Construction of very high order residual distribution schemes for compressible flow problems.

Rémi Abgrall^{a}st, Arnaud Krust[†], Adam Larat[†], Pascal Jacq[†], Guillaume Baurin[†] and Mario Ricchiuto[†]

[*] Team BACCHUS, INRIA Bordeaux Sud Ouest and Institut de Mathématiques
341 Cours de la Libération, 33 405 Talence cedex
e-mail: Remi.Abgrall@inria.fr

[†] Team BACCHUS, INRIA Bordeaux Sud Ouest
341 Cours de la Libération, 33 405 Talence cedex
e-mail: {Adam.Larat, Pascal.Jacq, Mario.Ricchiuto}@inria.fr

ABSTRACT

We are interested in the numerical approximation of steady and unsteady hyperbolic problems which are defined on an open set $\Omega \subset \mathbb{R}^d$, $d = 2, 3$ with weak Dirichlet boundary conditions defined on the inflow boundary. The target example we aim at is the system of the Euler equations with a perfect gas equation of state, but the method can be extended to other problems (see e.g. [1]). In the recent years, there have been many researches to produce really robust and high order schemes for these equations. One of the very popular methods is the class of Discontinuous Galerkin schemes. The solution is approximated by a reconstruction function that is generally discontinuous across the interface of the computation cells of the mesh. The solution of is updated thanks to a local weak form of the problem combined with a polynomial representation of the data in computational cells and a numerical flux. The formulation is very local and also very flexible but the number of degrees of freedom grows very quickly as the degree of the polynomial in cells increases, and the non linear stabilisation at discontinuities is not perfect. We have chosen to develop a different strategy, the Residual Distribution schemes, where the stencil stays very local, as in the DG methods, but the number of degrees of freedom grows less quickly, even in 3D. The price to pay is to impose the continuity of the approximation as in standard finite element methods. So far, this class of scheme has mainly been developed for second order accuracy only (see [2,3,4]). Here, we are interested in showing how these methods can be extended to very high accuracy, even in the case of the Euler equation, at a relatively moderate price. Details can be found in [5].

Currently, the solver is able to handle 3D Euler problems and 2D viscous ones. During the conference, we will present 3D Navier Stokes results for subsonic, transonic and supersonic flows, and possibly turbulent results.

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