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INVESTIGATION OF SLIP BOUNDARY CONDITIONS OF HYPERSONIC FLOW OVER MICROPOROUS SURFACES

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Abstract. Since hypersonic boundary-layer-transition is dominated by acoustic second mode instabilities, so-called Mack modes, one strategy to delay or prevent the transition process is to damp these modes passively by acoustic absorptive coatings which are realised in practise by porous walls. In the present paper a second mode stability analysis is performed for a boundary layer flow at Mach 6 over various porous walls. The influence of rarefied gas behaviour of the flow inside the pores is investigated by comparing a slip boundary condition with finite Knudsen numbers with a non-slip boundary condition for different radii and depths of the pores. For this study two different approaches are used: Linear stability analysis compared with direct numerical simulations including the modelling and resolution by a 4th order Navier-Stokes solver. Good agreement for different porous wall cases with adiabatic as well as cold wall condition is demonstrated. The approach of a slip boundary condition for rarefied gases, modelling a Knudsen-layer along the pore walls, results in all cases in an additional damping effect that can be explained physically.

1 INTRODUCTION

The ability to stabilize a hypersonic boundary layer for re-entry vehicles during flight to prevent transition is of high importance since skin friction drag and heat transfer rates in a turbulent boundary layer can be several times higher than those of a laminar one. A lot of different strategies are used to delay or prevent the transition process. In this work the manipulation of the transition by the use of porous surfaces is performed to influence the growth of the second mode in a passive way. The second or so called Mack mode¹ is the dominant mode for the transition process at hypersonic Mach numbers.

Malmuth et al.² have shown by inviscid linear stability theory a strong stabilisation effect of the second mode using a passive porous surface. In a continuation study Fedorov et al.³ confirmed the results by the use of viscous linear stability theory. Experiments of Rasheed and Hornung⁴ on a 5° half-angle sharp cone ($M_{\infty} \sim 5 - 6$) with micropores qualitatively confirmed the theoretical prediction. The working group around Fedorov performed further analyses and experiments. A good summary, which also includes an overview of the slip boundary condition for rarefied gases, is given by Maslov et al.⁵. Twodimensional direct numerical simulation studies that resolve the pores are presented by Bres et al.^{6,7}, where the linear stability theory code was verified. A study of the effect by direct numerical simulations with a three-dimensional pore flow is presented by Sandham and Lüdeke⁸.

In this paper two simulation techniques are used: Direct Numerical Simulation (DNS) by a 4th order finite difference version of the DLR FLOWer $code^{8,9}$, which is used to provide complete solutions of the boundary layer flow over and the flow inside the pores, compared with the DLR spatial linear stability code NOLOT¹⁰. Former investigations, performed with non-slip boundary conditions inside the pores, show good agreement between these codes for different porous walls¹¹. The present study investigates the influence of rarefied gases inside the pores modelled by a slip boundary conditions at finite Knudsen number Kn, which is the ratio of characteristic length scales. As shown by Maslov et al.⁵, the flow inside the pores cannot always be treated as a continuum, since the mean free path of the fluid particles inside the cavities has a significant value in comparison with the pore diameter, so generally the Knudsen number based on this diameter is no longer small. Within the present study, the influence of the slip boundary condition at different Knudsen numbers in comparison with the non-slip boundary condition calculated by direct numerical simulation and linear stability theory will be shown. Good agreement of both methods by a comparison for Mach 6 boundary layer flow over different porous walls with adiabatic as well as cold walls is demonstrated. The slip boundary condition generated in all cases an additional damping effect which will be explained physically in the following.

