Numerical Simulation of Three-dimensional Convection

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Introduction

Many researchers studied thermal Benard convection using numerical simulation [1-3]. As rule, they used the pseudo-spectral methods with periodic boundary conditions. Some authors performed 3-D simulations for high supercriticality with rigid [1] and free [3] boundary conditions on the horizontal planes. The fully developed turbulence is characterized by developed space spectrums. Moreover, the existence of well-defined spectrums in turbulent flow denotes the role of corresponding physical mechanisms.

In experiments on turbulent convection with cryogenic gaseous He the Bolgiano-Obukhov $k^{-7/5}$, Kolmogorov $k^{-5/3}$ and $k^{-2.4}$ spectrums have been observed for the temperature pulsations [4,5] and k^{-1}, $k^{-7/5}$, k^{-3} and k^{-7} - in numerical simulations [1,2]. For velocity pulsations, the Bolgiano-Obukhov spectrum $k^{-11/5}$ in experiments on turbulent convection with SF6 have been derived [6] and $k^{-5/3}$, k^{-3} - in numerical simulations [1,3].

The turbulent spectrums of turbulent convection have been investigated insufficiently in numerical simulations, for example, no Kolmogorov $k^{-5/3}$ and $k^{-2.4}$ spectrums were derived in numerical simulations for temperature pulsations. Also no Bolgiano-Obukhov spectrum $k^{-11/5}$ was derived in numerical simulations for velocity pulsations.

The aim of this work is investigation of the temperature and velocity pulsation spectrums in numerical 3-D simulations of turbulent Benard convection with free boundary conditions on horizontal planes and studying of mean values scaling.

Problem Formulation and Numerical Method

The turbulent convective flow in a horizontal layer numerically is simulated at heating from below. The fluid is viscous and incompressible. The flow is time-dependent and three-dimensional. The layer boundaries are isothermal and free from shearing stresses. The model Boussinesq is used without any semiempirical relationships (DNS). The dimensionless set of equations given in terms of deviations from an equilibrium solution is [7]:

$$\begin{aligned}
&div\,\vec{v} = 0 \\
&\vec{v}_t + \frac{1}{\Pr}(\vec{v}\cdot\nabla)\vec{v} + \nabla P = \Delta\vec{v} + Q\cdot\vec{e}_z \qquad (1)\\
&Q_t + \frac{1}{\Pr}(\vec{v}\cdot\nabla)Q = \frac{1}{\Pr}\Delta Q + \frac{(\vec{v}\cdot\vec{e}_z)}{\Pr}
\end{aligned}$$

where **v** = (u,v,w) and P is velocity and pressure, Q is the temperature deviation from equilibrium profile (the total temperature being T = 1 - y + Q), $\mathbf{e}_z$ = (0,0,1), $\Delta f = f_{xx} + f_{yy}$ is the Laplace operator, $Ra = g\beta H^3 dQ/\chi\nu$ is the Rayleigh number, $\Pr = \nu/\chi$ is the Prandtl number, g is the gravitational acceleration, β, ν, χ are the coefficients of thermal expansion, kinematics viscosity and thermal conductivity, respectively, H and dQ is the layer height and the temperature difference on the horizontal boundaries.

Let $r = Ra / Ra_{cr}$, $Ra_{cr} = 657.5$ is supercriticality. The stress-free boundary conditions on horizontal boundaries are: $u_z = v_z = w = Q = 0$. Such boundary conditions have been realized in the experiment [8]. The solution of system (1) is seeking in form of the sum of trigonometric functions, the size of flow domain is equal to π in the both horizontal directions. The sidewall boundary conditions are “soft” and following from form of solution. We use special pseudo-spectral method with resolution 65^3 of harmonics [7]. To proof the sufficient resolution, the two test simulations with resolutions 33^3 and 129^3 have been performed with r = 950 and Pr = 10. The mean values, the temperature and velocity pulsation profiles are close in test simulations.

Spectrums of Convective Turbulence

Fig.1 represents the time spectrum of temperature pulsations in center of cell, the solid line is result of present simulation (r = 410, Pr = 0.8), the points are experimental data [5]. The normalization of results, supercriticality and Prandtl number are same. The frequency f is in units of ν/H^2, f_d is the dissipation frequency. The essential deviation may be seen only in the neighbourhood of dissipation frequency f_d.

The fully developed spectrums at $r \geq 500$ (under always Pr = 10) are seen in our numerical simulations. Figs.2,3 represent the log-log one-dimensional energy space spectrums of the temperature pulsations in both horizontal directions with r = 950. The Kolmogorov $k^{-5/3}$ and $k^{-2.4}$ spectrums observed earlier in experimental works [4,5] are seen.

