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GLOBAL AND MULTIDISCIPLINARY AERODYNAMICAL OPTIMAL SHAPE'S DESIGN, INCLUDING DEFORMATION

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Abstract. *The global optimal design of the shape of a flying configuration (FC) in order to have minimum drag at cruise, leads to an extended variational problem with free boundaries. In some previous papers the author has developed an optimum-optimorum theory, which enables to determine, simultaneously, the camber, twist and thickness distributions and also the similarity parameters of the planform of global optimized FC, belonging to a class of admissible FCs defined by their common chosen properties. This theory was used to perform the inviscid global optimization of the FC's shape.*

A refined evolutionary iterative optimum-optimorum theory is proposed here, which uses the inviscid global optimized shape of the FC, as the first step of iteration. A computational checking of this shape is made, by using the own, new developed, hybrid, meshless solutions for the three-dimensional compressible Navier-Stokes layer (NSL), in improved form. These NSL's solutions use analytical potential solutions of the flow on the same FC twice, namely: at the NSL's edge (instead of parallel flow used by Prandtl in his boundary layer theory) and in the structure of the velocity's components, which are expressed inside the NSL, as products between the corresponding potential velocity's components with polynomes with arbitrary coefficients, versus a spectral variable. These coefficients are used to satisfy the NSL's partial-differential equations, in an arbitrary chosen number of points. The use of analytical elliptical potential leads to subsonic and of hyperbolical potential leads to supersonic NSL's solutions. The proposed reinforced numerical NSL's solutions are split, have important analytical properties, are accurate and rapid convergent.

The friction drag coefficient is computed and the FC's shape is checked for the structure point of view. The magnitude of thickness of FC is controlled in the critical zones like its central section and in its rear part. The limitation of the magnitude of twist of the FC at its rear part can also be requested. A weak interaction aerodynamics/structure is proposed, via new and/or modified constraints, requested for structure point of view. Up the second step of the iterative optimization, the total drag is the new functional, which is minimized and all the constraints are taken into consideration in the multidisciplinary aerodynamical global optimal design of the external FC's shape. Additionally, the aerodynamical pressure, together with the structure's load, produce the deformation of the structure. For the flattened FCs the modeling of the deformation can be obtained by using the solution of Sophie Germain partial-differential equation. The deformation must be substracted from the aerodynamical global optimized FC's shape, in order to have the wished final form, after the deformation.

1 INTRODUCTION

The usual aerodynamical optimal design (OD) of the shape of a flying configuration (FC) with given planform (i.e. the optimization of its camber, twist and its thickness distributions) with the aim to reach a minimum drag, used also by the author in her early papers, leads to a classical variational problem with given boundaries. The author has solved this problem in one shoot, by using the variational method with free Lagrangians. This classical optimization strategy was two times enlarged by the author in order to be able to perform the aerodynamical, global optimal design (GOD) and to include the friction effect in the computation of the total drag functional and in the GOD and, additionally, to be multidisciplinary.

The first enlargement consists in the inviscid GOD of the shape of FC (namely, the optimization of its camber, twist, thickness and *also* of its similarity parameters of the planform), which leads to an enlarged variational problem with free boundaries. An own optimum-optimorum (OO) theory was developed in order to solve this enlarged variational problem. The GOD of FC's shape is chosen inside of a class of admissible FCs, which is defined by the following common chosen properties:

- their surfaces are piecewise described in form of superposition of homogeneous polynomes of the same maximal degree and with free coefficients;
- their planforms are polygones, which can be related with affine transformations and
- they fulfill all the constraints of the variational problem.

A lower-limit hypersurface of the inviscid drag functional $C_d^{(i)}$ as function of the similarity parameters ν_i of the planform is defined, namely,

$$(C_d^{(i)})_{opt} = f(\nu_1, \nu_2, ..., \nu_n) \quad . \tag{1}$$

Each point of this hypersurface is obtained by solving of a classical variational problem with given boundaries (i.e. a given set of similarity parameters). The minimum of the hypersurface, it is,

$$(C_d^{(i)})_{opt\ opt} = \min\ (C_d^{(i)})_{opt} \tag{2}$$

and the position of this minimum are numerically determined and give us the best set of the similarity parameters. The FC's optimal shape, which corresponds to this set, is in the same time the global optimized FC's shape of the class.

