PARAMETRIZED DISCRETE EMPIRICAL INTERPOLATION OF NONLINEAR IMPLICIT EVOLUTION OPERATORS

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ABSTRACT

Many important problems from physics, chemistry or biology can be modeled by non-linear partial differential equations. If such problems are parameter dependent, additional terms can appear through possibly non-linear parametrization functions. Typically, in such situations numerical computations for parameter studies are quite time-consuming.

In this presentation, we introduce a new discrete empirical interpolation technique for the approximation of non-linear spatial differential operators that are used in fully implicit time discretizations. Such interpolation techniques are of particular interest when the reduced basis method is applied to reduce the complexity of the approximation scheme. With the help of empirical interpolation for non-linear functions [1], reduced basis methods have already been successfully applied to finite element schemes of elliptic and parabolic problems [2].

Here, we consider general numerical schemes with a focus on finite volume discretizations of evolution problems. In [3] it is shown how to extend the empirical interpolation idea to non-linear spatial operators in order to apply the reduced basis framework to explicit finite volume schemes with non-affine parameter dependence. The main ingredients are a reduction of the numerical grid to a smaller subset and the storing of a local representation of the reduced basis vectors on this subgrid to be used in the online phase. For implicit discretizations, however, the empirical interpolation needs to be processable by a nonlinear system solver with access to a Fréchet derivative. We show how the derivative of an interpolated differential operator can be formulated efficiently by a reduction to the subgrid under reasonable sparsity assumptions on the operator and the high dimensional discrete function space.

Experimental results with error convergence and runtime studies are given for a finite volume discretization of a parametrized convection-diffusion-reaction problem.

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