Finally to the right the fitted and empirical punch force.

5 CONCLUSION

An insignificant difference in the identified parameters was observed, the term insignificant is used due to the close resemblances between the identified yield loci, they are close to identical, see figure 1 and table 3. Further, a very good fit between the experimental and fitted punch force was achieved, see figure 1, where the residuals were reduced to ± 0.1[kN]over the majority of the punch stroke, whereas the largest error (0.3 [kN]) was observed within the first 5[mm] of the punch displacement, the relatively large error in this region may indicate that the Coulomb's friction model based on a single friction coefficient is unsuitable in this region.

A difference between the uni-axial parameters and inverse parameters was observed, both with respect to anisotropy and hardening parameters. The uni-axial tension tests seem to underestimate the strength coefficient K and to overestimate the hardening coefficient n; and as a result the yield stress is underestimated, thus a significant difference between the uni-axially and inversely identified yield loci.

6 FURTHER WORK

Verification of the results; does the inverse procedure produce more reliable simulation results? - i.e. prediction of the thickness distribution, geometric properties, strain distribution etc. Furthermore an implicit version is under implementation, as we in the explicit implementation was forced to use a very modest time step, $\delta t = 2.0\ 10^{-7}$, to ensure stable convergence and sufficient quality of the gradients.

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