The author has used this optimum-optimorum theory for the design of three inviscid GOD of the shapes of three models, namely of Adela, a delta wing alone and, more recently, of two FCs Fadet I and Fadet II, respectively optimized at cruising Mach numbers $M_\infty = 2.0,\ 2.2,\ 3.0$. The GOD shape of the fully-integrated model Fadet I is presented here, as an exemplification of the use of optimum-optimorum theory.

2 THE INVISCID GLOBAL OPTIMAL DESIGN OF THE SHAPE OF MODEL FADET I

The model Fadet I is a delta wing with a central integrated fuselage zone. This FC is treated like a discontinuous integrated delta wing (IDW) fitted with two artificial ridges along the junction lines between the wing and the fuselage. The downwash w on the thin surface of the IDW is supposed to be continuous and expressed in the form of supposition of homogeneous polynomes in two variables, as in [1], namely:

$$w = \tilde{w} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k} \left|\tilde{y}\right|^k \quad . \tag{3}$$

The downwashes w^* and $\overline{w}'^*$ on the wing and on the fuselage zone are expressed in form of two different superpositions of homogeneous polynomes :

$$w^* = \tilde{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}^*_{m-k-1,k} \left|\tilde{y}\right|^k \quad , \tag{4a}$$

$$\overline{w}'^* = \overline{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \overline{w}^*_{m-k-1,k} \left|\tilde{y}\right|^k \quad . \tag{4b}$$

$$\left(\tilde{x}_1 = \frac{x_1}{h_1} \quad , \quad \tilde{x}_2 = \frac{x_2}{\ell_1} \quad , \quad \tilde{x}_3 = \frac{x_3}{h_1} \quad , \right)$$

$$\left(\tilde{y} = \frac{y}{\ell} \quad , \quad \ell = \frac{\ell_1}{h_1} \quad , \quad \nu = B\ell \quad , \quad B = \sqrt{M_\infty^2 - 1} \right)$$

The coefficients $\tilde{w}_{m-k-1,k}$, $\tilde{w}^*_{m-k-1,k}$ and $\overline{w}^*{}_{m-k-1,k}$, together with the similarity parameter ν of the planform of the IDW, are the free parameters of optimization and ℓ_1 and h_1 are the half span and the depth of IDW . The quotient between the similarity parameters of the planforms of the wing and of the fuselage of the IDW depends on the purpose of FC and is here considered constant.

The constraints of the inviscid GOD are the following:
- the given lift, pitching moment and the Kutta condition on the subsonic leading edges of the thin IDW component (in order to cancel the induced drag at cruise and to suppress the transversal conturnement of the flow around the leading edges) and
- the given relative volumes of the wing and of the fuselage zone and the new introduced integration conditions along the junction lines between the wing and fuselage zone of the thick-symmetrical IDW component (in order to avoid the detachment of the flow along these lines).

