

**NUMERICAL SIMULATION OF THE TISSUE ABLATION  
IN HIGH INTENSITY FOCUSED ULTRASOUND THERAPY  
WITH AN ARRAY TRANSDUCER**

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**Abstract.** *The objectives of the present study are the realization of the appropriate focus control by an array transducer and the support of the preoperative planning of HIFU therapy by the prediction of ablation regions. For these purposes, the HIFU simulator, which employs the voxel phantom constructed from CT/MRI data of a living human body, has been developed in the present study. To reproduce the ultrasound propagation through an inhomogeneous media, the mass and momentum equations for mixture with the equation of state of media are solved. The ablation of tissue is modeled as a phase transition by the phase field model. The heat equation with viscous dissipation as a heat source and the Allen-Cahn equation with a free energy model are solved to predict the development of the ablation region. The basic equations are discretized spatially by the 4th order finite difference method and are integrated temporally as FDTD. HIFU therapy with an array transducer for a liver cancer is reproduced numerically. Although the results without control shows the displacement of the focal point due to the inhomogeneity of the body, the clear focal point is obtained at a target by the array transducer with the appropriate phase and amplitude control provided by the pre-computation. Furthermore, the HIFU simulator reproduces the evolution of the tissue ablation due to the temperature rise around the focal point.*

## 1 INTRODUCTION

High Intensity Focused Ultrasound (HIFU) therapy is one of the valuable non-invasive cancer treatments, which provides the ablation of tissue around the focal point by heating. HIFU therapy has been already applied for the cancer close to the body surface such as the breast cancer and prostate cancer. Now, the development of the HIFU therapy for the deeply placed cancer such as the brain cancer and liver cancer is desired. One problem is the displacement of the focal point due to the reflection and refraction of ultrasounds at the interfaces of tissues. Array transducer is a solution for the problem. Array transducer enables to control the focal point by phase delay. However, it is difficult to estimate the phase delay preoperatively because of the ultrasound propagation through the inhomogeneous media, i.e., human body.

The objectives of the present study are the realization of the appropriate focus control by an array transducer and the support of the preoperative planning of HIFU therapy by the prediction of ablation region. For these purposes, the HIFU simulator, which employs the voxel phantom constructed from CT/MRI data of a living human body, has been developed. Figure 1 shows the flows of input data to the HIFU simulation. A volume model of human body (voxel phantom) is constructed from CT/MRI data of a human body. On the other hand, the shape of a transducer defined by CAD is represented by the volume data of Signed Distance Function (SDF). Then, the ultrasound propagation from the transducer through the voxel phantom is performed numerically. Our approach is to solve the mass and momentum equations for mixture with the equation of state of media. The ablation of tissue is modeled as a phase transition by the phase field model [1]. The heat equation with viscous dissipation as a heat source and the Allen-Cahn equation with a free energy model are solved to predict the development of the ablation region.

In this paper, the basic equations for HIFU therapy and numerical methods are mentioned briefly. Then the computational results of HIFU therapy for liver cancer with an array transducer are discussed.

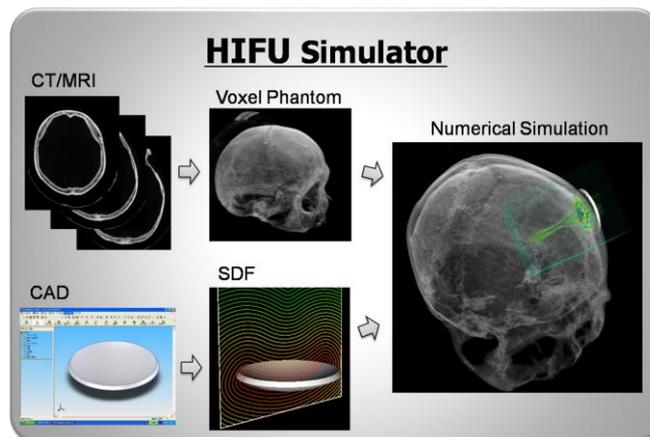


Figure 1: Schematic diagram of HIFU simulator. The voxel phantom constructed from CT/MRI data of the human body and the volume data of Signed Distance Function (SDF) representing the shapes defined by CAD are employed for numerical simulations.