2 Numerical methods

2.1 Direct Numerical Simulation by a 4th order finite difference method

For the DNS calculations a high-order variant of the DLR FLOWer code is used. The basic FLOWer code solves the compressible Reynolds-averaged Navier-Stokes equations, which are written in the conservation form, under perfect gas assumption on block-structured grids using second order finite volume techniques and cell-centred or cell vertex variables. The high-order version uses a fourth-order central differencing based on compact finite differences in a cell-centred formulation. Additional high-order compact filters are implemented that are applied at the end of each time step¹⁴. The DNS is initialised by an artificial disturbance at t = 0 to allow the development of the eigenmode within the calculation. The initial disturbance, which is applied for non-slip as well as for the slip boundary conditions, is a rough approximation of a second mode eigenfunction and is expressed as:

$$v = 0.0001 \exp\left[-4(y - 0.8)^2\right] \sin(2\pi x/L_x)$$
(1)

Due to the fact, that inside the pores with a small radius r the mean free path λ of the molecules becomes comparable to the relevant length scales of the shear layer, the flow cannot always be treated by a continuum approach⁵. Since the Knudsen layer thickness is of the order of the mean free path and outside this layer the gas can be treated again as a continuum, the influence of the Knudsen-layer is taken into account as an additional boundary condition. In a first-order approximation the boundary conditions for a small, finite Knudsen number are:

$$u_w = A_u Kn\left(\frac{\partial u}{\partial r}\right) , \quad T_w = A_e Kn\left(\frac{\partial T}{\partial r}\right)$$
 (2)

with u_w and T_w as fluid-velocity and temperature at the surface. The Knudsen number is defined as:

$$Kn = \frac{4\eta}{\rho \,\bar{c} \,r} \tag{3}$$

where η is the dynamic viscosity, ρ the density, $\bar{c} = \sqrt{8RT/\pi}$ the mean molecular velocity with the gas constant R and the gas temperature T. The dimensionless factors A_u and A_e are outcomes of the gas kinetic theory where they are a measure of the interaction of the gas molecules with the molecules of the wall¹². They can be expressed in terms of the molecular tangential impulse and the energy accommodation coefficients α_u and α_e .

$$A_u = \alpha_u^{-1} - 0.5$$
, $A_e = 2\gamma(\alpha_e^{-1} - 0.5)/(\gamma + 1)$ (4)

In this paper the accommodation coefficients for DNS as well as for linear stability theory are approximated by $\alpha_u = \alpha_e = 0.9$ which is taken from references^{5,13}.

The pores are described as rectangular cavities in a way that the grids match at the interface, removing interpolation as a possible source of error. For this study the simulations for the DNS were performed two-dimensional. Thus the pores are reduced to spanwise grooves with parallel sides and the half-width b.

2.2 Linear Stability Theory and absorptive boundary conditions

The NOLOT code¹⁰, which is a spatial linear stability code, was developed in cooperation between DLR and FOI and can be used for local as well as non-local analyses. In this work the local linear spatial approach is used which is a subset of the nonlocal stability equations. The equations are derived from the equations of conservation of mass, momentum and energy, which govern the flow of a viscous, compressible, ideal gas, formulated in primitive variables. All flow and material quantities are decomposed into a steady laminar basic flow \bar{q} and an unsteady disturbance flow \tilde{q} .

$$q(x, y, z, t) = \bar{q}(x, y) + \tilde{q}(x, y, z, t)$$
(5)

The disturbance \tilde{q} is represented as a harmonic wave

$$\tilde{q}(x, y, z, t) = \hat{q}(x, y) \exp[i(\alpha x + \beta z - \omega t)]$$
(6)

with the complex-valued amplitude function \hat{q} . In the following the hat over a variable denotes an amplitude function. Since NOLOT is a spatial code the wavenumbers α and β are complex quantities and the frequency ω is a real value. $-\alpha_i$ is the complex growth rate. The given boundary conditions in NOLOT for a smooth wall (at y = 0) are:

$$\hat{u}_w, \, \hat{v}_w, \, \hat{w}_w, \, \hat{T}_w = 0.$$
 (7)