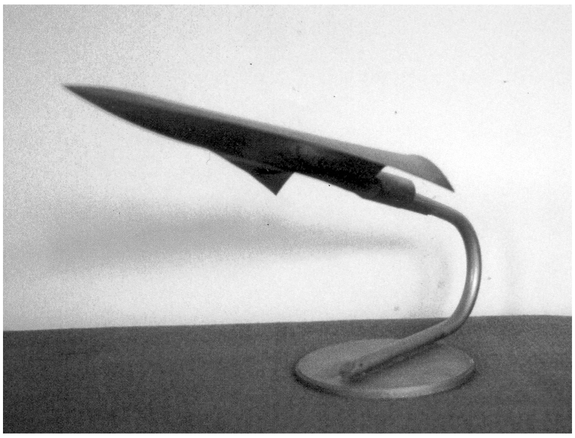


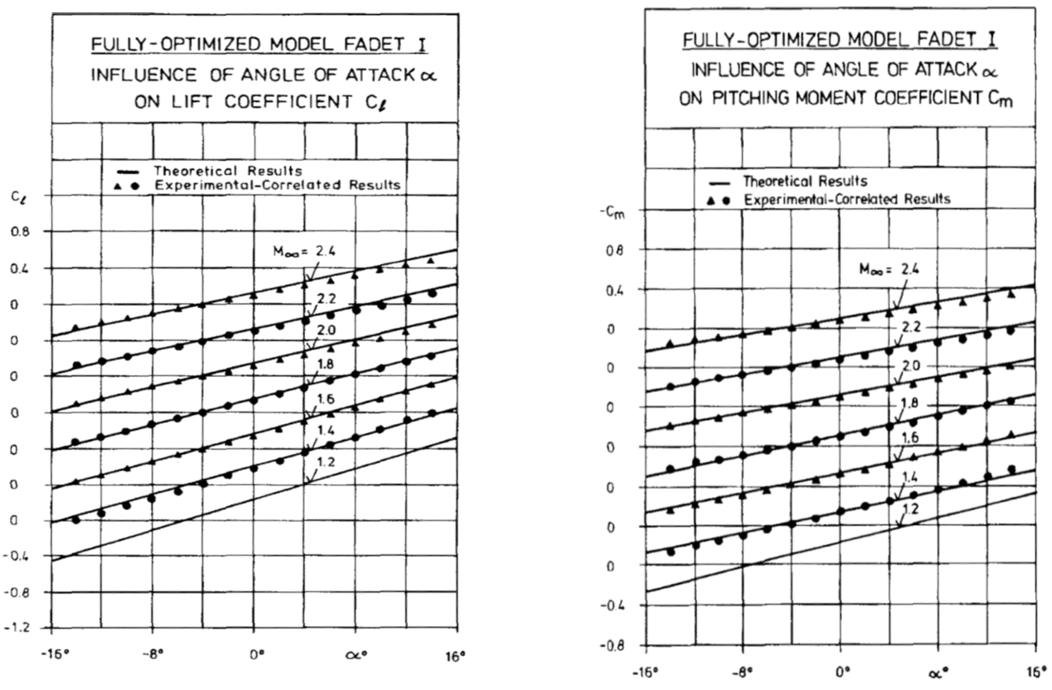
Figure 1: The View of the Global Optimized and Fully-Integrated Model Fadet I

A hybrid analytical-numerical solution according to the optimum-optimorum theory is used. Each point of the lower limes line of the inviscid minimum drag functional, for a given value of the similarity parameter ν, is analytical determined, by solving of a corresponding linear algebraic system and the minimum of the lower limes line is numerical obtained, as in [1] .

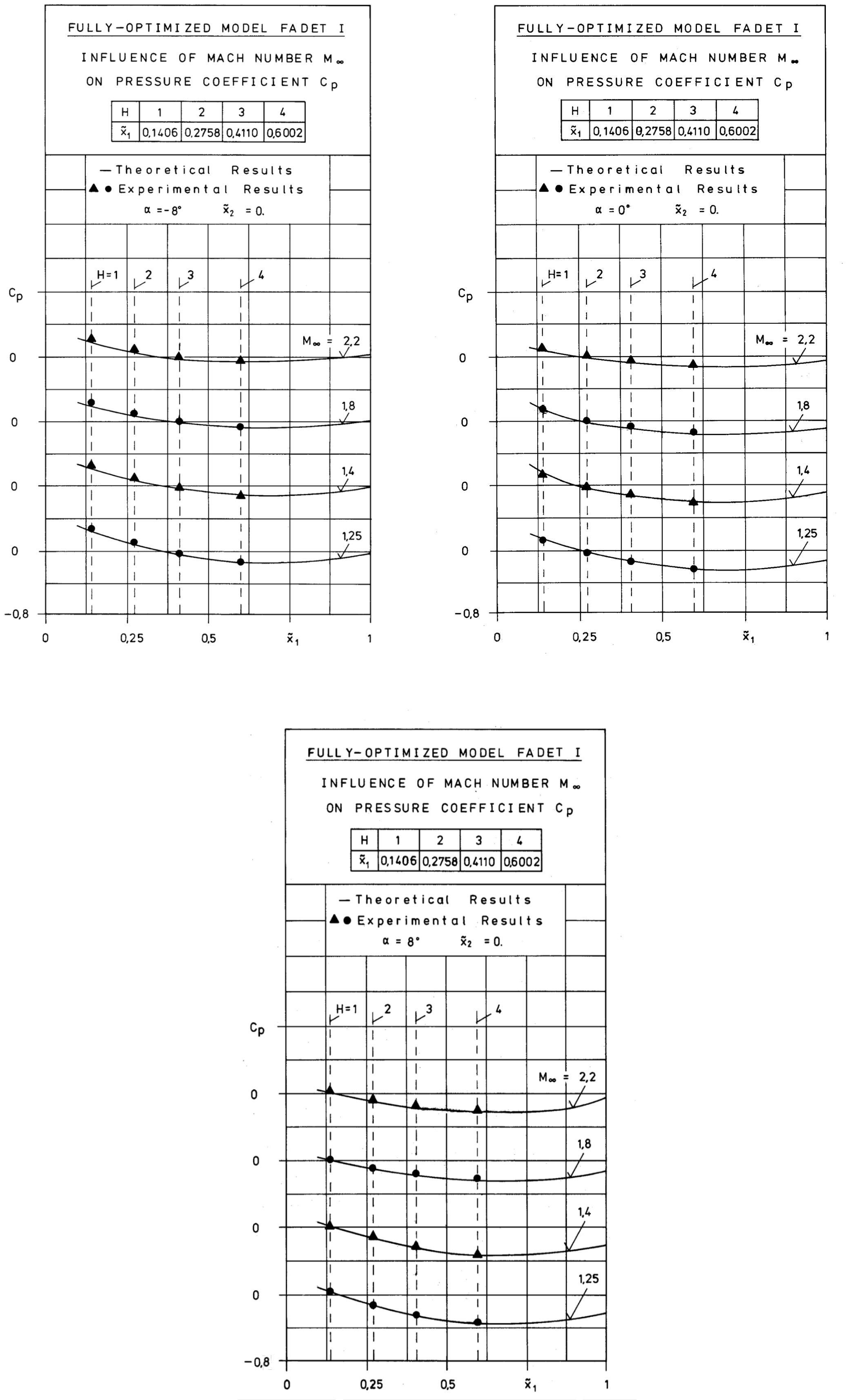
The pressure coefficients on the upper sides and the aerodynamical characteristics of these three models were measured in the trisonic wind tunnel of the DLR-Cologne, in the frame of some research contracts of the author, sponsored by the DFG. The determination of inviscid GOD of the shape of model Fadet I, presented in the (Fig. 1) and the good agreement of theoretical and experimental determined pressure, lift and pitching moment coefficients are presented below.

