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2D MICRO- AND MACROSCOPIC MODELS FOR SIMULATION OF HETEROGENEOUS TRAFFIC FLOWS

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Abstract. *The paper deals with the simulation of vehicular traffic flows in urban streets and at motorway. Original 2D macro- and microscopic models of multilane traffic are developed to predict flows for the real road geometry. The macroscopic model of synchronized traffic flow uses the kinetic approach by analogy with the quasi-gas-dynamic (QGD) system of equations. The microscopic model is based on the cellular automata theory. Test predictions demonstrate good agreement of the models in both qualitative and quantitative sense.*

1 INTRODUCTION

In the recent decades the problem of traffic congestion is becoming increasingly urgent. Vehicles often move slower than pedestrians during rush hours. Improving the traffic situations generally involves costly measures and is mostly inefficient. The mathematical modeling is the most economical tool for the simulation of traffic flows which develops strenuously nowadays. There are two basic types of the traffic flow models[1].

The "macroscopic" or hydrodynamical model treats traffic flow as slightly compressible fluid[2] and uses the continuum approach. This approach is applicable for long road intervals (much greater than vehicle sizes) with congested enough traffic, when all drivers have to follow the similar strategy. The notions of the flow density as the vehicle number per length unit per lane and the flow velocity are introduced. Similar to the gas dynamics, partial differential equations are derived to account for the mass and momentum conservation laws. These equations also include additional terms describing the human will.

In the so-called "microscopic" models vehicles are treated as separate particles, which interact according to certain laws[3,4,5] ensuring safety traffic, possibility of acceleration, deceleration, etc. Each vehicle has its own speed and destination point. This model type is mostly effective for modeling relatively short road intervals comparable with vehicle sizes, for example, while modeling traffic on crossroads.

Most of the present models are one-dimensional and do not account for parameter distribution across the road. The proposed paper deals with the development of original 2D macro- and microscopic models of multilane vehicular traffic to predict flows for the real road geometry.

In[6,7] the 2D multilane macroscopic traffic flow model was developed on the basis of the quasi-gas-dynamic (QGD) system of equations[8]. The main idea is that it makes no sense to consider scales less than the minimal reference length. For traffic flows the reference length equals the distance between vehicles for the given velocity. Contrary to the earlier traffic flow models, a variable transverse velocity is introduced. The present research generalizes the model to the case of multiphase flows, while a phase is a group of vehicles with identical features, i.e. the type of vehicle (car or lorry), destination, the speed limit.

The paper presents also an original 2D microscopic model based on the cellular automata theory[9]. Such models are rather flexible owing to the possibility of implementing any driving strategy without substantial algorithmic costs. As well as the macroscopic model, this model is modified to simulate heterogeneous traffic flows.

The 2D models described above are compared by a large number of test predictions for situations when both models are applicable. The results obtained demonstrated good agreement of the models in both qualitative and quantitative sense.

2 2D MACROSCOPIC TRAFFIC FLOW MODEL

Macroscopic traffic flow models describe synchronized traffic when the speeds are far from the free flow speed. Under these conditions it is possible to use the continuum approach on long intervals of the road and derive equations similar to the gas dynamics equations. The notions of the density ρ as the quantity of vehicles per lane in a distance unit, the velocity u as the average speed of vehicles and the flux $q=\rho u$ as the function of the density and velocity are employed.

When deriving the model, the vehicle ability to accelerate or decelerate is taken into account. The accelerating/decelerating force is:

$$f - a\rho \tag{1}$$

the acceleration is

$$a = \frac{u_{eq} - u}{T} \tag{2}$$

the equilibrium speed is

$$u_{eq} = u_f \left(1 - \frac{\rho}{\rho_{jam}} \right) \tag{3}$$

where u_f is the speed of the free motion of vehicles, ρ_{jam} is the density, at which the vehicles stop moving ("traffic jam"), and the relaxation time is

$$T = t_0 \left(1 + \frac{\tau\rho}{\rho_{jam} - \tau\rho} \right) \tag{4}$$

t_0 and r are the phenomenological constants, the speed limit is: $0 \le u \le u_{\max}$ where u_{max} is the maximum allowed speed limit.

The analogue of pressure shows the density gradient influencing the traffic flow

$$P = \lambda_x \frac{\rho^{\beta_x}}{\beta_x} \tag{5}$$

where λ_x and β_x are phenomenological constants.

