# EVALUATION OF AN INDUCED MAGNETOHYDRDYNAMIC VELOCITY POTENTIAL USING DUAL RECIROCITY BOUNDARY ELEMENT METHOD (ECCOMAS CFD 2010)

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**Abstract.** The main claim of this paper is to evaluate velocity potential caused by an applied magnetic field on a stationary fluid flow at rest. The fundamental governing equations of the problem can be found using a coupled Maxwell and flow potential equations. A rectangular duct with the section of unity in any direction of coordinate system has been chosen as the problem's domain. Some mathematical manipulation has been carried out to make the governing equations decoupled, mathematically. Then the final form of these equations may be stated in general form of Poisson equation so that the right hand side of the equation "b" is a function of nodes position vector and also function of new introduced potential parameter. Then powerful and practical approach of Dual Reciprocity Boundary Element Method has been employed to treat the problem. The radial basis function is a linear function of radial displacement which does not change throughout the problem.

The behavior of the flow is highly affected by changing of Hartmann number. The special case of interest could be investigation of boundary layer thickness on the walls and also figuring out the induced velocity component which both have been studied here. Furthermore Although the dual reciprocity boundary element method can solve several types of nonhomogeneous partial differential equation, here well known as Poisson equation, the method plied a set of interpolation function which causing the problem to be very sensitive to the nodes gird configuration. This configuration includes the number of interior and boundary nodes, the type of elements (constant, linear, etc), the boundary condition, etc. Therefore some typical cases are studied to get the best result.

The conclusion depicts that some other more complex radial basis functions may provide better results but with making the numerical initialization more difficult. Anyhow, as it expected the result tending to the exact solution as the number of nodes or degree of boundary elements increases. Boundary layer formation obviously is also illustrated. Moreover, different physical aspects of the problem investigated via several figures to give the reader a fast and reasonable discernment.

# **INTRODUCTION:**

The magnetohydrodynamic flow has lots of practical uses in different industries including MHD pumps, MHD shock absorber, MHD flow driver system, etc. The basis of all devices working based on MHD is almost the same. An applied constant or variant magnetic field causes the conducting fluid to flow. This procedure is usually followed by an induced current. Several researches have been carried out on simulating the MHD flow by virtue of various numerical and analytical techniques. The analytical solution of the problem is expressed by different authors. The problem was first studied by Hartmann [1] for an incompressible, viscous flow between two flat plates. The analytical researches made by Dragos [2] and Shercliff [3] may be mentioned as two typical analytical ones. The numerical approaches also include almost all well known numerical method including FDM, FDM and BEM as well. Numerically investigation of the problem using BEM is considered here. The domain of the problem is a one by one square which in Cartesian coordinate system can be shown as  $0 \le x, y \le 1$  as it is common in different text books. The domain of the problem is intentionally chosen like this to make the authors able to validate numerical analysis with the exact solution of the problem. The all sides of the squares are assumed to be insulated. Now, one can decouple the equations by method of change of variables.

After the equations are decoupled, governing equations will be in form of Poisson equations which their right hand sides (RHS) are a function of potential field only. Hence the dual reciprocity method has been employed to solve the problem numerically, the fundamental solution of the problem remains unchanged and is exactly the same as Laplace equation's fundamental solution. The DRBEM [4] evaluates domain integrals appearing in boundary integral equation by series of interpolation functions based on different radial basis functions. Several different RBFs are introduced and investigated to verify the accuracy of the results. The work of Ramachandran and Karur [5] is a good example. However the simplest radial basis function i.e. 1+r is used here to solve the problem. Then different interior and exterior node configuration has studied to show that the results are independent of the nodes number. Lots of comparisons showed in the results section depicts the solution has a good agreement with the exact solution. The conclusion shows that the numerical results tending to the exact solution when the number of interior nodes increases, as it expected.

# **GOVERNING EQUATION**

When a constant or variant magnetic field applies to an incompressible non-viscous conducting fluid, induced current forces the fluid to flow and the flow is now called a magnetic driven flow. The Maxwell electromagnetic equation and also flow momentum equation lead to a coupled system of equations. Additionally it is supposed that no vortex shedding mechanism presents. Thus the flow should be classified as fully developed laminar flow. The non-dimensional set of equation is presented by Dragos[2] and Shercliff [3] as following:

$$\begin{cases} \nabla^2 V + M \frac{\partial B}{\partial x} = -1 \\ \nabla^2 B + M \frac{\partial V}{\partial x} = 0 \end{cases}$$
(1)

Where here  $\nabla$  is Laplace operator, M is the Hartmann number, V(x,y) and B(x,y) are velocity and magnetic fields, respectively. Both of the equations are defined on the domain  $\Omega$ . The domain of the problem illustrated in Fig.1 showing both the main domain geometry and applied boundary condition.