3 COMPARISON OF THEORETICAL PREDICTED AERODYNAMICAL CHARACTERISTICS WITH EXPERIMENTAL RESULTS

The lift, pitching moment and the pressure coefficients C_ℓ , C_m and C_p of the model Fadet I were measured in the trisonic wind tunnel of the DLR-Cologne, in the frame of research projects of the author, sponsored by the DFG. The very good agreements between the theoretical and experimental results for the lift and pitching moment coefficients C_ℓ and C_m are shown in the (Fig. 2a,b), for all the ranges of Mach numbers $M_\infty = 1.4 \div 2.4$ and angles of attack $\alpha = -12^\circ \div 12^\circ$. For these ranges, the model Fadet I has subsonic leading edges.



Figures 2a,b: The lift and pitching moment coefficients of the global optimized and fully-integrated model Fadet I



Figures 3a-c: Variation of pressure coefficients C_p in longitudinal central section

The good agreement between the measured and theoretical predicted pressure coefficients is presented in the (Fig. 3a-c) for the longitudinal central section and for the values of angles of attack $\alpha = -8°;\ 0°;\ 8°$.

4. THE HYBRID SOLUTIONS FOR THE NAVIER-STOKES LAYER

The starting point for the developing of the own hybrid solutions for the computation of the flow over flattened FCs, are the partial-differential equations (PDEs) of the three-dimensional stationary compressible NSL, without any simplifications. A new coordinate η is defined:

$$\eta = (x_3 - Z(x_1, x_2)) / \delta(x_1, x_2) \quad . \tag{5}$$

Hereby $Z(x_1, x_2)$ is the equation of the surface of the flattened FC and $\delta(x_1, x_2)$ is the NSL's thickness distribution. The spectral forms of the axial, lateral and vertical velocity's components u_δ, v_δ and w_δ, the density function $R = \ln \rho$ and the absolute temperature T are here given as follows:

$$u_\delta = u_e \sum_{i=1}^{N} u_i \eta^i, \quad v_\delta = v_e \sum_{i=1}^{N} v_i \eta^i, \quad w_\delta = w_e \sum_{i=1}^{N} w_i \eta^i,$$
$$R = R_w + (R_e - R_w) \sum_{i=1}^{N} r_i \eta^i \quad , \quad T = T_w + (T_e - T_w) \sum_{i=1}^{N} t_i \eta^i \ . \tag{6a-e}$$

Hereby R_w and T_w are the given values of R and T at the wall and u_e , v_e , w_e, R_e and T_e are the values of u, v, w, R and T at the NSL's edge, obtained from an inviscid reinforced potential solver, used here also as outer flow of the same FC (instead of the parallel undisturbed flow used by Prandtl in his boundary layer theory) and u_i, v_i, w_i, r_i and t_i are their free spectral coefficients, which are used to fulfill the NSL's PDEs. These hybrid solutions use the potential flow, over the same FC, twice: as outer flow until the NSL's edge and to reinforce the numerical solutions, which are obtained as product between numerical and analytical solutions.

