Coupled Contact Problems in Piezoelectricity – Mathematical Modelling and Boundary Element Approximation of Higher Order

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ABSTRACT

In this contribution we are concerned with unilateral contact problems with friction in piezoelectricity.

First we extend the mathematical modelling and solvability analysis given in [1] for linear transmission problems for piezoelectric elastic materials to this class of nonlinear free boundary value problems. Similar to [2] for micropolar hemitropic elasticity, we apply potential methods which transform the weak formation of these contact problems to boundary variational inequalities involving boundary integral operators. Based on our boundary variational inequality approach we prove existence and uniqueness theorems for weak solutions. We prove that the solutions continuously depend on the data of the original problem and on the friction coefficient. We treat also the case when the body is not fixed, but only submitted to some forces along some part of the boundary and is in unilateral frictional contact with a rigid foundation. In this situation we present necessary and sufficient conditions of solvability.

Then we study the numerical approximation of these coupled contact problems. In virtue of our boundary variational inequality approach we can reduce the spatial dimension and employ finite element discretization on the boundary, only, what leads to the numerical treatment by the well-known boundary element method (BEM). Since we here admit approximations of higher order, we are confronted with nonconforming approximation. To this end we extend the numerical analysis given in [3] for a scalar model problem to the full vector case of coupled piezoelectricity.

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