Traffic flow models are conventionally one-dimensional, describing vehicles moving in one lane. To take into account the neighboring lanes (multilane traffic), some models can, for instance, use the corresponding sources on the right-hand sides of the equations[1]. The challenge of developing a fully two-dimensional model is that it is impossible to generalize a one-dimensional model for the two-dimensional case in the usual way, since motions along and across the road are not equivalent. Earlier in[6,7] the model providing the possibility of the transverse motion was proposed. In this model vehicles can move to the lane with a faster speed or a lower density while driving towards their destination. Then the transverse velocity v is described by the sum of the following terms:

movement to the lane with a higher speed — $v_u = k_u \cdot \rho \dfrac{\partial u}{\partial y}$;

movement to the lane with a lower density — $v_\rho = -k_\rho \cdot u \dfrac{\partial \rho}{\partial y}$;

movement to the destination — $v_{des} = k_{des} \dfrac{u^2}{\left(x_{des} - x\right)^2} \left(y_{des} - y\right)$

Here k_u, k_ρ, k_{des} are phenomenological constants, (x_{des}, y_{des}) — the destination coordinates.

In the final form the 2D system of equations describing traffic flows was obtained by analogy with the QGD system of equations[8]. One of the criteria of gas flows is the Knudsen number Kn that is the ratio between the reference length of the medium (the free path length) and the reference length of the flow. In gas dynamics $\mathrm{Kn} \le 10^{-3}$. For traffic flow under the free path length the average distance between the vehicles is assumed therefore Kn≈0.1. The QGD system works well within the wide range of Knudsen number values, which is the reason to use it for derivation of the traffic flow model.

One of the basic assumptions for the QGD system is existence of additional mass flux that ensures a smooth solution at the reference length of the medium. For traffic flows, the reference lengths along and across the road are different. Along the road, it is the distance $\delta(u)$ between the vehicles for the velocity u, and the reference length across the road is the width of one lane.

The reference time τ is also introduced in the continuum approach. The time interval of crossing the given point of the road by several vehicles can be treated as such a time.

$$\tau_x \approx \frac{\delta(u)}{u}, \qquad \tau_y \approx \frac{1}{v} \tag{6}$$

Note that τ is rather conservative and does not change significantly at different velocities. For simplification, τ_x and τ_y can be treated as constants.

An additional flux W_x is introduced on the right-hand side of the continuity equation to ensure smoothing along the road

$$W_x = \frac{\tau_x}{2}\frac{\partial}{\partial x}\left(\rho u^2 + P\right) \tag{7}$$

The diffusion flux associated with the transverse motion of vehicles is

$$W_y = \frac{\tau_y}{2}\left(\frac{\partial \rho v^2}{\partial y} + \lambda_y \rho^{\beta_x}\frac{\partial \rho}{\partial y}\right) \tag{8}$$

There are smoothing terms in the momentum equation as well.
Generalizing the above assumptions, the following system of equations for the traffic flow dynamics is obtained:

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} &= \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial x}\left(\rho u^2 + P\right)\right) + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial y}\left(\rho uv\right)\right) + \\ &+ \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho v^2\right) + \lambda_y \rho^{\beta_y}\frac{\partial \rho}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\frac{\partial}{\partial x}\left(\rho uv\right)\right) - \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}f\right)\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho uv}{\partial y} &= f - \frac{\partial}{\partial x}P + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial x}\left(\rho u^3 + 3Pu\right)\right) + \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}\frac{\partial}{\partial y}\left(\rho u^2 v\right)\right) + \\ &+ \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\left(\frac{\partial}{\partial y}\left(\rho uv^2\right) + \lambda_y \rho^{\beta_y}\frac{\partial \rho u}{\partial y}\right)\right) + \frac{\partial}{\partial y}\left(\frac{\tau_y}{2}\frac{\partial}{\partial x}\left(\rho u^2 v\right)\right) - \frac{\partial}{\partial x}\left(\frac{\tau_x}{2}fu\right)\end{aligned} \tag{10}$$

$$v = k_u \rho \frac{\partial u}{\partial y} - k_\rho u \frac{\partial \rho}{\partial y} + k_{des}\frac{u^2}{\left(x_{des} - x\right)^2}\left(y_{des} - y\right) \tag{11}$$

As the model possesses a great number of coefficients and parameters, which can be chosen arbitrarily, the problem of the model calibration evolves. Furthermore, it is important to take into account the anisotropy and statistical and experimental data. The following values of parameters were used to perform calculations:

$$\lambda_x = 60, \quad \lambda_y = 4, \quad \beta_x = 2, \quad \beta_y = 0, \quad \tau_x = 2\cdot 10^{-3}, \quad \tau_y = 3\cdot 10^{-4}$$

$$t_0 = 50, \quad r = 0.95, \quad \rho_{jam} = 120, \quad u_f = 120, \quad u_{\max} = 90$$