These hybrid numerical solutions have also analytical properties, namely: a correct asymptotical behavior along the singular lines like leading edges, junction lines wing/ fuselage, leading and hinge lines of the leading edge flaps, according to the minimum singularities principle of van Dyke, a correct last behavior (at infinity) and are stable.

For the supersonic flow, the additional boundary condition on the characteristical surface is also correct fulfilled, due to the outer hyperbolical potential flow, as in [1–4].

The spectral forms (6a-e) automatically satisfy the boundary conditions at wall $(\eta = 0)$. The boundary conditions at the NSL's edge are written in explicit forms and are eliminated by fixing seven spectral coefficients of the velocity's components, namely u_{N-2}, u_{N-1}, u_N, v_{N-2}, v_{N-1}, v_N and w_N.

Further the physical equation of ideal gas for the pressure p and an exponential law of the viscosity μ versus T are used:

$$p \equiv R_g \rho T = R_g e^R T\ , \quad \mu = \mu_\infty \left[\frac{T}{T_\infty} \right]^{n_1} . \tag{7a,b}$$

Here are: R_g and T_∞ the universal gas constant and the absolute temperature of the undisturbed flow and n_1 is the viscosity exponent.

By using of a logarithmic density function $R = \ln\rho$ in the continuity and temperature PDEs it was possible to express all the physical entities only as functions of the spectral coefficients of the velocity's components. This splitting of NSL's PDEs contributes to speed up the computation, as in [1–4].

The impulse partial differential equations are used for the computation of the velocity's components inside the NSL. For this purpose, the spectral forms given in (6a-c) are introduced in the NSL's PDEs of impulse and the collocations method is used. The spectral coefficients of the velocity's components u_i , v_i and w_i are obtained by the iterative solving of a linear algebraic system with variable coefficients, it is:

$$\sum_{i=1}^{N-3} (\tilde{A}_{ik}^{(1)} u_i + \tilde{B}_{ik}^{(1)} v_i) + \sum_{i=1}^{N-1} \tilde{C}_{ik}^{(1)} w_i = -\tilde{D}_k^{(1)} + \sum_{i=1}^{N-3} u_i \left[\sum_{j=1}^{N-3} (\tilde{A}_{ijk}^{(1)} u_j + \tilde{B}_{ijk}^{(1)} v_j) + \sum_{j=1}^{N-1} \tilde{C}_{ijk}^{(1)} w_j \right]$$

$$\sum_{i=1}^{N-3} (\tilde{A}_{ik}^{(2)} u_i + \tilde{B}_{ik}^{(2)} v_i) + \sum_{i=1}^{N-1} \tilde{C}_{ik}^{(2)} w_i = -\tilde{D}_k^{(2)} + \sum_{i=1}^{N-3} v_i \left[\sum_{j=1}^{N-3} (\tilde{A}_{ijk}^{(2)} u_j + \tilde{B}_{ijk}^{(2)} v_j) + \sum_{j=1}^{N-1} \tilde{C}_{ijk}^{(2)} w_j \right],$$

$$\sum_{i=1}^{N-3} (\tilde{A}_{ik_1}^{(3)} u_i + \tilde{B}_{ik_1}^{(3)} v_i) + \sum_{i=1}^{N-1} \tilde{C}_{ik_1}^{(3)} w_i = -\tilde{D}_{k_1}^{(3)} + \sum_{i=1}^{N-3} w_i \left[\sum_{j=1}^{N-3} (\tilde{A}_{ijk_1}^{(3)} u_j + \tilde{B}_{ijk_1}^{(3)} v_j) + \sum_{j=1}^{N-1} \tilde{C}_{ijk_1}^{(3)} w_j \right] \tag{8a-c}$$

$$(k = 1,\ 2,..,N-3 \text{ and } k_1 = 1,\ 2,..,N-1)$$

The values of its variable coefficients are taken for the precedent step of iteration. The iteration is going to an end, when the maximal difference between the velocity's components in two consecutive iteration steps is smaller than a chosen small value.