The NOLOT code is validated with the help of several test cases against published results, including DNS, PSE (parabolized stability equations), multiple scales methods and LST. A good summary of the validation is given by Hein et al.¹⁰. For the treatment of porous walls with non-slip boundary condition and slip boundary condition, additional boundary conditions were implemented within this study. The validation of the non-slip boundary condition is given in¹¹. Both boundary conditions for porous walls are taken from Maslov et al.⁵ and Koslov et al.¹³, a complete derivation can be found in these references. The conditions are written by:

$$\hat{u}_w, \ \hat{v}_w = 0, \quad \hat{w}_w = A\hat{p}_w, \quad \hat{T}_w = B\hat{p}_w \tag{8}$$

The thermal admittance B was found to have a marginal effect $(< 1\%)^3$ and is neglected in the present work. The admittance A is:

$$A = \frac{n}{Z_0} \tanh(md) \tag{9}$$

The investigated pores are equally spaced blind pores with a depth d, which is normalised by the displacement thickness δ , and the porosity n. The characteristic impedance Z_0 and the propagation constant m are:

$$Z_0 = -\frac{\sqrt{\frac{\hat{\rho}}{\hat{C}}}}{Ma\sqrt{T_w}} , \qquad m = \frac{i\omega Ma\sqrt{\hat{\rho}\hat{C}}}{\sqrt{T_w}}$$
(10)

where the dimensionless complex dynamic density $\hat{\rho}$ and dynamic compressibility \hat{C} are expressed in following form

$$\hat{\rho} = \frac{1}{1 - F_1(B_u, \Lambda)} , \qquad \hat{C} = 1 + (\gamma - 1)F_2(B_E, \hat{\Lambda})$$
(11)

The functions F depend on the shape of the pores. As described above the DNS simulations here were performed as 2D, thus the pores are reduced to spanwise grooves. Corresponding to this approach a split boundary condition for the linear stability solver was chosen:

$$F_1(B_v,\Lambda) = \frac{\tan\Lambda}{\Lambda[1 - B_u\Lambda\tan\Lambda]} , \quad F_2(B_E,\hat{\Lambda}) = \frac{\tan\hat{\Lambda}}{\hat{\Lambda}[1 - B_E\hat{\Lambda}\tan\hat{\Lambda}]}$$
(12)

The macroscopic parameter Λ is $b\sqrt{i\omega\rho_0/\eta}$ and $\hat{\Lambda}$ is $\Lambda\sqrt{Pr}$. The pore half-width b is normalised with the displacement thickness. While the dimensionless flow quantities are normalised by the values of the boundary layer edge. ω represents the dimensionless angular frequency. The factors B_u and B_e are expressed in terms of the accommodation coefficients α_u and α_e :

$$B_u = (\alpha_u^{-1} - 0.5)Kn , \quad B_e = (2\gamma(\alpha_e^{-1} - 0.5)/(Pr(\gamma + 1))Kn$$
(13)

It has to be pointed out, that this boundary condition just includes the absorptive effect of the porous layer without taking into account any roughness effects of the pores. Nevertheless, the influence of the acoustic absorption on the first modes can be investigated well by this approach.

2.3 Introduction of the Gaster Transformation

Due to the fact that the LST code is a spatial code while the DNS uses a temporal approach for the mode-development, obviously a transformation for the comparison of the growth rate is necessary, which is well known as Gaster transformation¹:

$$\omega_{i,temporal} = \sigma_{spatial} \ c_{gr} \tag{14}$$

where ω_i is the temporal growth rate, σ the spatial growth rate and c_{qr} the group velocity:

$$c_{gr} = \frac{\partial \omega_r}{\partial \alpha_r} \tag{15}$$

The Gaster transformation is an approximation that is valid for small growth rates. Within the approximation, the real part of the frequency and the real part of the wavenumber of the spatial wave are the same as for the temporal wave.

3 Grids and flow conditions

The basic flow parameters for the studied porous wall cases and, for completeness, for a smooth wall are: $M_{\infty} = 6$ at a Reynolds number of Re = 20000, a Prandtl number of Pr = 0.72 and a ratio of specific heats of $\gamma = 1.4$. The viscosity μ is prescribed by Sutherland's law with a constant of 110.4K and a reference temperature of 216.65K, leading to an adiabatic wall temperature of 1522.44K.

