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KINETIC APPROACH TO SIMULATION OF MULTIPHASE POROUS MEDIA FLOWS

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Abstract. *On the basis of the kinetic approach a new model of porous media flows is developed taking into consideration the fluid compressibility, capillary and gravity forces. The continuity equation is modified taking into account the minimal scales of averaging on space and on time, as a result the regularizing term and the time derivation of the second order with small parameters are present in the equation. For approximation the three-level explicit difference scheme with a mild stability condition is proposed. The model is generalized to the case of two-phase fluid flow. A computational algorithm of the explicit type is developed to achieve efficient implementation on high-performance computer systems with hybrid architectures. Computations of test problems on fluid inflow to oil-producing wells and on contaminant infiltration into the soil have been executed to validate the model.*

1 INTRODUCTION

The work deals with the development of a new approach for simulation of multiphase fluid flows in porous media. Computations of such flows are extremely important while developing technologies of oil recovery, constructing hydraulic structures, solving ecological problems aimed at prevention of the soil and groundwater contamination by petroleum products etc. It is well known that numerical simulation of these large-scale processes is very time-consuming and practically impossible without the use of high-performance computer systems. On the one hand modern supercomputers give unique opportunities for solving scientific and industrial problems on the other hand hybrid architectures of these computers cause great difficulties in their efficient employment[1]. These difficulties reflect fundamental problems of the software development for computer systems with the superhigh performance[2]. At present supercomputers combine shared and distributed memory, multicore CPUs and different accelerators[3] (GPU and EPLD). Therefore the necessity of new general-purpose computational approaches with high accuracy and logical simplicity arises to exploit hybrid systems most efficiently.

One of the algorithms of a simple structure is explicit finite-difference schemes. Logical simplicity of explicit schemes harmonizes well with such advanced techniques as the dynamic adaptation of computational meshes. However explicit schemes have got rather a strong time-step restriction. This restriction becomes crucial while increasing the number of processor nodes and decreasing the step of the spatial mesh. In the present paper a new approach to porous medium flow simulation is proposed assuming implementation by explicit different schemes and allowing improvement of the scheme stability.

In many previous papers[4,5,6] the authors investigated multiphase porous media flows at the assumption that fluids were incompressible. The corresponding traditional governing models[7,8,9] and different variants of IMPES method[7] for their implementation were used. Unfortunately these algorithms were not quite economical and possessed moderate (not very high) parallelization efficiency due to the necessity of solving the elliptic pressure equation in the framework of IMPES method. The model of another type was constructed by the authors[10,11] on the basis of the kinetic approach at assumptions that fluids were slightly compressible. Nowadays kinetic schemes such as lattice Boltzmann schemes[12] and kinetically-consistent finite difference (KCFD) schemes[13] are the most perspective algorithms in hydro- and gas dynamics. The main idea used at deriving KCFD schemes and the related Quasi-Gas Dynamic (QGD) system of equations consists in the next: there is no sense to consider scales less than the minimal reference length. This principle is extended to porous media flows where the reference length equals approximately to a hundred rock grains. At that the mass conservation equation includes an additional diffusion term with a small parameter in the right-hand side what provides new computational opportunities. Test predictions of single-phase flows validated the model[10,11,13].

In the present paper the continuity equation is transformed from the parabolic to hyperbolic type in order to provide the high solution accuracy at the sufficient scheme stability. For the first time the new model is generalized to the case of two-phase fluid flow taking into account the capillary and gravity forces. A fully explicit algorithm is developed for the model implementation. Computations of test problems on fluid inflow to oil-producing wells and also on contaminant infiltration from the earth surface into the soil have been performed using both kinetic and traditional approaches. Good agreement of results has been observed while the kinetic approach allowed to increase the time step and to reduce significantly computational costs.

2 SINGLE FLUID FLOW IN A POROUS MEDIUM

2.1 Hyperbolic model

The classical model of single fluid flow in a porous medium at the assumption of slight compressibility of the fluid[7] is written as follows:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\mathbf{u} = 0 \tag{1}$$

$$\mathbf{u} = -\frac{K}{\mu}\mathrm{grad}\, p \tag{2}$$

$$p = p_0 + \beta\left(\rho - \rho_0\right) \tag{3}$$

Here ρ is the density, p is the pressure, $\mathbf{u}$ is the Darcy velocity, K is the absolute permeability, μ is the dynamic viscosity, β is the compressibility factor, p_0 and ρ_0 are constant reference values of the pressure and the density.