The inviscid global optimized shape of FC, obtained by using the hyperbolical potential solutions as start solutions for the optimization and of the OO theory as strategy for the optimization represents now the first optimization step of a more refined iterative global optimization strategy, which uses the own reinforced hybrid NSL solutions, presented here, up the first computational checking and up the second step of optimization, as presented below.

5 THE ITERATIVE OPTIMUM-OPTIMORUM THEORY AND THE STRUCTURE'S DEFORMATION

The second enlargement of the variational method consists in the development of an iterative OO theory, in order to introduce also the influence of friction in the drag functional and in the aerodynamical GOD of the FC's shape. The previous inviscid global optimized shape of the FC represents now the first step in the iterative viscous shape optimization process, as described in the (Fig. 4) .

An intermediate computational checking of the inviscid GOD of the FC's shape is made with own zonal spectral viscous solvers, for the three-dimensional NSL. The friction drag coefficient $C_d^{(f)}$ of the FC is determined. This aerodynamical inviscid global optimized shape of FC is also checked for the structural point of view. A weak interaction aerodynamics/structure via additional or modified constraints, introduced in order to control the camber, twist and thickness distributions of the aerodynamical, global optimized FC's shape, for structure reasons, is here proposed. Up the second step of optimization, the predicted inviscid optimized shape of the FC is corrected, by including all the constraints in the variational problem and of the friction drag coefficient in the drag functional. The iterative optimization process is repeated, until the maximal local modification of the shape, in two consecutive optimization steps, presents no significant change. The final aerodynamical global optimized FC's shape is good for the aerodynamical point of view and can satisfy also the stiffness requirements of the structure.

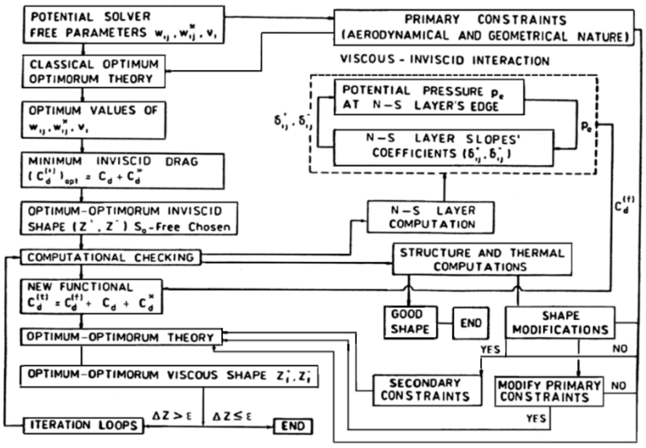


Figure 4: The iterative optimum-optimorum theory

A new enlargement of the aerodynamical global optimization of the FC's shape, consists in including the effect of the deformation of the structure.

For the flattened FCs the modeling of the deformation Z_d can be obtained by using the solution of Sophie Germain PDE, namely:

$$D\left(\frac{\partial^4 Z_d}{\partial x_1^4} + 2\frac{\partial^4 Z_d}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 Z_d}{\partial x_2^4}\right) = p_t \,. \tag{9}$$

$$\left(p_t = p_w + p_a \quad , \qquad D = \frac{Eh^3}{12\left(1-\mu^2\right)}\right)$$

Hereby are: p_w the unit weight, p_a the aerodynamical pressure and p_t is the total unit load, h is the thickness of the structure, E is the module of elasticity of Young and μ is the Poisson coefficient. Due to the coupling aerodynamics/structure, the structure is more deformed due to aerodynamical pressure and the total deformation must be substracted from the aerodynamical global optimized FC's shape, in order to have the wished final form, after the deformation.

6 CONCLUSIONS

The evolutionary iterative optimum-optimorum theory, proposed here:

- allows the multidisciplinary aerodynamical GOD with weak interactions, via modified and additional constraints, requested for the structure purposes;
- allows the multipoint design by morphing, by using movable leading edge flaps;
- is flexible (it can use different start solutions, drag functionals and constraints, which can be changed in the different beginning steps of iterations);
- is a *deterministic* theory which has almost all attributes of genetic algorithms (like *migrations* in the drag functional and in the constraints, *mutations* in the start solutions and in the constraints, *crossover* by construction of hybrid analytical-numerical start solutions, *multiple selection* inside of a class of FCs and among different classes of FCs etc.);
- its hybrid numerical start solutions are more accurate than the fully-numerical solutions (are meshless, the partial derivatives can be exactly computed, have analytical properties);
- is economic and competitive (due to the splitting of NSL's PDEs and of analytical hybridization, a speed up of computing time occurs).

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