Fig.1: The problem domain and applied boundary condition

As it shown in Fig.1 the flow direction is now in the opposite direction of positive Z axis, say, here driven down. Since here the applied boundary condition simulate physical insulation, one can now decouple the governing equation using a simple change of variable method. Let  $u_1=V+B$  and  $u_2=V-B$  so that the transformed equation presented by (1) converted to the following set of equations.

$$\begin{cases} u_1 = V + B \\ u_2 = V - B \end{cases} \rightarrow \begin{cases} \nabla^2 u_1 + M \frac{\partial u_1}{\partial x} = -1 \\ \nabla^2 u_2 - M \frac{\partial u_2}{\partial x} = -1 \end{cases} \equiv \begin{cases} \nabla^2 u_1 = -1 - M \frac{\partial u_2}{\partial x} \\ \nabla^2 u_2 = -1 + M \frac{\partial u_2}{\partial x} \end{cases}$$
(2)

The new transformed set of equation now is decoupled with respect to two new introduced parameters and in general form of Poisson equation which the RHS is only a function of spatial potential derivate respect to x component. Although the DRBEM can solve the aforementioned equations in an iterative manner recommended by Ramachandran [6], the set of equations can be further simplified by using second change of variable. Now define the new variables as  $U_1=exp(0.5Mx)\times u_1$  and  $U_2=exp(0.5Mx)\times u_2$ . Thus we have

$$\begin{cases} U_1 = e^{\frac{M}{2}x} \cdot u_1 \\ U_2 = e^{-\frac{M}{2}x} \cdot u_2 \end{cases} \xrightarrow{\longrightarrow} \begin{cases} \nabla^2 U_1 = \frac{M^2}{4} U_1 - e^{\left(\frac{M}{2}\right)x} \\ \nabla^2 U_2 = \frac{M^2}{4} U_2 - e^{-\left(\frac{M}{2}\right)x} \end{cases}$$
(3)

Now the RHS of Poisson equations are only a function of potentials and the positional components of any desired points and not a function of potentials derivatives. As it discussed later, the numerical solution can be carried out much easier in this situation. If the domain discretized via several nodes, the spatial terms in foregoing equation are always known for every node. However the potential is not determined and should be evaluated. The RHSs of (3) may be emphasized and rewritten as:

$$\begin{cases} b_1(x, y, U_1) = \frac{M^2}{4} U_1 - e^{\left(\frac{M}{2}\right)x} \\ b_2(x, y, U_2) = \frac{M^2}{4} U_2 - e^{-\left(\frac{M}{2}\right)x} \end{cases}$$
(4)

Readily one can result that  $b_2=b_1(-M)$ . This relation decreases the needed CPU time much less than solving two equations separately. The new boundary conditions of the problem with considering two changes of variable can be straightly forward obtained as  $U_1=U_2=0$  on all boundaries. Once the different values of  $U_1$  and  $U_2$  are calculated, the origin V and B field values will be computed by two simple equations arising from the changes of variable and may be expressed as (5).

$$\begin{cases} V = \frac{1}{2} \left[ \exp\left(-\frac{M}{2}x\right) U_1 + \exp\left(\frac{M}{2}x\right) U_2 \right] \\ B = \frac{1}{2} \left[ \exp\left(-\frac{M}{2}x\right) U_1 - \exp\left(\frac{M}{2}x\right) U_2 \right] \end{cases}$$
(5)

#### NUMERICAL SOLUTION PROCEDURE

As it mentioned before, the Dual Reciprocity Boundary Element Method has been employed to solve the problem numerically. It is well known that the direct boundary element method was firstly formulated and specified for solving only the Laplace equations and has a drawback of solving Poisson equation because of domain integral which appears in this class of partial differential equation. Although the domain integral may be evaluated by surface numerical integration, this approach cannot be known as boundary method. Then the new modified boundary element method first presented by Nardini and Brebbia [4] and Partridge [7] disappear the weakness of direct boundary element method. This approach utilizes a set of interpolation functions  $\phi_k$  to evaluate the values of *b* function on every node. Interpolation function  $\phi_k$  sometimes called radial basis function because it is only a function of radial distances between the nodes. The main advantage of radial basis function is that they are only a function of single variable independent of problem dimensions. There are several proposed radial basis function which some of them can be mentioned here as following [5,7]:

- 1. Linear function:  $\phi_k = 1 + r$
- 2. Duchon radial cubic:  $\phi_k = r^3$
- 3. Radial quadratic plus cubic:  $\phi_k = 1 + r^2 + r^3$
- 4. Thin plate spline:  $\phi_k = r^2 \ln r$
- 5. Hardy multiquadrics:  $\phi_k = (r^2 + C^2)^{n/2}$
- 6. Inverse multiquadrics:  $\phi_k = 1/(\sqrt{(r^2+C^2)})$
- 7. Gaussian:  $\phi_k = \exp(-r^2/C^2)$

Using interpolation function the RHS of the Poisson equation can be rearranged as

$$b_m = \sum_{k=1}^{N+L} \phi_{m,k} \alpha_k \tag{6}$$

Where subscript *m* is the counter parameter on nodes,  $\alpha$  is the interpolation coefficient and N and L are the number of boundary and interior nodes, respectively. The values of matrix  $\phi$  elements are always constant if the position of all nodes sustained unchanged. So the interpolation function is easily found by taking inverse of interpolation function as following

$$\alpha_k = \sum_{m=1}^{N+L} E_{km} b_m \tag{7}$$

Here *E* in the matrix inverse of interpolation function. Now if a set of functions  $f_k$  can be found such that the Laplacian of  $f_k$  is equal to  $\phi_k$ , then the domain integral can be converted to the boundary integral [8] therefore we have

$$\nabla^2 f_k = \phi_k, \ \phi_k = 1 + r \tag{8}$$

Now because we assumed that the interpolation functions are a single variable function of radial distance, the Laplace operator can be rewritten in polar coordinates to find the  $f_k$ .

$$\nabla^2 f_k = \frac{1}{r} \frac{d}{dr} \left( r \frac{df}{dr} \right) = 1 + r \quad \rightarrow \quad d \left( r \frac{df}{dr} \right) = (r + r^2) dr \quad \rightarrow \quad \frac{df}{dr} = \frac{r}{2} + \frac{r^2}{3}$$
$$\rightarrow \quad f = \frac{r^2}{4} + \frac{r^3}{9} \tag{10}$$

Where here r(x,y) in our two dimensional problem has a familiar definition of a vector length.

$$r = \sqrt{(x - x_k)^2 + (y - y_k)^2}$$
(11)

Note that the governing equation is now discretized by virtue of interpolation functions as following:

$$\nabla^2 U = b = \sum_{i=1}^{N+L} \nabla^2 f_k. \, \alpha_k \tag{12}$$

Multiplying by fundamental solution and integrating over the whole domain arises

$$\int_{\Omega} \nu \nabla^2 U \, d\Omega = \int_{\Omega} \nu (\sum_{i=1}^{N+L} \nabla^2 f_k. \, \alpha_k) d\Omega \tag{13}$$

Hence we use DRBEM for solving the problem, the fundamental solution remains unchanged. The fundamental solution is a particular solution which satisfies following relation.

$$\nabla^2 U = -\delta(x, y) \tag{14}$$

Where  $\delta$  is the Dirac delta function has an infinity value at applied source points and zero value at any other points. One can obtain the fundamental solution as Katsekadelis [9] did.

$$v = -\frac{1}{2\pi} ln r = \frac{1}{2\pi} ln \left(\frac{1}{r}\right)$$
(15)

Applying Green-Gaussian theory for LHS of (12) gives

$$LHS = \int_{\Gamma} \left[ v \frac{\partial U}{\partial n} - \frac{\partial v}{\partial n} U \right] d\Gamma - \epsilon_i U_i$$
(16)

Here  $\epsilon$  is a coefficient depends on the degrees between elements. The LHS can be discretized and written in matrix form.

$$LHS = \sum_{j=1}^{N} G_{ij} U_n^j - \sum_{j=1}^{N} \widehat{H}_{ij} U_j - \epsilon_i U_i$$
(17)

The LHS is only the boundary integral. That's why the summation performed only on N boundary element nodes. Let's now apply Green-Gauss theory to the RHS of equation (12).

$$RHS = \sum_{k=1}^{N+L} \left[ \int_{\Gamma} \left( \nu_i \frac{\partial f_i}{\partial n} - \frac{\partial \nu_i}{\partial n} f_k \right) d\Gamma - \epsilon_i f_{ik} \right] \alpha_k$$
(18)

The normal derivative of *f* function can be simply found by rule of chain.