## 2 BASIC EQUATIONS

To represent the ultrasound propagation through inhomogeneous media such as a human body, the mass and momentum equations for mixture with the equation of state of media are solved. Because of the small acoustic Mach number, we can assume the

adiabatic change of media and neglect the advection term during the ultrasound propagation. The mass conservation equation for mixture then becomes

$$\frac{1}{\rho_m c_{sm}^2} \frac{\partial p}{\partial t} + \frac{\partial u_j}{\partial x_j} = 0, \quad (1)$$

where the mixture density  $\rho_m$  and the adiabatic sound speed of mixture  $c_{sm}$  are defined by the volume fraction of media  $f_k$  as  $\rho_m = \sum_k f_k \rho_k$  and  $1/\rho_m c_{sm}^2 = \sum_k f_k / \rho_k c_{sk}^2$ , respectively. The sound source,  $f_p A \cos(\omega t)$ , is added to the right hand side of Eq.(1) to represent the ultrasound generated by a piezo transducer.

With considering pressure waves with a viscous attenuation, the momentum equation is described as

$$\rho_m \frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left\{ \zeta_m e_{kk} \delta_{ij} + 2\mu_m \left( e_{ij} - \frac{1}{3} e_{kk} \delta_{ij} \right) \right\}, \quad (2)$$

where  $e_{ij} = 1/2(\partial u_i / \partial x_j + \partial u_j / \partial x_i)$  is the deformation tensor,  $\zeta_m$  and  $\mu_m$  are the bulk and shear viscosity.

To take into account the nonlinear effect due to the equation of state of media, Tait's equation is employed,

$$\rho = \rho_0 \left( \frac{p + A_1}{p_0 + A_1} \right)^{1/\gamma}, \quad (3)$$

where  $A_1 = \rho_0 c_{s0}^2 / \gamma - p_0$  and  $\gamma = B/A + 1$ . Here,  $B/A$  is the non-linearity coefficient [2][3].

In addition, the heat equation for mixture with a viscous dissipation as a heat source is solved,

$$\rho_m C_{pm} \frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x_j} \left( \kappa_m \frac{\partial T}{\partial x_j} \right) + \zeta_m (e_{kk})^2 + 2\mu_m \left\{ e_{ij} e_{ij} - \frac{1}{3} (e_{kk})^2 \right\}. \quad (4)$$

The ablation of tissue, which is the heat denaturation of protein from higher structure to primary structure, is modeled as a phase transition by the phase field approach [1]. The development of a non-conserved order parameter  $\phi$  corresponding to the denaturation region is described by the Allen-Cahn equation with a free energy model

$$\frac{\partial \phi}{\partial \tau} = -M_D \left[ W \left\{ \phi(\phi-1) \left( \phi - \frac{1}{2} \right) + \frac{\beta(T)}{2} \phi(1-\phi) + \gamma(T) \phi \right\} - \varepsilon^2 \nabla^2 \phi \right], \quad (5)$$

where  $M_D$ ,  $W$  and  $\varepsilon$  are constants. Here  $\beta(T)$  and  $\gamma(T)$  are defined as

$$\beta(T) = \begin{cases} 0 & \text{for } T \leq T_m \\ \frac{T - T_m}{T_c - T_m} & \text{for } T_m < T < T_c \\ 1 & \text{for } T \geq T_c \end{cases} \quad \text{and} \quad \gamma(T) = \begin{cases} 0 & \text{for } T \leq T_c \\ \frac{T - T_c}{T_c - T_m} & \text{for } T > T_c \end{cases}, \quad (6)$$

respectively. The bulk free energy density of one state ( $\phi = 1$ ) is getting higher than that of the other state ( $\phi = 0$ ) with increasing the temperature from  $T_m$  to  $T_c$ . The shape of

bulk free energy density function finally changes from the double well to the single well for  $T > T_c$ . Then the order parameter changes from 1 to 0 to decrease the free energy of a system and represents the heat denaturation region. In the present study,  $M_D = 1$ ,  $W = 2$ ,  $T_m = T_0 + 13$ ,  $T_c = T_m + 5$  are employed currently. The constants will be corrected by the comparison with experiments in future work.