The model described above account for homogeneous traffic flows, which means that vehicles differ in their coordinates only. However in reality there are vehicles of different types (for example, passenger cars or lorries) on roads. They can differ not only in their characteristics (the free flow speed, the relaxation time) but also in their behaviour on roads (for example, lorry can take aim at the right lane). Vehicles have got different routes and different destinations. As a result, a homogeneous flow is initially divided into a number of components in a complicated manner. For simulation of such compound statistically-heterogeneous flows the concept of multiphase traffic is introduced, where under a phase a set of vehicles with some identical features is assumed. The proposed model has been generalized to the multiphase case. Equations (9)-(11) are modified taking into account different characteristics of phases: each phase has got its own density, velocity, transverse velocity, destination etc. Consequently now the model comprises a set of equations (the continuity equation, the momentum equation and the transverse velocity equation) for each phase.

3 2D MICROSCOPIC TRAFFIC FLOW MODEL

The proposed microscopic model is based on the cellular automata (CA) theory[9]. CA are idealization of the physical system with discrete space and time, each of interacting units of the system has got a finite number of discrete states. For the description of vehicular traffic the CA concept is being developed since 1980s.

In the classical approach, a lane is represented by a one-dimensional lattice. Each cell of the lattice can be either empty or occupied by one particle, which represents a vehicle. Vehicles can skip from one cell to another (which must be empty) in one direction and cannot overtake one another. Since space and time are quantified, the speed and the acceleration take on only discrete values. In such models particle movement is regulated by special laws of the cell state update incorporating stochastic observations. The update rules are identical for all cells and are applied to all cells in parallel. Therefore for modelling parallel computer codes can be developed to run on high-performance multiprocessors.

The CA rules feature the property of locality. In other words, to obtain the current state of the cell, it is necessary to know only the states of some of its neighbours called the cell vicinity. The cell length equals the length of the road interval occupied by a vehicle in the traffic jam that is the length of a vehicle and the distance between neighbouring vehicles. Usually it is 7.5 m. The speed denotes how many cells the vehicle overpasses during a time step. The cell length, the maximal speed and the time step describe the model completely.

One of the well-known CA-based microscopic model is the Nagel-Schreckenberg model[2]. Its original variant is one-dimensional. In this model the speed v of each vehicle can take one of the integer values $v = 0, 1,\dots, v_{max}$.

If x_n and v_n are the position and the speed of the current n-th vehicle, d_n is the distance between the current vehicle and the vehicle in front of it, then at each time step $t \to t+1$ the algorithm of the vehicle arrangement update consists of the next stages:

1. *Acceleration* $v_n \to \min(v_n+1, v_{max})$
2. *Deceleration* $v_n \to \min(v_n, d_n-1)$
3. *Randomization* $v_n \to \max(v_n-1, 0)$ with some probability
4. *Vehicle movement* $x_n \to x_n + v_n$

The first stage reflects the common tendency of all drivers to move as fast as possible, the second one guarantees avoiding collisions, the third one takes into account randomness in driver behavior, and the skip itself takes place on the fourth stage – each vehicle is moved forward according to its new velocity.

The Nagel-Schreckenberg model is a minimal model because it reproduces only primary features of real traffic flows. The present research takes this model as a basis to describe movements on relatively short road intervals with a high probability of traffic congestion.

Aiming at simulation of multilane traffic, the authors have generalized the above model to the two-dimensional case. In this case the computational domain is the 2D lattice. The number of cells in the transverse direction corresponds to the number of lanes. Such a model allows vehicles to change lanes and to overtake one another. The algorithm of the cell state update is formed by two components:

1) lane change (if it is necessary and possible);
2) movement along the road by the rules of one-lane traffic (stages 1-4 above).

Change of lanes should happen during a time step. If there are more than two lanes in one direction, a conflict can occur when two vehicles from extreme lanes tend to the inner lane and try to occupy one and the same cell. The rule like the next one could help to resolve such a situation: vehicles change to the right only on even time steps and change to the left on odd steps.

In general reasons and conditions for changing lanes are as follows:

1) the vehicle is located in the domain where lane change is allowed;
2) lane change leads to increase of the speed (decrease of the density) or is necessary to reach the destination (to achieve the object);
3) the target cell is empty;
4) the safety condition is satisfied - on the target lane the distance behind the vehicle is greater or equal to v_{max}, in front of the vehicle it is greater or equal to v_n, then the change takes place with some probability.

The algorithm proposed in the present work ensures the possibility to achieve the destination. For example, the side road exit or the appointed turning at traffic lights can be assumed as destinations in multilane traffic. In any case, starting from the certain time moment vehicles aim at the target lane and ignore the density and velocity values on it. However drivers cannot disregard the safety condition. If the destination is not far, vehicles change to the target lane at the first opportunity and do not quit it anymore. The situation is possible when a driver is not able to turn to the required lane up to the destination achievement. In such cases the vehicle has to stop near by the target lane and to wait for the opportunity of wedging itself in the lane. In doing so, it can disable forward movement of other vehicles on the current lane.

