Adaptive Isogeometric Finite Element Analysis of Groundwater Flow in Heterogeneous Porous Media

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ABSTRACT

Steady-state groundwater flow in heterogeneous porous media is studied numerically using isogeometric finite element analysis with adaptive mesh refinement. The governing equations are presented in the strong and weak forms. Galerkin's method is used for the discretization of the weak forms.

B-splines and NURBS do not allow for local refinement due to the tensor product structure in the mathematical formulations. Hence, LR B-splines, first proposed by Dokken et al. (2013) and later applied in isogemetric analysis by Johannessen et al. (2014), are used in the adaptive analyses. A brief introduction to the mathematical background behind LR B-splines is presented. The associated adaptive refinement strategies are also discussed.

The numerical simulation is performed on a square domain with a heterogeneous hydraulic conductivity field. The hydraulic conductivity values are assumed to obey a lognormal distribution and a correlated random field is generated for the numerical simulation domain.

The need for adaptive mesh refinement is determined by a posteriori error estimates in the analysis. The error estimates are obtained based on recovery of the projected finite element solution. The numerical study is performed for varying polynomial degree orders. The convergence rates in the energy norms from adaptive simulations are compared with results from simulations based on uniform mesh refinement.

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