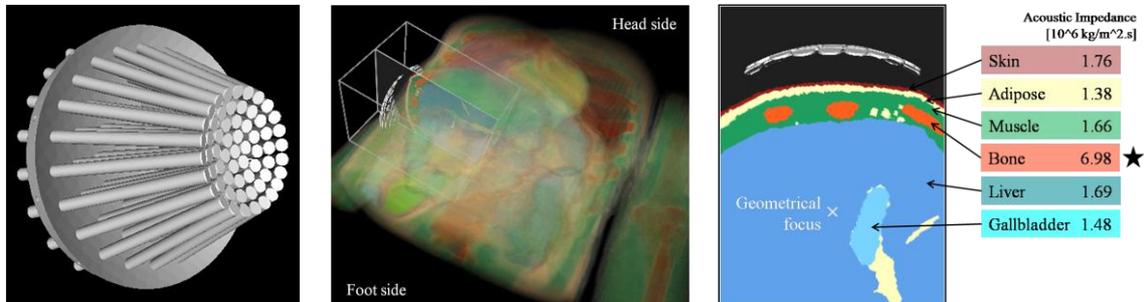
From the view point of the propagation time scale, temperatures for each media change adiabatically. But the temperature fluctuation of such adiabatic change is much smaller than the temperature rise required for the tissue ablation. Although the viscous dissipation oscillates in the propagation time scale, it linearly increases in the time scale of ablation. Therefore, the ablation time scale  $\tau$ , which is larger than the propagation time scale  $t$ , is introduced for equations (4) and (5) to reproduce the ablation of tissue by the reasonable computational time.

### 3 NUMERICAL METHODS

The numerical method is based on the finite difference method in the present study. The basic equations are discretized spatially by the 4th-order central finite differential method. The finite-difference time-domain (FDTD) technique [4] is employed for the time marching. Perfectly Matched Layer [5] is applied to avoid the reflection of ultrasound at the boundary. In addition, the parallelization approach of the domain decomposition with MPI is employed for the large scale parallel computing.

### 4 HIFU THERAPY FOR A LIVER CANCER WITH AN ARRAY TRANSDUCER

The HIFU therapy for a liver cancer with an array transducer is considered as shown in Fig. 2. The array transducer includes 61 pen type transducers with the diameter of 10mm. The focal length is 80mm. The numerical domain with a simplified model of the array transducers is arranged for the voxel phantom of a human body as shown in Fig. 2(b). The numerical domain is the size of 120mm  $\times$  160mm  $\times$  120mm and is divided by 600  $\times$  800  $\times$  600 meshes. The voxel phantom, which was constructed by tagging ID of each tissue from both CT and MRI data in RIKEN Computational Biomechanics Project, enables to use the physical properties for each tissue. For example, Fig. 2(c) shows tissues on the ultrasound propagation path and their acoustic impedance. The physical properties in the book [3] are employed except for the viscosity. The viscosity of tissues is estimated from the viscosity of water by comparing the attenuation coefficient of tissue with that of water.



(a) Shape of an array transducer including 61 pen type transducers with the diameter of 10mm.

(b) Arrangement of the transducer with a numerical domain for the voxel phantom of a human body.

(c) Distribution of tissues on the ultrasound propagation path and their acoustic impedances.

Figure 2: HIFU therapy for a liver cancer with an array transducer.

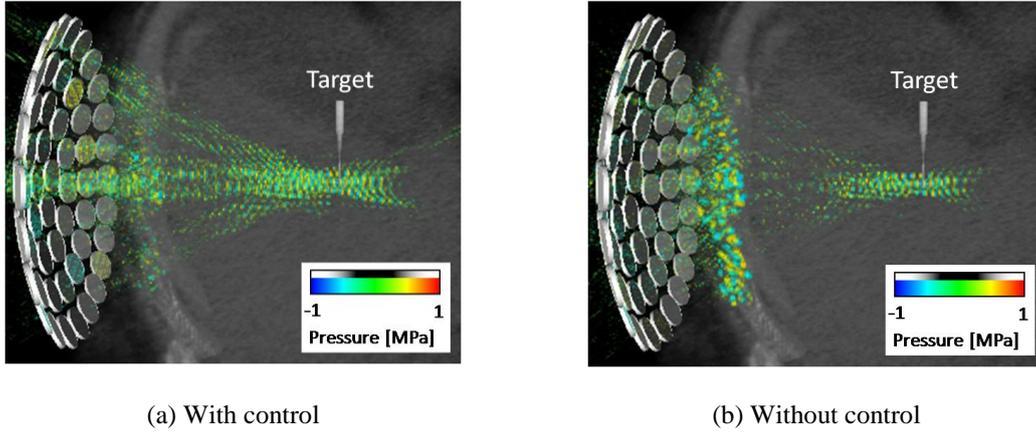


Figure 3: Instantaneous ultrasound fields with and without control of the array transducer in the HIFU therapy for the liver cancer through ribs.