For the wall normal grid distribution a stretching function is used which places most points near the wall in an analytical way. The distribution is given by a sinh function with an iteratively determined stretching factor. In the wall normal direction 401 points are set for both codes in the same way. For the DNS a 2D grid with 64 equally spaced points in streamwise direction is generated. The number of points for y > 0 is again 401 and the additional points within the pores (y < 0) depend on the depth of the pores. The pore depth and all coordinates are normalised by the displacement thickness δ . As already mentioned the calculations of pores were performed as two-dimensional spanwise grooves with parallel side walls. The porosity n for all cases is 0.25.

4 Simulations with adiabatic wall

For this study two different Knudsen numbers (0.1 and 0.2) were chosen. These deliberately selected values for the Knudsen numbers result from planned wind tunnel tests at DLR facilities, using cones with porous surfaces. The wind tunnel conditions corresponding to a flight altitude of 28 km, where the Knudsen number calculated by equation 3 is in a range of 0.1. For the first case the artificial Knudsen number is set to 0.2. The pore half-width b is 0.0234375, corresponding to 16 pores for the DNS, and a pore depth d of 0.8 is chosen. The pore half-width of the second case is with 0.046875 twice as large as of the first test case and corresponds to 8 pores for the DNS. To be consistent to the previous Knudsen number and equation 3 ($\mathbf{Kn} = 4\eta/(\rho \bar{c} \mathbf{b})$), for the second case, with a double pore half-width b, the Knudsen number is set to 0.1. A pore depth of 1.5 is chosen. The reason for the selected pore depths will be shown in section 4.3. There the influence of the pore depth will be described and that only with a certain pore depth a nearly constant growth rate is reach, which is obtain in these cases with the chosen pore depths. An overview of the test cases, including the smooth wall case with d = 0, is shown in table 1.

4.1 Test cases with 16 pores - slip boundary condition at Kn = 0.2

In this subsection a detailed view of the test cases with 16 pores with a Knudsen number of 0.2 for the slip boundary condition is given.

Figures, that reapply in the following sections, like test cases with 8 pores and cold wall cases, will be explained here in detail and won't be repeated there.

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	boundary	number	pore half-width	pore depth
	condition	of pores	b	d
smooth wall	-	0	0	0
porous wall	non-slip	16	0.0234375	0.8
porous wall	slip at $Kn = 0.2$	16	0.0234375	0.8
porous wall	non-slip	8	0.0468750	1.5
porous wall	slip at $Kn = 0.1$	8	0.0468750	1.5

Table 1: Overview of the test cases with adiabatic wall

4.1.1 Spatial growth rate - LST results

For the further detailed comparison of both codes the maximum growth rate of the second mode have to be found, because for this value the highest damping effect is expected. With the LST a fast method is provided to calculate the whole spectrum of the amplified growth rates, which is given in figure 1. The stability diagram shows for the first mode of the smooth wall a maximum growth rate σ of 0.0033. Due to the fact that the chosen pores are small enough, the absorption effect on the first mode generates only a slight increase with the porous walls. The most unstable mode is the second mode with a maximum growth rate of 0.0453 at a wave number of $\alpha_r = 2.2$. The figure demonstrates the reduction of the second mode amplitude due to the pore effect of absorbing parts of the disturbance energy: With the chosen wall porosity and non-slip boundary condition the maximum growth rate is reduced by 36.4% in comparison with the smooth wall case. The modelling of rarefied gas effects by the slip boundary condition leads to another reduction of 4.5% or related to the maximum rate for non-slip boundary conditions: The maximum growth rate with slip is 7.1% smaller than the non-slip value. Another effect, visible in figure 1, is a shift in the wavenumber for the respective cases with- and without wall porosity. To keep the following DNS calculations comparable, a fixed wave number α of 2.094, which is near the second mode maxima, is chosen.