For many problems of continuum mechanics there are some minimal sizes (so-called reference scales) which act as the lower bounds for the description details. In other words, it makes no sense to consider sizes smaller than the minimal reference scale. This principle is a cornerstone in deriving KCFD schemes and the related QGD system in gas dynamics[13]. The free path length of a molecule is such a scale in gas dynamics.

For porous media flows the minimal reference scale l is a distance of the order of a hundred rock grain sizes. If to take into consideration distances less than l it would be necessary to describe fluid flows with the account of the real geometry of rock grains. But the experimental Darcy law is valid for averages and follows directly from the equation of motion in the Navier-Stokes system averaged over the given length l. Using this reference length and the analogy with KCFD schemes the modified model has been obtained with the next continuity equation[10]:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\rho\mathbf{u} = \mathrm{div}\frac{l\,c}{2}\mathrm{grad}\rho \tag{4}$$

where c is the magnitude of the order of the sound speed in fluid. The additional diffusion term in the right-hand side guarantees solution smoothing on length l.

In contrast to equation (1) equation (4) can be approximated by explicit schemes with central differences for discretization of the convective term $\mathrm{div}\rho\mathbf{u}$ [10,11]. In some cases of porous media flow modeling employment of the central-difference approximation is very convenient. At the same time usage of explicit schemes for equation (4) as well as for initial equation (1) leads to the strong time-step restriction:

$$\Delta t \;\le\; h^2 \tag{5}$$

Let us consider the further development of the model (4), (2), (3). The difference approximation of the time derivation can be presented as given below (j is the time level number):

$$\frac{\rho^{j+1} - \rho^{j}}{\Delta t} = \frac{\partial \rho}{\partial t} + \frac{\Delta t}{2}\frac{\partial^2 \rho}{\partial t^2} + O\left(\Delta t^2\right) \tag{6}$$

If to base on this formulation equation (4) can be replaced by the hyperbolic equation

$$\frac{\partial \rho}{\partial t} + \tau\frac{\partial^2 \rho}{\partial t^2} + \mathrm{div}\,\rho\mathbf{u} = \mathrm{div}\frac{l\,c}{2}\mathrm{grad}\,\rho \tag{7}$$

with an unknown else parameter τ evaluated as the minimal reference time. In gas dynamics the time interval between molecule collisions is taken as such a scale. This parameter can be also interpreted as the time for inner equilibrium establishing in the volume with the reference size *l*.

The sound speed in fluid is much greater than the Darcy velocity. Then at the given step of the spatial discretization *h* the minimal time scale has the next order of magnitude:

$$\tau \sim h/c \tag{8}$$

For computations a value depending linearly on *h* can be chosen as τ.

The additional term $\tau \frac{\partial^2 \rho}{\partial t^2}$ in equation (7) by the order of magnitude is much smaller than the term $\frac{\partial \rho}{\partial t}$. Taking into account (8) one can get the following evaluation:

$$\left[\tau \frac{\partial^2 \rho}{\partial t^2} \right] \Big/ \left[\frac{\partial \rho}{\partial t} \right] \sim \frac{h}{L} \tag{9}$$

where *L* is the reference size of the problem.

2.2 Three-level explicit difference scheme

In this subsection let us consider the 1D case for simplicity. Substituting (2) and (3) into equation (7) one can rewrite this equation schematically in the model form:

$$\frac{\partial \rho}{\partial t} + \tau \frac{\partial^2 \rho}{\partial t^2} = \kappa \frac{\partial^2 \rho}{\partial x^2} \tag{10}$$

The three-level difference scheme for its approximation is as follows:

$$\frac{\rho_i^{j+1} - \rho_i^{j-1}}{2\Delta t} + \tau \frac{\rho_i^{j+1} - 2\rho_i^j + \rho_i^{j-1}}{\Delta t^2} = \kappa \frac{\rho_{i+1}^j - 2\rho_i^j + \rho_{i-1}^j}{h^2} \tag{11}$$