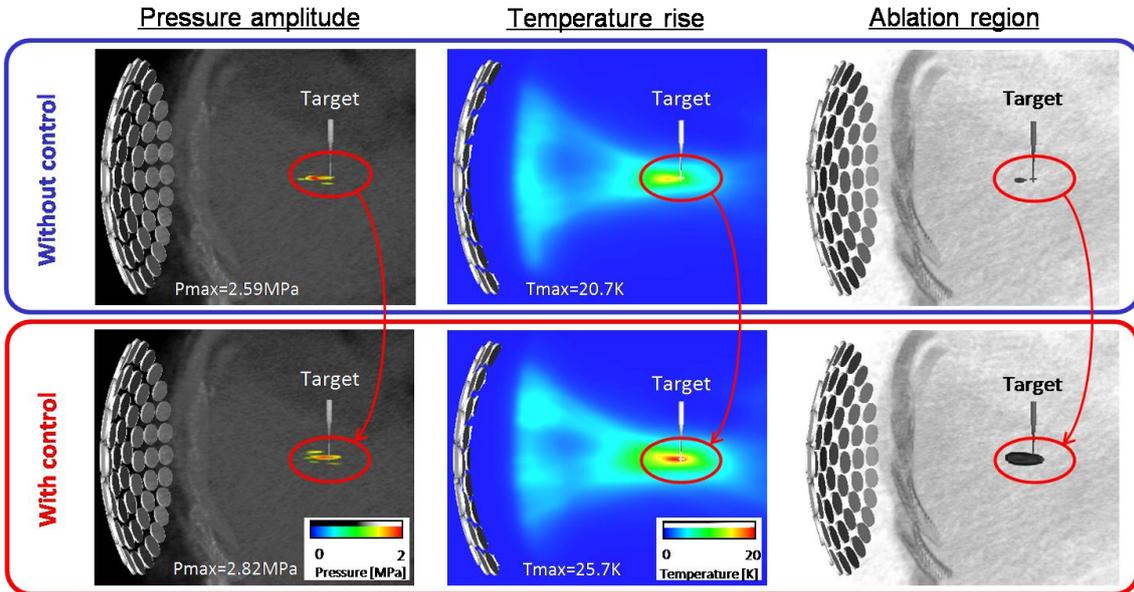


Figure 4: Controllability of the focal point and efficient tissue ablation by the array transducer in the HIFU therapy for the liver cancer through the ribs. The distribution of maximum pressure amplitude, the distribution of instantaneous temperature rise from the initial temperature on a cross section and profiles of instantaneous denaturation region at  $\tau = 135$ [s] are compared for with and without control.

In the present study, both phase delay and amplitude of the transducers are controlled by estimating as follows. First, the propagation of ultrasound emitted from the target point is simulated, in which each transducer receives the ultrasound. Next, we calculate the cross-correlations of the received ultrasound profiles of the every transducer element with that of the center transducer element. Then, the phase delay  $\tau$  is chosen to take the cross-correlation maximum. The amplitude is also decided from the maximum cross correlations for each transducer by multiplying the ratio to keep the sum of squared amplitude constant.

Figure 3 shows the instantaneous ultrasound field with and without control of the array transducer. In the result with control, the ultrasounds propagate through the space between ribs and focus on the target correctly. Especially, the pressure fluctuations in the ribs, which are observed in the result without control, are almost disappeared, even though the pressure amplitude around the focal point increases.

Figure 4 shows the distribution of the maximum pressure amplitude, the distribution of instantaneous temperature rise on a cross section and the profile of the instantaneous ablation region for with and without control. As shown in figures without control, the focal point moves to the transducer away from the target due to the reflection and refraction of ultrasounds. The ablation region develops around not the target but the focal point. On the other hand, the focal point is assigned to the target adequately by the control of the array transducer. The ablation region with control develops close to the target and is larger than that without control because of the increase of the maximum pressure amplitude. Thus, the appropriate control of the array transducer efficiently provides the tissue ablation.

## 5 CONCLUSION

The HIFU simulator, which employs both the voxel phantom constructed from the CT/MRI images of the living human body and the volume data of the signed distance function representing the transducer defined by CAD, was developed to reproduce the tissue ablation in HIFU therapy with the array transducer.

The HIFU therapy for the liver cancer with the array transducer was reproduced numerically. In the results without control, the displacement of the focal point due to the inhomogeneity of the body is observed. The array transducer with the appropriate phase and amplitude control, which are estimated by the pre-computation, enables to assign the focal point to the target adequately and to produce the tissue ablation efficiently. The simulation shows the feasibility of the non-invasive treatment for liver cancers by the array transducer.

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