In the following subsections all spatial LST results are changed by Gaster Transfor-



Figure 1: Spatial growth rate against real part of the wavenumber α calculated by LST - 16 pores

mation (equation 14) into temporal results to compare with DNS.

4.1.2 Mack mode developing - DNS results

The chosen fixed wave number of 2.094 corresponds to a domain length L_x of 3, calculated by the following equation:

$$L_x = \frac{2\pi}{\alpha} \tag{16}$$

That means it is a wavelength of 3 times the displacement thickness. Using this fixed domain length L_x , figure 2 shows DNS results from Mack modes developing on a smooth surface in comparison with the porous surfaces. The left side contour plot shows the normal velocity in a boundary layer over a smooth wall, the middle and right figure demonstrate the reduction of the Mack mode amplitude. This is visible by comparing the legends of the figures: The values with pores are one order of magnitude smaller. For the case with slip boundary condition at Kn = 0.2 (right) an additional reduction is visible by comparing the ends of the legends with the non-slip boundary condition (middle). The flow through the pores using slip boundary condition are comparable with a non-slip boundary condition but an effectively larger diameter. So practically the pores widened for the instabilities which results in an increasing damping effect by taken into account the Knudsen number, as described by Maslov et al.⁵.



Figure 2: DNS (contours of v): (a) Mack mode developing above a smooth wall, (b) a porous surface with non-slip boundary condition and (c) slip boundary condition at Kn = 0.2 at nearly the same time step

4.1.3 Comparison of LST with DNS: Eigenfunctions

It has to be pointed out once more, that both codes provide completely different approaches, so the comparability of the eigenfunctions has to be shown explicitly. This is done in figure 3 for smooth wall, porous surface and the additional consideration of the Knudsen-layer. The eigenfunctions compare well, though for the slip boundary-condition in figure c differences in the second maximum are visible.



Figure 3: Comparison of the amplitude functions - 16 pores: (a) smooth wall, (b) porous surface with non-slip boundary condition and (c) slip boundary condition at Kn = 0.2

4.1.4 Comparison of LST with DNS: Growth rates

Figure 4 shows the development of the natural logarithm of the v-perturbation root mean square, integrated in y direction. As the DNS simulation proceeds after some time steps the unstable mode has emerged from the rough approximation, given as startup definition and grows strongly over the remainder of the simulation. The growth rates are extracted from the slopes of the lines. In all cases the DNS predicts slightly lower growth rates than LST (for exact values see table 2). A very good agreement between the methods for a smooth wall at 0.8% difference, as well as for the porous wall case with non-slip at 3.1% difference can be seen. A difference of 7.6% for the case with slip boundary condition is also in a acceptable range for two completely different calculation methods. For a better comparison of the damping effect, table 3 shows the percental damping, dependent on the growth rate of the smooth wall case and on the porous wall case with non-slip boundary condition. A significant additional damping effect, resulting from the modelling of Knudsen layer, is visible for both prediction methods: The damping, expressed in relation to the porous wall case using non-slip boundary conditions is 10.8% for the LST simulation.



Figure 4: DNS values of disturbance amplitude and LST growth rates - 16 pores: (a) smooth wall, (b) porous surface with non-slip boundary condition and (c) slip boundary condition at Kn = 0.2

ſ		smoo	th wall	porous wall	porous wall
		0	ω_i	ω_i with non-slip	ω_i with slip at $Kn = 0.2$
Ì	DNS	0.0	3344	0.02464	0.02199
	LST	0.0	3372	0.02541	0.02366
	difference	0.	8%	3.1%	7.6%
L	Table 2: Comparison of the growth rate - 16 pores - pore depth $d = 0.8$			pores - pore depth $d = 0.8$	
	bound	lary	ω_i		ω_i
	condi	tion	(% of smooth wall case)		(% of porous non-slip wall case
DN	S non-s	lip		73.7	-
LSI	Γ non-s	lip		75.4	-
DN	$S \mid slip Kn$	= 0.2	65.8		89.2
LST	$\Gamma \mid \text{slip } Kn$	= 0.2		70.2	93.1

Table 3: Comparison of the damping effect - 16 pores - pore depth d = 0.8

4.2 Test cases with 8 pores at Kn = 0.1

The next test cases has been performed with 8 pores, which results to a pore half-width twice as large as of the former ones with 16 pores, as described at the beginning of section 4. For this reason the Knudsen number has with 0.1 half the size of the previous 16 pore geometry.

