

# Approximate Block Factorization Preconditioners for Scalable Solution of Multiphysics MHD Systems.<sup>#</sup>

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## *Abstract*

The magnetohydrodynamics (MHD) model describes the dynamics of charged fluids in the presence of electromagnetic fields. The mathematical basis for the continuum modeling of these systems is the solution of the governing partial differential equations (PDEs) describing conservation of mass, momentum, and energy, augmented by the low-frequency Maxwell's equations. This PDE system is non-self adjoint, strongly-coupled, highly-nonlinear, and characterized by physical phenomena that span a very large range of length- and time-scales. To enable accurate and stable approximation of these systems a range of spatial and temporal discretization methods are commonly employed. In the context of finite element spatial discretization methods these include mixed integration, stabilized methods and structure-preserving (physics compatible) approaches. For effective time integration for these systems some form of implicitness is required. Two well-structured approaches, of recent interest, are fully-implicit and implicit-explicit (IMEX) type methods. Clearly the desire to accommodate disparate spatial discretizations, and allow the flexible assignment of mechanisms as explicit or implicit operators, implies a wide variation in unknown coupling, ordering, the nonzero block structure, and the conditioning of the implicit system. These characteristics make the scalable and efficient iterative solution of these systems extremely challenging.

In this talk we consider the development and evaluation of approaches based on approximate block factorization (ABF) [1,2] and physics-based preconditioning approaches [2,3]. These methods reduce the coupled systems into a set of simplified systems to which multilevel methods are applied. A critical aspect of these methods is the development of approximate Schur complement operators that encode the critical cross-coupling physics of the system. To demonstrate the flexibility and performance of these methods we consider application of these techniques to resistive MHD and multiphysics MHD models for challenging prototype systems. In this context robustness, efficiency, and the parallel and algorithmic scaling of the preconditioning methods are discussed.

[1] E. G. Phillips, H. C. Elman, E. C. Cyr, J. N. Shadid, R.P. Pawlowski, A Block Preconditioner for an Exact Penalty Formulation for Stationary MHD, accepted in SISC

[2] E. C. Cyr, J. N. Shadid, R. S. Tuminaro, R. P. Pawlowski, and L. Chacon, A new approximate block factorization preconditioner for 2D incompressible (reduced) resistive mhd, SIAM Journal on Scientific Computing, 35:B701-B730, 2013

[3] L. Chacon, An optimal, parallel, fully implicit Newton-Krylov solver for three-dimensional visco-resistive magnetohydrodynamics, Phys. Plasmas, 15, 056103 (2008)

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