Under the condition

$$\tau / \Delta t^2 = \kappa / h^2 \tag{12}$$

scheme (11) changes into the absolutely stable Duffort-Frankel scheme[14]. It is known that the accuracy of solutions obtained with the use of the Duffort-Frankel scheme in many cases is unsatisfactory. However in the present research parameter τ (8) is chosen from the special considerations: the additional term with the second time derivation in (7) causes minimal changes in the solution of (4). Combination of (8) and (12) results in the stability condition of the three-level scheme for solving equation (7):

$$\Delta t \ \leq \ h^{3/2} \tag{13}$$

This condition is more acceptable in comparison with the classical restriction (5). Advantages of the mild condition (13) become particularly evident on the finest computational meshes which are applicable while implementing algorithms on multiprocessor/multicore computer systems.

Equation (7) can be approximated directly in its initial form by the conditionally stable three-level explicit scheme of the second order of approximation on time and on space using central differences for the convective term discretization:

$$\frac{\rho_i^{j+1}-\rho_i^{j-1}}{2\Delta t}+\tau\frac{\rho_i^{j+1}-2\rho_i^{j}+\rho_i^{j-1}}{\Delta t^2}+\frac{(\rho u)_{i+1}^{j}-(\rho u)_{i-1}^{j}}{2\Delta x}=\left(\frac{l\,c}{2}\rho_{\bar{x}}\right)_x^j \qquad (14)$$

where $\left(\frac{l\,c}{2}\rho_{\bar{x}}\right)_x^j$ is the standard difference approximation[14] of the term $\operatorname{div}\frac{l\,c}{2}\operatorname{grad}\rho$.

Or with the account of (2)-(3) the next scheme for equation (7) can be presented (compare with (11)):

$$\frac{\rho_i^{j+1}-\rho_i^{j-1}}{2\Delta t}+\tau\frac{\rho_i^{j+1}-2\rho_i^{j}+\rho_i^{j-1}}{\Delta t^2}=\left[\left(\rho_i^j\frac{K}{\mu}\beta+\frac{l\,c}{2}\right)\rho_{\bar{x}}\right]_x^j \qquad (15)$$

2.3 Test predictions

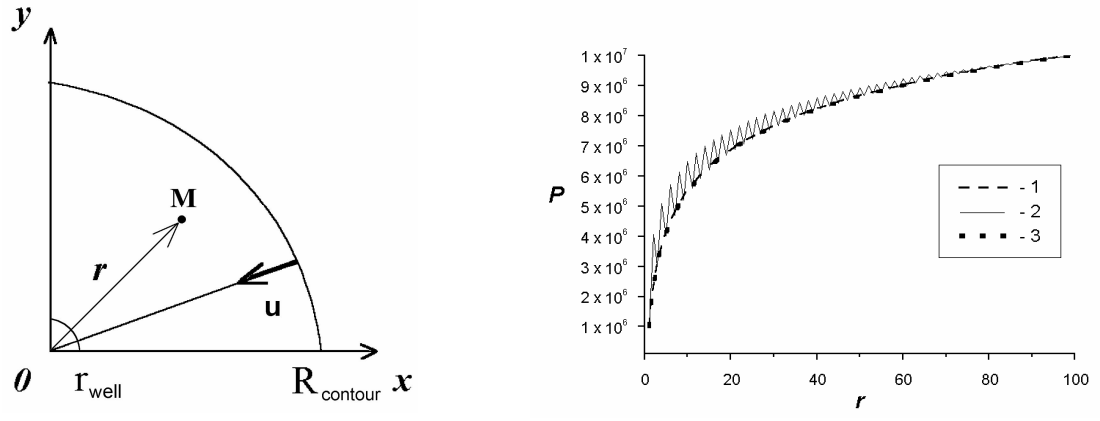
First of all the plane-radial fluid flow to a single vertical producing well (see Figure 1a) was predicted to validate the modified hyperbolic model (7), (2)-(3). The well is centered at the origin, r_{well} is the well radius, $R_{contour}$ is the radius of the feeding contour. Streamlines are radial and directed to the well. The given test problem can be formulated in the polar coordinates as a one-dimensional problem on interval $[r_{well}, R_{contour}]$ [10,11,13] in view of the axial symmetry. Constant values of the pressure and the density are set on the well and the contour. In computations the next values were used:

$$\begin{gathered} p_{well}=10^6\frac{g}{cm\cdot s^2},\quad p_{contour}=10^7\frac{g}{cm\cdot s^2},\quad \rho_{well}=1\frac{g}{cm^3},\quad \rho_{contour}=1.009\frac{g}{cm^3} \\ r_{well}=0.5cm,\quad R_{contour}=10^4 cm,\quad \beta=10^8\frac{cm^2}{s^2},\quad \frac{K}{\mu}=10^{-7}\frac{cm^3\cdot s}{g},\quad c=10^5\frac{cm}{s} \end{gathered} \qquad (16)$$

The analytical formula for the steady-state plane-radial flow around a well is known:

$$p=p_{well}+\frac{p_{contour}-p_{well}}{\ln\left(R_{contour}/r_{well}\right)}\ln\frac{r}{r_{well}} \qquad (17)$$

The subscripts *well* and *contour* indicate whether values of the corresponding quantities are taken on the well or on the contour.

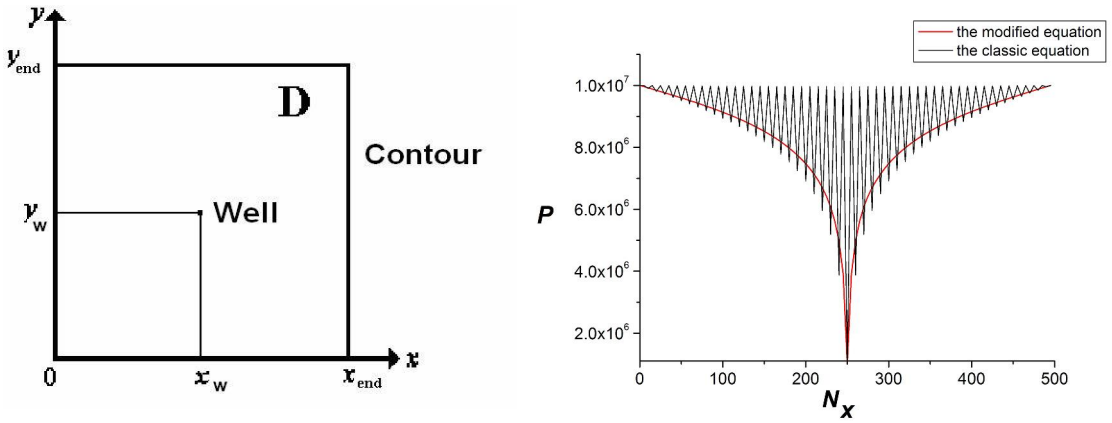


(a) (b)

Figure 1: Plane-radial flow to a well – the problem statement (a) and the obtained pressure (b).

Let us compare the exact solution (17) with results obtained via the classical model (1)-(3) (Model I), via the modified model with the regularizing diffusion term (4), (2)-(3) (Model II) and via the new hyperbolic model (7), (2)-(3) (Model III). Figure 1b reflects the pressure and illustrates this comparison. All the models were transformed into the polar coordinates. At numerical implementations by explicit schemes convective terms of all these models were approximated by central differences like in (14). Model I gives oscillations in the solution at any time step (Figure 1b, line 2). The use of Model II as well as Model III leads to smooth solutions which coincide one with another (Figure 1b, line 3) and with the exact solution (Figure 1b, line 1). In these predictions the next values of small parameters were chosen: the minimal reference length $l \geq 10^{-4}\, cm$ and the minimal reference time $\tau = 2\, s$. However to comply with the stability condition the highest possible time step of the two-level scheme for Model II equals to $\Delta t = 2 \cdot 10^{-4}\, s$ while the three-level scheme for Model III ensures the much greater time step $\Delta t = 2 \cdot 10^{-2}\, s$ at one and the same step of the spatial grid.