The amplitude growth rates are illustrated in figure 5 including the smooth wall case



Figure 5: DNS values of disturbance amplitude and LST growth rates: (a) smooth wall, (b) porous surface with non-slip boundary condition and (c) slip boundary condition

from figure 4 for a better visual comparison of the slope. In addition the previous test cases are plotted in grey in the diagrams for an easier comparison. As in the previous test cases, the predicted growth rates, extracted of figure 5, with LST are slightly larger than with DNS (for exact values see table 4). Due to the larger diameter the damping effect is higher for 8 pores than for 16. The additional damping effect of the slip boundary condition is naturally lower for the smaller Knudsen number, but still visible for both prediction methods: The damping for the DNS, using slip boundary conditions, expressed in relation to the porous wall case using non-slip boundary conditions is 4.7% (see table 5).

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ſ		smoo	th wall	porous wall	porous wall
		ω_i		ω_i with non-slip	ω_i with slip at $Kn = 0.1$
ľ	DNS	0.03344		0.01769	0.01686
	LST	0.0	3372	0.01834	0.01768
	difference	0.	8%	3.7%	4.9%
L	Table 4: Comparison of the growth rate - 8 pores - pore depth $d = 1.5$				
	bound	lary		ω_i	ω_i
	condit	tion	(% of s)	mooth wall case)	(% of porous non-slip wall cas
DNS	S non-s	slip		52.9	_
LST	non-s	lip		50.4	-
DNS	$S \mid slip Kn$	slip $Kn = 0.1$		54.4	95.3
LST	$\Gamma \mid \text{slip } Kn$	u = 0.1		52.4	96.4

Table 5: Comparison of the damping effect - 8 pores - pore depth d = 1.5

4.3 Influence of the pore depth

A detailed study of the influence of the pore depth, including the just listed cases, has been performed by varying the pore depth for DNS as well as LST. The result is visible in figure 6, where the growth rate as a function of the pore depth is plotted: Figure 6a shows the variation of the pore depth d for the test cases with 16 pores and figure 6b for 8 pores.



Figure 6: Variation of the pore depth: Temporal growth rate ω_i versus pore depth d(a) 16 pores and (b) 8 pores

The points at d = 0 denote the previously shown smooth wall case while the points at d = 0.8 (6a) and at d = 1.5 (6b) represent the porous ones for which a detailed comparison was given. As mentioned before, DNS predicts a little bit stronger damping effect of the pore depth than LST, but the trend of the function is in all cases identical. For deeper pores the resulting growth rates saturate towards a limit value. Fedorov³ described for an example test case with cylindrical pores that a pore depth of about 5 diameters is

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	boundary	16 pores	8 pores
	condition	limit-pore depth	limit-pore depth
LST	non-slip	0.4	1.2
LST	$_{\rm slip}$	0.5	1.4
DNS	non-slip	0.5	1.2
DNS	slip	0.6	1.5

Table 6: Comparison of the limit-pore depths

sufficient for the appropriate damping rate, which is within the range of our results. In addition to the pore width, the disturbance frequency plays a certain role. In the following the minimum pore depth for which the variation of the growth rate is less than 0.5% for all larger *d*-values, is denoted as the limit-pore depth and the corresponding growth rate as the limit-growth rate. This defined limit-pore depth increase with larger pore width.

By modelling of Knudsen layer the pore depth shows a stronger effect in all cases because of the effectively larger pore width, as described in section 4.1.2. With the effectively widened pores the transient oscillation behaviour, visible for example in the shifts of the first minima and second maxima in figure a and b, changes and consequently the limit pore depth increase, as described before. For the test cases with 8 pores the limit-pore depth increase from 1.2 to 1.4 (LST) and for the DNS from 1.2 to 1.5. The same trend can be seen for test case with 16 pores in table 6.