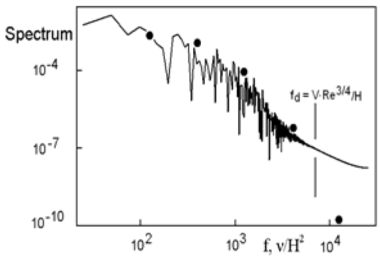

Fig.1. Time spectrum of temperature pulsations

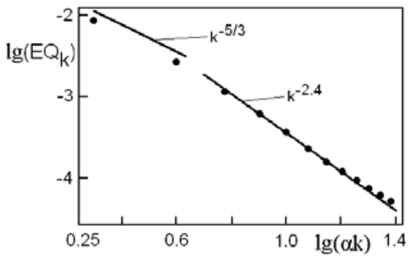

Fig.2. Temperature space spectrum in x-horizontal direction

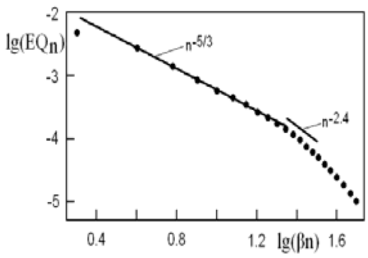

Fig.3. Temperature space spectrum in y-horizontal direction

The physical nature of $k^{-2.4}$ spectrum has no clear explanation. But, the typical for temperature pulsations Kolmogorov $k^{-5/3}$ spectrum indicates the temperature behave as passive scalar, without essential role of buoyancy.

Fig.4,5 represent the log-log one-dimensional energy space spectrums of the velocity pulsations in the both horizontal directions, also with r = 950. At fig.4 the Bolgiano-Obukhov $k^{-11/5}$ spectrum observed in experimental works [6] are seen. The spectrum n^{-3} has been found in numerical simulations [1,3]. These spectrums are typical for stratification flows and indicate the essential role of buoyancy [9].

Scaling of Mean Values

Fig.6 represents the temperature pulsations (between the planes) versus supercriticality r. We can see that data of our simulations ($q' \sim r^{-0.133}$) has the exponent close to experimental ([10], $q' \sim r^{-2/15}$) and numerical ([1], $q' \sim r^{-1/7}$). Fig.7 represents the vertical velocity pulsations (between the planes) versus supercriticality r. We can see that data of our simulations ($q' \sim r^{0.40}$) has the exponent close to experimental ([10], $v' \sim r^{0.42}$) and numerical ([1], $v' \sim r^{0.46}$).

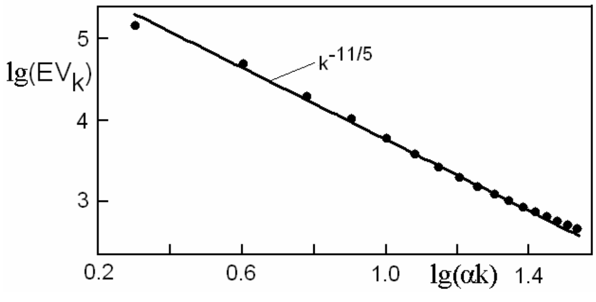

Fig.4. Velocity space spectrum in x-horizontal direction

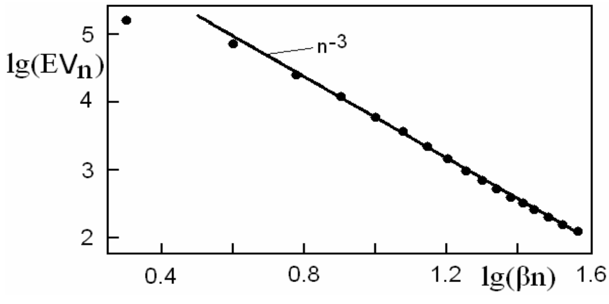

Fig.5. Velocity space spectrum in y-horizontal direction

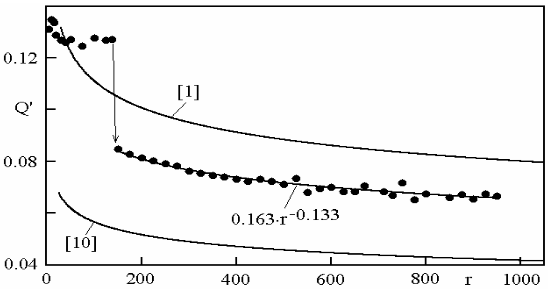

Fig.6. Temperature pulsations versus r

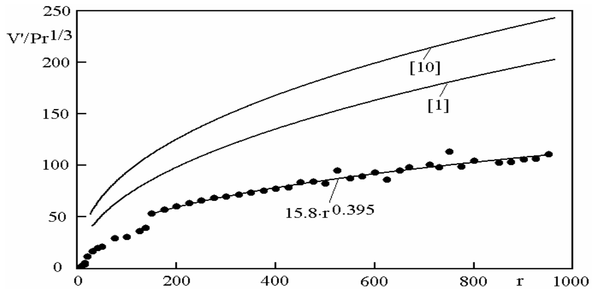

Fig.7. Vertical velocity pulsations versus r

Conclusion

Figs.4,5 show an anisotropy of horizontal directions because of the moderate values of supercriticality and Reynolds number ($r \leq 950$ and $Re \leq 44$). But, despite it, the existence of the developed space spectrums for temperature and velocity pulsations at $r \geq 500$ and $Pr = 10$ denotes the developed turbulence in our simulations.

The physical nature of $k^{-2.4}$ spectrum for temperature pulsations has no clear explanation. But, the typical for temperature pulsations Kolmogorov $k^{-5/3}$ spectrum indicates the temperature behave as passive scalar, without essential role of buoyancy.

But for velocity pulsations the stratification flow spectrums $k^{-11/5}$ and k^{-3} are typical and indicate the essential role of buoyancy.

The dependences of temperature and velocity pulsations (between the planes) versus supercriticality r show the exponents close to experimental [10] and numerical [1].

References

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