General considerations help to evaluate the distance to the destination at which drivers start trying to change to the target lane. As a matter of fact, this distance depends on the flow density. In different problems it is 75-150 m. Thus the developed model keeps the destination parameter for each vehicle. Vehicle destinations are obligatory, they cannot be modified.

The microscopic model was generalized to the multiphase traffic flow too. In this case the algorithm of cell state update includes the initial vehicle destination control, the other stages are fulfilled according to this destination. Each phase can have its own v_{max}.

4 TEST PREDICTIONS

Figures 1, 2 illustrate the first test problem – the quasi-one-dimensional traffic flow.

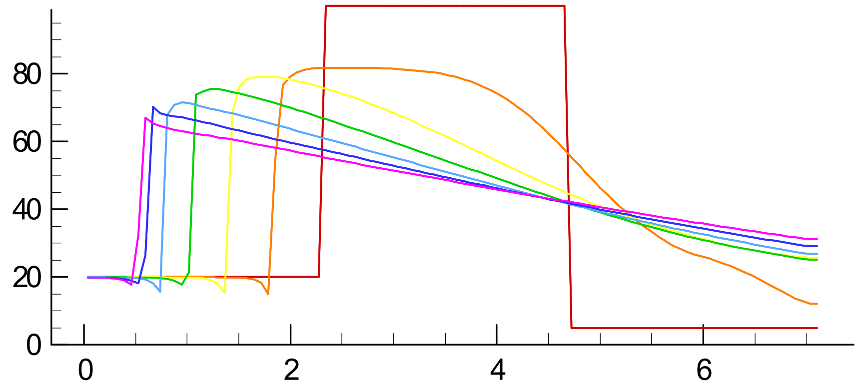

Figure 1: Time evolution of the high density step
(profiles of the density along the road) obtained via the macroscopic model

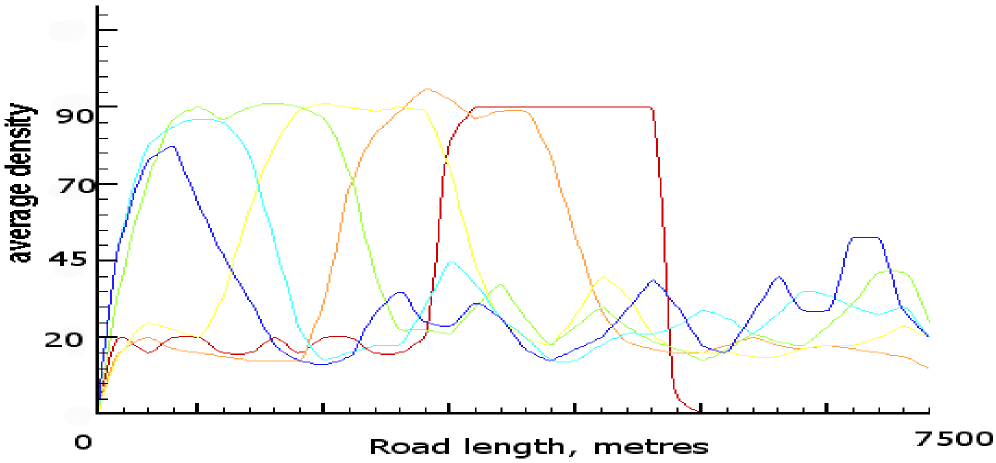

Figure 2: Time evolution of the high density step obtained via
the microscopic model

The road interval of 7500 m length is under consideration. The traffic within some part of this interval is strongly congested. Figures 1 and 2 show the time evolution of the high density “step” obtained by the macroscopic and microscopic models respectively. The density in the microscopic case is the implication of averaging discrete values namely the number of cars per a length unit. Therefore graphic lines in Figure 2 have the oscillating behaviour. Nevertheless one can see the qualitative agreement of results. The initial “step” is red coloured. Then the step is moving backward until its density falls below some certain value. The propagation of such density jumps often leads to the jam.

The second test problem is the vehicle movement on the road with local widening. The corresponding road configuration is shown in Figure 3.