5 Simulations with cold wall

Because hypersonic wind-tunnel tests are usually carried out in blowdown facilities in which the measurement time is too short to increase the wall temperature significantly, simulations with a low constant wall temperature were carried out. The wall temperature was chosen with $T_w = 1.35 \cdot T_{\infty}$. In all cold wall cases the pore half-width is set to b = 0.046875, corresponding to 13 pores for the DNS, and the Knudsen number has a value of 0.1. The reason for this change in the pore number is given in 5.1.1.

5.1 Cold wall test cases with 13 pores in comparison with the previous adiabatic wall cases with 8 pores

In this subsection a detailed investigation of test cases at a pore depth of d = 6.0, where a nearly constant growth rate is reached, is carried out. The chosen pore diameters of these cases are comparable to the adiabatic ones with 8 pores. In table 7 the compared test cases of this section are listed. The reason for the chosen higher pore depth in comparison with the adiabatic wall cases results of an increase of the limit-pore depth. The explanation for that behaviour is following in section 5.2 (Influence of the pore depth).

	-	-			
	wall	boundary	number	pore half-width	pore depth
	condition	condition	of pores	b	d
smooth wall	cold	-	0	0	0
porous wall	cold	non-slip	13	0.0468750	6.0
porous wall	cold	slip at $Kn = 0.1$	13	0.0468750	6.0
These pore quantities correspond to the adiabatic wall case with:					ith:
smooth wall	adiabatic	-	0	0	0
porous wall	adiabatic	non-slip	8	0.0468750	1.5
porous wall	adiabatic	slip at $Kn = 0.1$	8	0.0468750	1.5

Table 7: Overview of the test cases with cold wall including the comparison cases with adiabatic wall

5.1.1 Spatial growth rate - LST results

Figure 7 shows the whole spectrum of the amplified growth rate spectrum of the second mode using cold wall condition. The first mode is omitted because with cold wall condition the maximum growth rate of the first mode of the smooth wall case is completely damped and even with pores the value is below 10^{-4} . For the smooth wall case the second mode has a maximum growth rate of 0.0486 at the wave number $\alpha_r = 1.36$ and decrease strongly to a value of 0.0123 using non-slip or 0.0119 by using the slip boundary condition. The maximum growth rate wavenumber of 1.36 for the smooth wall is nearly 1.6 times lower than for the adiabatic wall case. According to this, the disturbance frequency is also 1.6 times lower. The reason for that behaviour could be found in the constant wall temperature: As mentioned before, the adiabatic wall temperature has a value of 1522.44K, over 5 times larger than the cold wall temperature of 292.48K. Thus for the cold wall the local viscosity change strongly and, as a consequence, the boundary layer profiles of the flow quantities. This results in a change of wavenumber and frequency.

Due to the change in the position of the maximum, the wave number α_r for all cold wall cases is set to a fixed value of 1.289, resulting in a domain length L_x of 4.875, calculated by equation 16. For holding the porosity and the pore width of the previous adiabatic wall cases, the pore number for the DNS was set to 13 to provide comparability with the adiabatic 8 pore cases (see table 7).

5.1.2 Comparison of LST and DNS: Limit-Growth rate

Figure 8 shows once more the development of the natural logarithm of the root mean square of the v perturbations. For a better visual comparison of the slope, the previous adiabatic test cases are plotted in grey in the diagrams. With cold wall condition the DNS needs more time steps till the unstable mode has emerged from the startup definition. In contrast to the adiabatic wall, the slope for the cold wall, corresponding to the extracted growth rates of the DNS, are a little bit higher than those predicted by LST. Furthermore, differences between DNS and LST for porous walls are significantly higher than for the



Figure 7: Spatial growth rate against real part of the wavenumber α calculated by LST - 13 pores



Figure 8: DNS values of disturbance amplitude and LST growth rates: (a) smooth wall, (b) porous surface with non-slip boundary condition and (c) slip boundary condition with Kn = 0.1

adiabatic wall cases (see table 8). The reason is the extremely high damping effect for cold walls: With the porous wall (non-slip condition) the growth rate has only 30.5% of the original value with LST and 32.9% for DNS (see table 9).