As the second example the similar test problem was considered in the two-dimensional geometry in the square domain where the well was placed in the center and the boundaries served as the feeding contour (see Figure 2a). Figure 2b shows the pressure profiles across the well computed on the basis of Model I (the oscillating curve) and on the basis of Model III (the smooth solution). Thus the smoothing effect of the regularizing term in the right-hand side of (7) is demonstrated visually.



(a) (b)

Figure 2: 2D flow to a well – the problem statement (a) and the obtained pressure profiles (b).

Here noticeable increase of the stability threshold is observed at employment of Model III. Table 1 cites data obtained for Models II and III at different steps of the spatial grid when $l = 10^{-5}\, cm$. It is follows from this table that for the three-level scheme the time step $\Delta t \sim h^{3/2}$ is really valid.

h, cm	0.2	0.5	1.0
Two-level scheme: Δt, s	0.008	0.05	0.2
Three-level scheme: Δt, s	0.09	0.35	1.0
τ, s	0.66	1.59	3.26

Table 1: Dependence of the time step from the spatial step for Models II and III.

3 TWO-PHASE FLUID FLOW IN A POROUS MEDIUM

3.1 The governing system of equations

Practically important applications concerned with the porous media flow simulation usually require considering multiphase fluids. For example, ecological problems aimed at prevention of the soil and groundwater contamination by different pollutants assume simulation of the three-phase flow of water, soil air and a so-called Non-Aqueous Phase Liquid (NAPL)[9] – it can be petrol or diesel oil or tetrachloroethylene etc. Besides that the capillary and gravity forces should be taking into account.

In the present paper the case of two-phase fluid flow is investigated. Generalizing results of Section 2 the next system of equations can be written (the subscript $\alpha=w, n$ indicates water or NAPL correspondingly):

$$m\frac{\partial(\rho_\alpha S_\alpha)}{\partial t}+\operatorname{div}(\rho_\alpha \mathbf{u}_\alpha)=q_\alpha+\operatorname{div}\frac{l_\alpha c_\alpha}{2}\operatorname{grad}(\rho_\alpha S_\alpha) \tag{18}$$

$$\mathbf{u}_\alpha=-K\frac{k_\alpha(S_w)}{\mu_\alpha}(\operatorname{grad} p_\alpha-\rho_\alpha \mathbf{g}) \tag{19}$$

$$\rho_\alpha=\rho_{0\alpha}\left[1+\beta_\alpha(p_\alpha-p_{0\alpha})\right] \tag{20}$$

$$S_w+S_n=1 \tag{21}$$

$$p_n-p_w=p_c(S_w) \tag{22}$$

where m is the porosity, S_α is the α-phase saturation, $k_\alpha(S_w)$ is the relative phase permeability, c_α is the sound speed in the α-phase, $\mathbf{g}$ is the gravity vector, q_α is the source of the fluid, $p_c(S_w)$ is the capillary pressure.

The above system is the analog of Model II from the previous section of the paper. Note that here the regularizing term depends on the phase saturation S_α. The minimal reference scales ($l_\alpha = l_w, l_n$) differ for the phases.

The capillary pressure and the relative phase permeability are strongly non-linear functions of the saturation. An analytical determination of the capillary pressure – saturation relation is impossible because of the irregular pore geometry. In the current research the Brooks & Corey constitutive relationships[9] are chosen as the most famous correlations fitted to experimental data:

$$p_c(S)=P_d S_e^{-\frac{1}{\lambda}} \tag{23}$$

$$S_e=\frac{S_w-S_{wr}}{1-S_{wr}} \tag{24}$$

$$k_w(S)=S_e^{\frac{2+3\lambda}{\lambda}},\qquad k_n(S)=(1-S_e)^2\left(1-S_e^{\frac{2+\lambda}{\lambda}}\right) \tag{25}$$

where S_e is the so-called effective saturation, S_{wr} is the residual saturation, P_d is the entry pressure, λ is the indicator of the pour size distribution for the given medium.