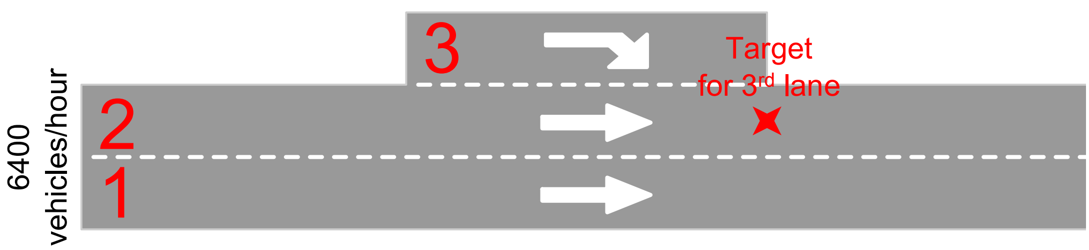

Figure 3: Statement of the “local widening” problem

Figure 4 demonstrates the density field obtained with the use of the macroscopic model. The red colour corresponds to the maximal density while the minimal density is blue. The density of vehicles falls at the wide part of the road but when the traffic flow shrinks back from three to two lanes, the speed falls significantly. Thus, the total time required to pass the given road interval grows as compared with the road without widening.

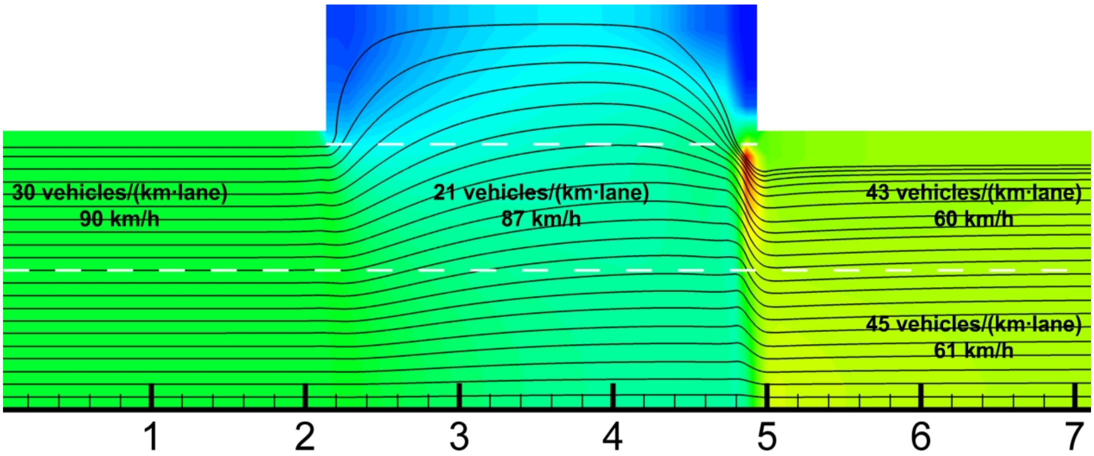

Figure 4: The density field obtained via the macroscopic model

The same test problem has been solved using the microscopic model. The average density is depicted in Figure 5.

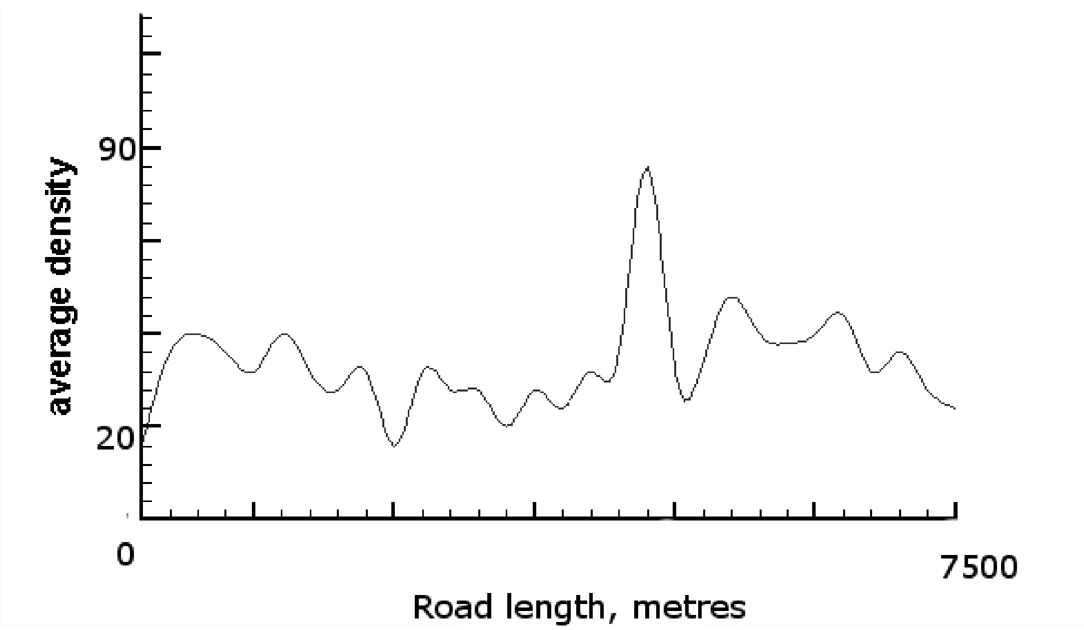

Figure 5: The traffic density obtained via the microscopic model

One can observe the same tendency: the maximal density is located at the end of widening. Moreover the density at the road exit exceeds the enter density.