As shown by Fedorov³, the damping effect of the pores increases as the wall temperature decrease at the same free stream condition. This trend is consistent with the admittance asymptotic behaviour associated with equations (9) - (12). For deep pores (|md| >> 1) of relative small pore width ($|\Lambda| << 1$), the admittance A is proportional to $Ma\sqrt{T_w}$ and decrease with the wall temperature.

Because of the high damping rate the effect of the slip boundary condition using cold wall condition is on the first glance smaller in comparison with the adiabatic wall case, but expressed in relation to the porous wall case using non-slip boundary condition, the damping for the LST with adiabatic wall is 3.6% while it is for the cold walls 2.1% which is within the same range.

5.2 Influence of pore depth

To investigate the influence of the pore depth for cold walls, figure 9 shows the growth rate as a function of the pore depth. Due to the significantly larger number of points along the boundary layer for cold walls, only the spatial linear stability results without Gaster Transformation are calculated.

Since the boundary layer profiles change due to the local viscosity as well as the

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	smooth wall	porous wall	porous wall
	ω_i	ω_i with non-slip	ω_i with slip at $Kn = 0.1$
DNS	0.03188	0.01050	0.01021
LST	0.03092	0.00942	0.00922
difference	3.0%	10.3%	9.7%

Table 8: Comparison of the growth rate - 13 pores - pore depth d = 6.0

	boundary	ω_i	ω_i
	condition	(% of smooth wall case)	(% of porous non-slip wall case)
DNS	non-slip	32.9	-
LST	non-slip	30.5	-
DNS	slip $Kn = 0.1$	32.0	97.2
LST	slip $Kn = 0.1$	29.8	97.9

Table 9: Comparison of the damping effect - 13 pores - pore depth d = 6.0

wavenumbers and frequencies, the limit-pore depth for cold walls is with a value of 4.9 much higher than the value calculated with adiabatic walls. An additional increase up to 5.8 results for the slip boundary condition from the effectively widened pores. However, the same trend as before for the adiabatic wall is visible: With modelling of the Knudsen layer the pore depth has a stronger effect in all cases because of the effectively larger pore width. For a better comparison the envelopes are plotted as dashed lines. An interesting detail in this case are even negative σ values, so there are ranges of pore depth where the modes are fully damped. Furthermore the wavelength for the oscillations of σ is comparable with the wavelength for adiabatic cases (see figure 6) which is explicable by the physical fact, that the wall-normal extent of the modes is not strongly affected by the wall temperature. So mainly the damping part in the admittance dependency on d is affected by the variation of the flow quantities.

6 Conclusion

In the present study a second mode stability analysis was performed for a boundary layer flow at Mach 6 over different porous walls with and without slip boundary condition to consider the rarefied gas effect inside the pores. The influence of the Knudsen boundary condition was investigated for different pore diameters, depths and Knudsen numbers with adiabatic as well as cold walls. Two different approaches were compared successfully: The linear stability results, calculated by the DLR code NOLOT in comparison with direct numerical simulations including the resolution of the pores. It has been shown that the trends of the predicted functions of both codes are identical, the limit-pore depths are nearly the same and differences for the predicted limit-growth rate are in a reasonable range.

The investigation of the slip boundary condition shows in all cases an additional damp-



Figure 9: Variation of the pore depth - spatial LST results: Spatial growth rate σ versus pore depth d

ing effect compared with the non-slip approach, resulting from an effective increase of the pore diameter by this effect. For Knudsen numbers below 0.1 and high porous damping rates, a negligence of the slip boundary condition could be demonstrated. For higher Knudsen number the additional damping effect becomes significant and has therefore to be taken into account.

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