The analog of hyperbolic Model III from the previous section of the paper is obtained via replacing equation (18) by the next equation:

$$m\frac{\partial(\rho_\alpha S_\alpha)}{\partial t}+\tau\frac{\partial^2(\rho_\alpha S_\alpha)}{\partial t^2}+\operatorname{div}(\rho_\alpha \mathbf{u}_\alpha)=q_\alpha+\operatorname{div}\frac{l_\alpha c_\alpha}{2}\operatorname{grad}(\rho_\alpha S_\alpha) \qquad (26)$$

3.2 Numerical implementation

Let us develop the computational algorithm for implementation of model (18)-(22) on the base of explicit methods. The sought-for quantities are the NAPL saturation and the water pressure. For simplicity the one-dimensional case is presented.

The two-level scheme with central differences for approximation of equation (18) is written as follows (the subscript α is omitted):

$$m\frac{(\rho S)_i^{j+1}-(\rho S)_i^j}{\Delta t}+\frac{(\rho u)_{i+1}^j-(\rho u)_{i-1}^j}{2\Delta x}=q_i+\left(\frac{lc}{2}(\rho S)_{\bar{x}}\right)_x^j \qquad (27)$$

If to base on equation (26) the three-level scheme similar to (14) should be used:

$$\begin{aligned} m\frac{(\rho S)_i^{j+1}-(\rho S)_i^{j-1}}{2\Delta t}+\tau\frac{(\rho S)_i^{j+1}-2(\rho S)_i^j+(\rho S)_i^{j-1}}{\Delta t^2}+ \\ +\frac{(\rho u)_{i+1}^j-(\rho u)_{i-1}^j}{2\Delta x}=q_i+\left(\frac{lc}{2}(\rho S)_{\bar{x}}\right)_x^j \end{aligned} \qquad (28)$$

Then the algorithm consists of the next stages ($\alpha = w, n$).

1. Calculation of the Darcy velocities for the both phases on the current time level:

$$u_{\alpha\, i}^{\ j}=-K\frac{k_\alpha\left(S_{w\, i}^{\ j}\right)}{\mu_\alpha}\left(\frac{p_{\alpha\, i+1}^{\ j}-p_{\alpha\, i-1}^{\ j}}{2h}-\rho_\alpha \mathrm{g}\right) \qquad (29)$$

2. Searching for $(\rho_\alpha S_\alpha)_i^{j+1}$ on the new time level with the use of scheme (27) (or (28) if to base on equation (26)):

$$(\rho_\alpha S_\alpha)_i^{j+1}=\frac{\Delta t}{m}\left[q_i+\left(\frac{l_\alpha c_\alpha}{2}(\rho_\alpha S_\alpha)_{\bar{x}}\right)_x^j-\frac{(\rho_\alpha u_\alpha)_{i+1}^j-(\rho_\alpha u_\alpha)_{i-1}^j}{2\Delta x}\right]+(\rho_\alpha S_\alpha)_i^j=A_\alpha \qquad (30)$$

3. Obtaining the NAPL saturation and the water pressure on the new time level by means of the next system solution in each node of the spatial grid:

$$\begin{cases} \rho_{0w}\left[1+\beta_w\left(p_{w\, i}^{\ j+1}-p_{0w}\right)\right]\left(1-S_{n\, i}^{\ j+1}\right)=A_w \\ \rho_{0n}\left[1+\beta_n\left(p_{w\, i}^{\ j+1}+p_c\left(1-S_{n\, i}^{\ j+1}\right)-p_{0n}\right)\right]S_{n\, i}^{\ j+1}=A_n \end{cases} \qquad (31)$$

This system can be solved, for example, by Newton's method what takes only a few iterations.

3.3 Test predictions

Among test problems on multiphase porous media flows one of the most interesting and difficult for modeling is the problem on Dense NAPL infiltration into a reservoir filled with several different kinds of sand and fully saturated with water[9, 5]. DNAPL is a non-wetting phase with respect to water, so the process of displacement of the wetting fluid by the non-wetting one under the gravitation influence is considered. For this problem there are experimental data to be compared with numerical results. For the present the new kinetically based approach has been implemented to solve this problem

in a homogeneous medium in the quasi one-dimensional (see Figure 3a) and two-dimensional statements (see Figure 3b). The proposed approach is being further developed for predictions of heterogeneous (sandwich-type) media.