To validate multiphase variants of the models, the following test problem on heterogeneous flows has been considered. There are two types of vehicles differing in final destinations: phase 0 (it is 1/3 of all vehicles) is formed by the vehicles turning to the right at the side exit, phase 1 (correspondingly 2/3 of all vehicles) is the vehicles moving forward along the road (see Figure 6).

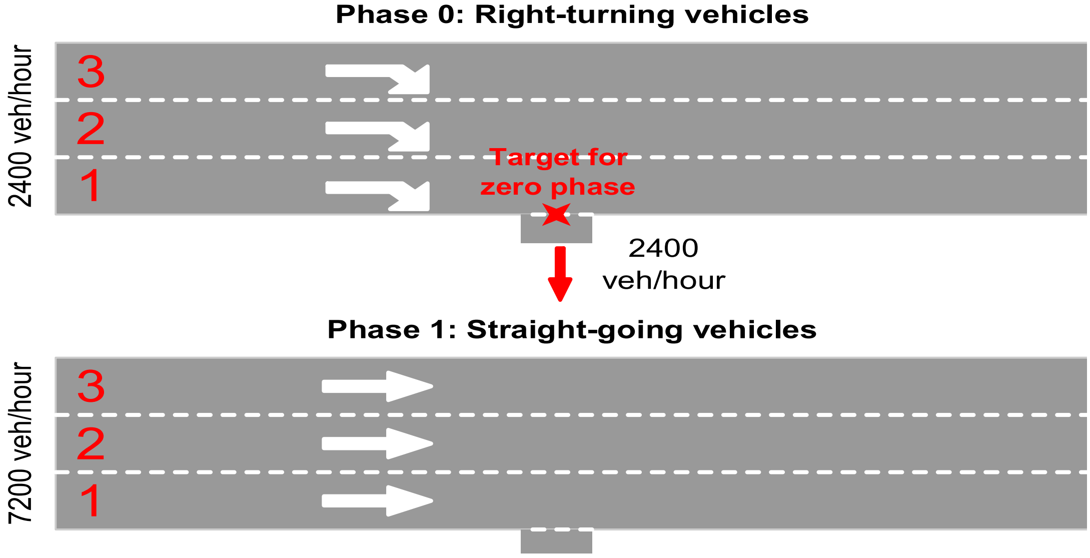

Figure 6: Statement of the multiphase flow problem

The comparison of results obtained with macro- and microscopic models is illustrated by Figures 7 and 8. In those one can see that over some interval phase 0 displaces phase 1 out of the first lane, in front of the exit the density jump occurs in both phases, and past the exit phase 0 vanishes from the road, and vehicles of phase 1 are distributed uniformly throughout the multilane road.

