A NEW APPROACH TO SOLVE MICROFLUIDIC SYSTEMS

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ABSTRACT

We study a widely used mathematical model for electrophoresis [1]. The model is naturally composed of 2 interacting blocks. The first is the Stokes system which governs the motion of the ambient fluid where the charged species of interest are. The second block is a nonlinear convection diffusion system for charged species and elliptic equation for electric potential. The real challenge comes from the physically reasonable charge neutrality assumption: charge distributions for different species add up to zero. This constraint makes the system essentially overdetermined, and leads immediately to problems with standard numerical methods which are designed to deal only with square systems (as many equations as unknowns).

In this work we propose a new method which allows us to deal with the charge neutrality constraint in a stable and efficient way. Similar approach has also proved to be succesful when dealing with Stokes problem [2].

The general idea is first to complete the system to involutive form. Since this is overdetermined, we use the compatibility operator to obtain an associated square system (called augmented system) which is actually solved numerically. Since all relevant constraints are included in the augmented system, any reasonable numerical scheme respects the constraints and produces good results. Indeed our simulations showed that with our method the charge neutrality error was approximately one order of magnitude smaller than with standard methods.

Implementing our method did not require a very big effort. Starting from an existing code it was relatively straightforward to modify it for the present purposes. Moreover the computational cost of solving the augmented system was about the same as the initial system.

Finally let us note that our approach is not restricted to microfluidic systems, but on the contrary can be applied in a wide variety of situations.

REFERENCES

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