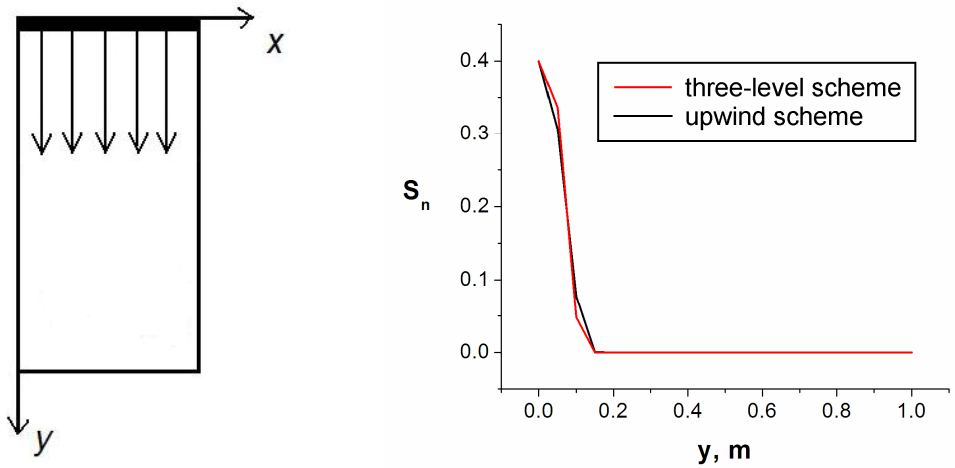
As DNAPL tetrachloroethylene is treated. The medium and fluid properties are taken from works by Prof. R. Helmig[9]:

$$m = 0.4, \quad \mathrm{K} = 6.64 \cdot 10^{-11} m^2, \quad \lambda = 2.7, \quad P_d = 755\,Pa, \quad S_{wr} = 0.09,$$
$$\rho_{0w} = 1000 \frac{kg}{m^3}, \quad \rho_{0n} = 1460 \frac{kg}{m^3}, \quad \mu_w = 10^{-3} \frac{kg}{m \cdot c}, \quad \mu_n = 9 \cdot 10^{-4} \frac{kg}{m \cdot c} \tag{32}$$

The model parameters in the current computations are as follows:

$$l_w = 10^{-6} m, \quad l_n = 10^{-7} m, \quad c_w = 1407 \frac{m}{s}, \quad c_n = 1225 \frac{m}{s},$$
$$\beta_w = \beta_n = 10^{-4} \frac{m \cdot s^2}{kg} \tag{33}$$

At the initial moment the water pressure has the hydrostatic distribution, DNAPL is absent in the domain. In the quasi 1D statement the whole top boundary is a source of DNAPL, in the 2D statement only some central part of the top boundary is a source which is described by the boundary condition S_n=0.4 while q_n=0. The pressure on the top boundary is atmospheric.



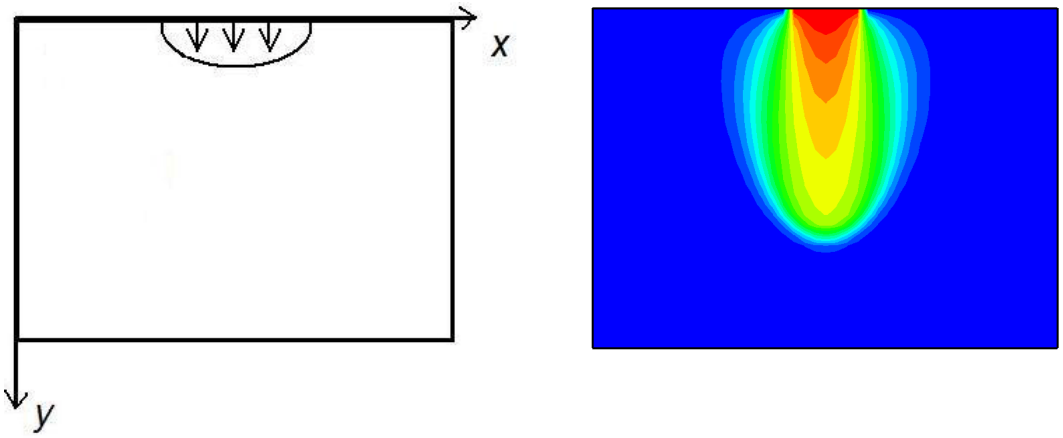
(a) (b)

Figure 3: Quasi 1D infiltration problem statement (a) and obtained DNAPL saturation profiles (b).

