APPROXIMATION OF THE THERMALLY COUPLED MHD PROBLEM USING A STABILIZED FINITE ELEMENT METHOD

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ABSTRACT

In this paper we present a numerical formulation to solve thermally coupled MHD flows. It is a *stabilized* finite element method, whose design is based on splitting the unknown into a finite element component and a subscale and on giving an approximation for the latter. The main features of the formulation are that it allows to use equal interpolation for all the unknowns and it is stable and optimally convergent in a norm that remains meaningful in the whole range of the physical parameters.

Regarding the interpolation issue, it is particularly relevant in problems involving the magnetic field. The method proposed allows us to approximate it using standard continuous interpolations. It has to be remarked that if the solution of the continuous problem exhibits singularities (in non-convex domains), the expression of the stabilization parameters needs to be modified. This is, however, a point that we do not analyze. Referring to the norm of the stability and error analysis, it gives some sort of control on the unknowns for all values of physical parameters for the linearized problem. Obviously, the fully nonlinear problem may display very complex physics, which need to be approximated not only by a robust formulation, but also by an appropriate discretization. Our objective in this paper is to address the first point.

The formulation proposed depends on some stabilization parameters. A key point of the method is that these parameters are designed based on the stability and convergence analysis of the method for the thermally uncoupled problem.

Several computational aspects of the final formulation, such as the time integration, the linearization and the block iterative coupling, will be discussed. Altogether this leads to a method for solving thermally coupled MHD problems that we believe is robust and easy to implement. Some simple numerical examples support these statements.