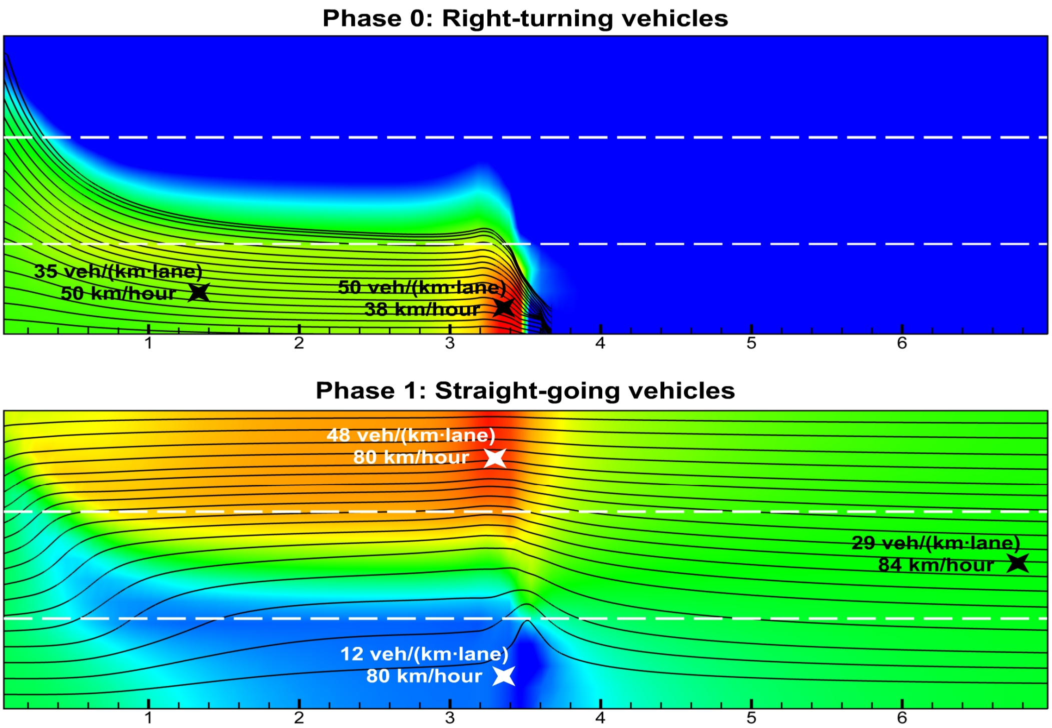

Figure 7: The density field obtained via the multiphase macroscopic model

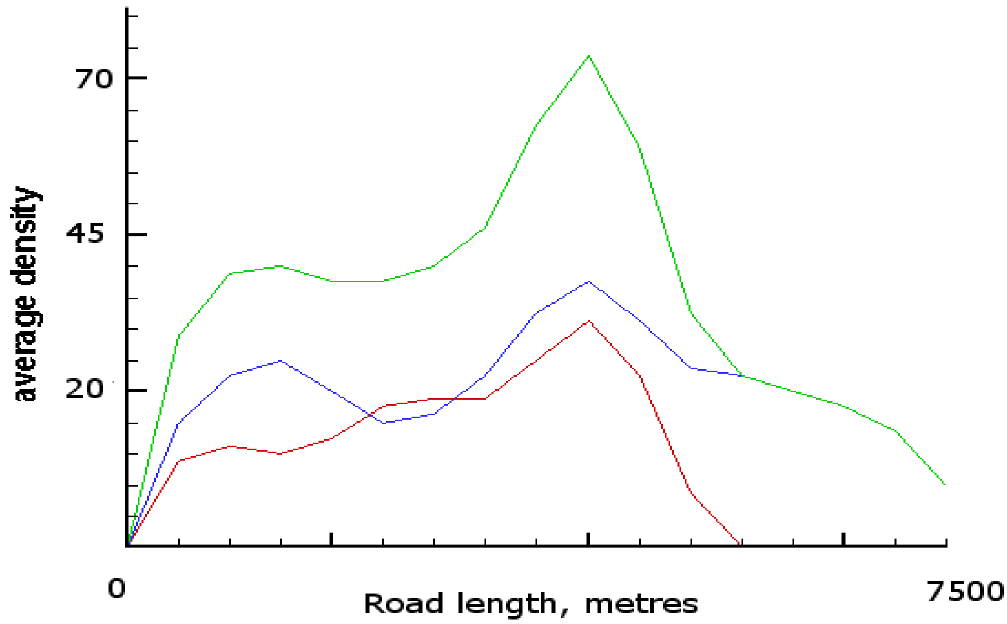

Figure 8: The traffic density obtained via the multiphase microscopic model:
Red line – right-turning vehicles
Blue line – straight-going vehicles
Green line – the total amount of vehicles

The final problem to be considered is the simulation of traffic lights regimes on a crossroad using the above microscopic model. The problem consists in obtaining the optimal traffic lights regime namely the signal durations to ensure the minimal time of stay on the crossroad for all traffic participants. Figure 9 illustrates the problem.