First of all on the example of the quasi 1D problem the comparison of approaches based on equation (18) and on equation (26) has been performed using the algorithm from Subsection 3.2. Obtained results have completely coincided what confirms the adequacy of model (26), (19)-(25).

Further this model and the corresponding three-level scheme (28) are verified by the comparison with the classical model without any regularizing terms (like equation (1)). The same computational algorithm is used for the classical model implementation but the phase continuity equations are approximated via the conditionally stable two-level scheme with upwind differences for convective term approximation what is rather traditional. Figure 3b shows very close agreement of DNAPL saturation profiles.

Figure 4b demonstrates the DNAPL saturation field at some time moment obtained on the basis of model (26), (19)-(25) for the 2D problem. This field has the typical qualitatively correct view. Red colour corresponds to the maximal value of the saturation.



(a) (b)

Figure 4: 2D infiltration problem statement (a) and the obtained DNAPL saturation field (b).

Comparison of the new hyperbolic model and the classical model in the 2D statement is reflected by Figure 5 where the DNAPL saturations in the middle cross section are presented. Good agreement is observed: solution fronts differ within one computational grid cell.

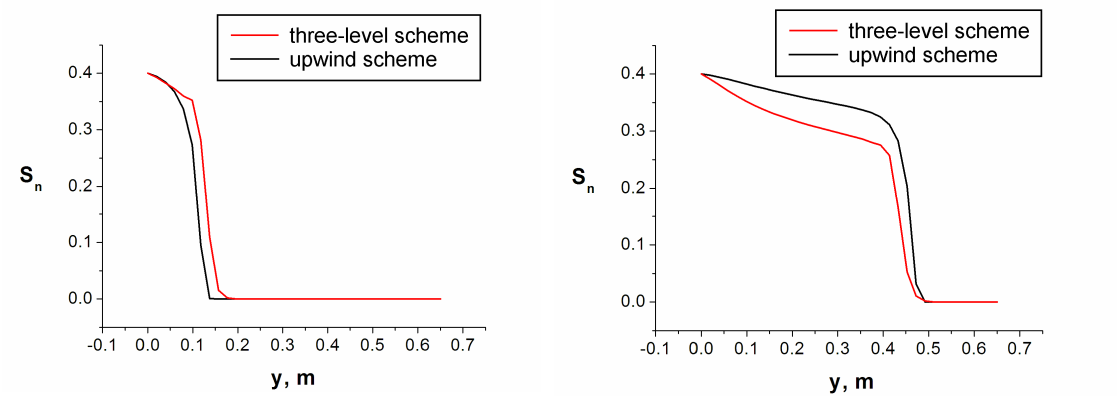


Figure 5: DNAPL saturation profiles at different time moments for the 2D infiltration problem.

Table 2 illustrates the time step restriction and τ-parameter values for the three-level scheme when solving the 2D problem on grids of different coarseness. In the current implementation the time step $\Delta t \sim h$ is achieved.

h_x, m	0.05	0.02045	0.1
h_y, m	0.05	0.01969	0.1
Δt, s	0.05	0.01	0.1
τ, s	1.0	0.02	0.2

Table 2: Dependence of the time step from spatial steps for the 3-level scheme on the 2D problem

4 CONCLUSIONS

A new model of the hyperbolic type has been proposed for simulation of porous media flows including two-phase fluid flows. The model is developed by the analogy with kinetically-consistent finite difference schemes and the related quasi-gas dynamic system of equations basing on the principle of minimal sizes i.e. taking into account minimal reference scales on space and on time. The explicit three-level scheme with a mild stability condition has been proposed for the modified continuity equation approximation. The new approach has been verified by comparison with traditional ones via numerical simulation of a number of test problems.

The proposed approach is very promising for HPC. At present the algorithms developed are being implemented on a graphic cluster using CUDA libraries. In the future the new model and parallel algorithms will be used for solution of large-scale applied problems based on real data.

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