IMEX Lagrangian and ALE Methods for Multiphysics Systems

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ABSTRACT

An appealing methodology for simulating multiphysics shock-hydro systems is the Arbitrary Lagrangian Eulerian (ALE) method. One approach is to implement this using an operator splitting type-procedure: (1) take Lagrangian step following the advected/pressure wave time-scale, (2) remesh and remap physical quantities, and (3) solve for the remaining physics on the remapped mesh. This approach has been successful in many practical applications of interest. However, for multiphysics regimes characterized by multiple interacting time and spatial-scales this splitting may have stability limitations that force the time step to be too small to efficiently follow the time scales of interest. An alternative formulation to ALE is to solve the system monolithically. For explicit time stepping this approach may have similar challenges with disparate and overlapping time-scales. Additionally fully implicit solution methods may ultimately be deemed too costly because of the expense and challenge of solving large-scale multiphysics systems.

An appealing approach to resolve these issues is to develop an Implicit-Explicit (IMEX) time stepping formulation of the monolithic ALE equations. IMEX schemes (see [1,2]) are mathematically well-structured approaches that permit the separation of timescales within a fully coupled residual formulation. These methods support the development of high-order in time integration methods and adjoint methods for sensitivity analysis, both of which can be challenging in the operator-split regime.

This talk will review the structure of IMEX Runge-Kutta schemes. These time integrators will be applied to a VMS stabilized Lagrangian formulation of the Euler equations (see [3,4,5]). We will also present results demonstrating the feasibility of a monolithic multiphysics ALE scheme using IMEX time integrators.

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