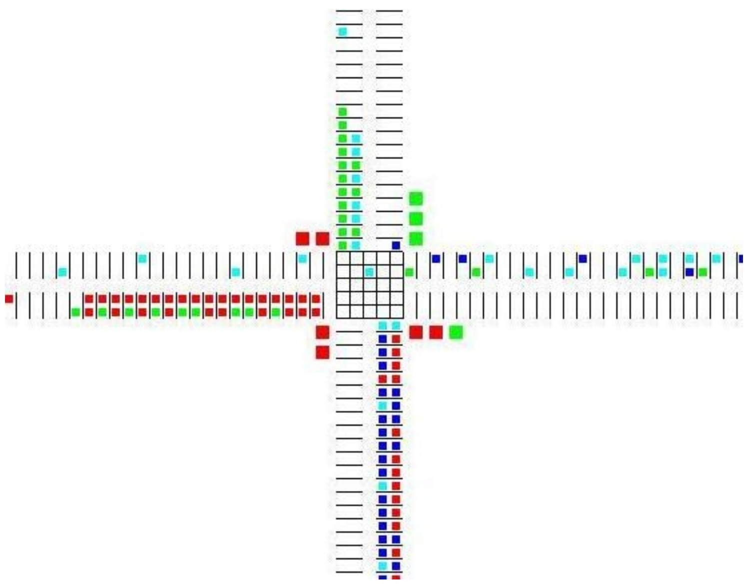

Figure 9: Problem statement of traffic lights on a crossroad

The lattice consists of four crossing roads each of them has four lanes: two in forward and two in reverse directions. The traffic flows are multiphase, incoming flow rates are non-uniform. The vehicle colour indicates its destination – to move forward or to turn to the right or to the left. Figure 10 shows incoming flow rates and allowed directions of movement on the crossroad.

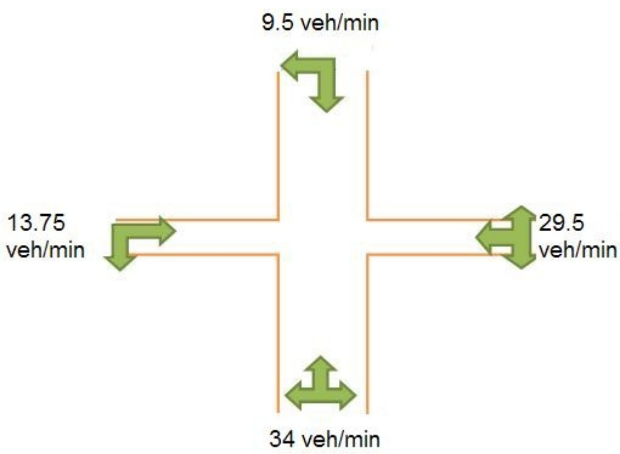

Figure 10: Incoming flow rates and allowed directions on the crossroad

The traffic lights has four operating modes which determine the order of traffic on the crossroad at the given time moment (see Figure 11). Each mode has its own duration coinciding or not with other ones. All data correspond to some real crossroad.

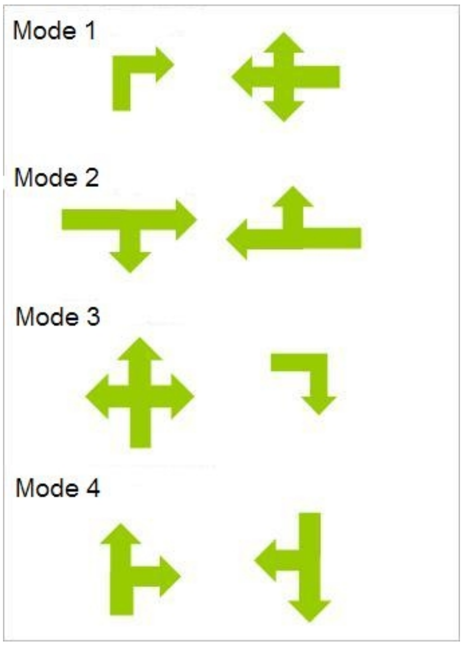

Figure 11: The traffic lights operating modes – allowed directions on the crossroad

Vehicles have more complicate behavior in the vicinity of the crossroad in comparison with straight intervals of the road. If the vehicle intends to turn it must decrease the speed till to allowed for turning. The vehicle must also take into account positions of vehicles in neighbour lanes and in other directions.

Table 1 demonstrates results of predictions. Varying mode durations one can increase the crossroad capacity i.e. the number of vehicles left the crossroad for 10 minutes. Obtained results allow to conclude the follows. Equal mode durations do not provide the best capacity (see row 2). Mode 1 allows passing over the crossroad by vehicles with the maximal incoming flow rate. But excessive increase of the Mode 1 duration do not lead to the capacity increase (compare rows 2 and 4).

	Mode 1 Duration, s	Mode 2 Duration, s	Mode 3 Duration, s	Mode 4 Duration, s	Capacity of the crossroad
1	90	90	45	45	490
2	60	60	60	60	545
3	90	60	90	60	545
4	45	45	60	60	575
5	45	45	90	90	575

Table 1: The traffic lights operating modes – allowed directions on the crossroad

5 CONCLUSION

The 2D micro- and macroscopic models have been developed to describe multilane homo- and heterogeneous traffic flows. The models are applicable in distinct cases and can be used for simulation of both congested and rarefied flow.

Predictions of several test problems were performed to validate the both models. Traffic flows on a signal-controlled crossroad were simulated using data on real traffic lights regimes to optimize them.

The tools developed will be implemented in program packages to be widely used in various engineering applications, including recommendations for the optimal motorway construction, road situation prediction and traffic flow control.

6 ACKNOWLEDGEMENTS

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