Coupled mathematical and computational models of autoregulation mechanisms in vascular districts

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ABSTRACT

It is well known that blood flow rate in vessels and tissue metabolism are tightly coupled processes. Increased oxygen demand due to metabolic activity in the tissues proportionally increases arterial blood flow (metabolic autoregulation); conversely, at constant metabolic rate, an altered arterial oxygen content reciprocally alters blood flow, leaving total oxygen delivery constant. In addition, blood flow remains constant over a certain range of perfusion pressure values (myogenic autoregulation). Alteration of blood flow is in any case obtained by an active increase or decrease in the vessels' diameter, occurring in muscular arterioles. The interaction of these mechanisms results into a complex vascular dynamics, mediated by vasoactive substances, mainly nitric oxide released by the vascular endothelium. Impaired autoregulation is recognized to be a major risk factor for several diseases, including hypertension, ischemic events and several celebral diseses.

In literature, there exist several mathematical models that address autoregulatory mechanisms in a single vessel [1] or based on a lumped representations [2]. However, very few examples exists of these studies on complete networks [3] and do not address in a consistent manner the tissue role.

In this work, we present a mathematical and computational model of autoregulationoccurring in a vascular network embedded in a metabolically active tissue. The vessel network is represented as a connected graph on which blood flow is approximated using a generalizedPoiseuille's law. Oxygen is advected throughout the network (in its free and haemoglobin-bounded forms) and is delivered to the surrounding tissue according to the actual metabolic requirements of this latter. The vessel and tissue sub-domains result thus to be coupled problems. In turn, active autoregulation, which enters into play to maintain an homeostatic oxygenation levels, is modeled as a forcing term in the mechanical balance of the arterial wall in radial direction (see also [4]), according to a chemical pathway triggered by oxygen and nitric oxide levels in the wall.

Several physiological and pathological conditions will be examined, outlining the influence of the different parameters of the model.

REFERENCES

[1] N.A. Kudryashov and I.L. Chernyavskii, "Numerical simulation of the process of autoregulation of the arterial blood flow",*J. Fluid Dynamics*, Vol. **43**(1), pp. 32-48, (2008).

[2] B.E. Carlson, J. C. Arciero, T.W. Secomb, "Relative influence of myogenic, shear-dependent, and conducted responses on vascular autoregulation ", *Am. J. Physiol. Heart. Circ. Physio.l*, Vol.**295**, pp. H1572-H1579, (2008)

[3] D.A. Boas, S.R. Jones, A.Devor, et al., "A vascular anatomical network model of the spatio-temporal response to brain activation", *NeuroImage*, Vol. **40**(30), pp. 1116-1129, (2007).

[4] P. Causin. F.Malgaroli and F.Zorzan, "A mathematical model for metabolic autoregulation bio-mechanical processes in microvessels: Application to the inner retinal circulation", *J. Mech. Med. Biol*,(2015, in press).