

# Analysis of Stress Partitioning in Biphase Mixtures Based on a Variational Purely-Macroscopic Theory of Compressible Porous Media: Recovery of Terzaghi's Law

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## ABSTRACT

In geotechnics, Terzaghi's law [1] is considered the 'experimental true form' of stress partitioning for saturated granular soils, and valid under the assumption that the intrinsic compressibility of the constituent materials is negligible when compared to the compressibility of the porous skeleton. However, when phase compressibility is taken into account, most of the currently available poroelastic theoretical frameworks predict stress partitioning mechanisms that deviate from Terzaghi's expression of effective stress [2].

In this contribution, the mechanics of stress partitioning in two-phase porous media is analysed as predicted by a variational purely-macroscopic theory of porous media (VMTPM) with compressible constituents [3]. Particular emphasis is given to those applications where an undrained behaviour is relevant (such as in, e.g., soil consolidation and fast deformations in the cartilage tissue).

It is shown, as a general property of rational continuum mechanics, that Terzaghi's effective stress law is recovered as the characteristic stress partitioning law, that a biphase medium naturally complies with, as limit undrained conditions are approached (low permeability or fast loading). This result is predicted with minimal constitutive hypotheses and no assumptions on internal microstructural features of a particular class of material, and, notably, turns out to be unrelated to phases intrinsic compressibility.

Simulations of compressive stress relaxation tests are also performed via a combined analytical-numerical integration technique, based on the employment of Laplace antitransforms carried out numerically via de Hoog et al.'s algorithm [4]. The behaviour of the system observed in the numerically computed transition from drained to undrained flow is also investigated. It is shown that Terzaghi's law is respected by the numerical solutions as the limit of undrained conditions is approached.

## REFERENCES

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