$$f'_{k} = \frac{\partial f_{k}}{\partial n} = \mathbf{n} \cdot \nabla f_{k} = n_{x} \left(\frac{\partial f_{k}}{\partial x}\right) + n_{y} \left(\frac{\partial f_{k}}{\partial y}\right) = \frac{\partial f_{k}}{\partial r} \left[n_{x} \frac{\partial r}{\partial x} + n_{y} \frac{\partial r}{\partial y}\right]$$
$$= \left(\frac{1}{2} + \frac{r}{3}\right) \left[n_{x}(x - x_{k}) + n_{y}(y - y_{k})\right]$$
(19)

Where here **n** is the normal vector outward the boundary. Thus the RHS of (12) can be rearranged to provide a new equation.

$$RHS = \sum_{k=1}^{N+L} \left( \sum_{j=1}^{N} G_{ij} f_{jk}' - \sum_{j=1}^{N} \widehat{H}_{ij} f_{jk} - \epsilon_i f_{ik} \right) \alpha_k$$
(20)

For sake of simplicity introduce a new parameter S defining as following [6]

$$S_{ik} = \sum_{j=1}^{N} G_{ij} f'_{jk} - \sum_{j=1}^{N} \widehat{H}_{ij} f_{jk} - \epsilon_i f_{ik}$$
(21)

Substituting the equation (7) for  $\alpha$  into latter equation gives

$$RHS = \sum_{k=1}^{N+L} S_{ik} \sum_{m=1}^{N+L} E_{km} b_m$$
(22)

It is convenient in programming to absorb the second summation into the first one in following manner using a new dummy variable M [6].

$$M_{im} = \sum_{k=1}^{N+L} S_{ik} E_{km}$$
(23)

Presenting the RHS much more practical for programming purposes.

$$RHS = \sum_{m=1}^{N+L} M_{im} b_m \tag{24}$$

Combining equations (18) and (20) and then substituting in equation (12) results the final discretized Poisson equation invoking DRBEM.

$$\sum_{j=1}^{N} G_{ij} U_n^j - \sum_{j=1}^{N} \widehat{H}_{ij} U_j - \epsilon_i U_i = \sum_{m=1}^{N+L} M_{im} b_m$$
(25)

As it shown in the final discretized equation, the summation on the LHS is performed only on N boundary nodes, whereas the RHS is on both boundary and interior nodes. The term  $\epsilon_i U_i$  intentionally is not absorbed in the H matrix. In contrast with the direct boundary element method, the values of  $\epsilon$  is not constant due to presence of interior nodes. The governing equations are now discretized and would ready to solve if the *b* function was constant. However, here *b* is a function of nodes position vectors and potential values (4). The position dependent terms i.e.  $\exp(\pm 0.5M)x$  are always known because the nodes position known in advance, whereas the potential dependent terms cannot be found simply. In this condition we have N unknown quantity (potential or flux depending on boundary conditions) and L unknown potential values for interior nodes. Thus we have to move the collocation or source points also to the interior nodes to close the problem.

In this case study we have to assume a set of initial values for the unknown potentials. Let's have a brief look on two common boundary conditions namely Neumann and Dirichlet. If the RHS of the Poisson equation i.e. *b* is a linear function of the potential such as  $b=C_{1m}+C_{2m}U$ , one can rewrite (25) and finds that

$$\sum_{j=1}^{N} G_{ij} U_n^j - \sum_{j=1}^{N} \widehat{H}_{ij} U_j - \epsilon_i U_i = \sum_{m=1}^{N+L} M_{im} (C_{1m} + C_{2m} U_m)$$
(26)

Where here  $C_{1m}$  and  $C_{2m}$  are two potential independent coefficients. Note that these parameters are not always constant and can vary with spatial coordinates of the nodes. Suppose that no boundary conditions applied, so we can move all unknown terms to the LHS of (26) and all known ones the RHS giving

$$\sum_{j=1}^{N} G_{ij} U_n^j - \sum_{j=1}^{N} \widehat{H}_{ij} U_j - \epsilon_i U_i - \sum_{m=1}^{N+L} C_{2m} U_m = \sum_{m=1}^{N+L} M_{im} C_{1m}$$
(27)

Depending on what types of boundary conditions applied on the boundaries, the procedure can be continued as it is done and common in direct boundary element method. A special care should be paid on the upper and lower limits of the summation over the term  $C_{2m}U_m$  for the convenience in programming. The summation can be divided into two terms as following.

$$\sum_{m=1}^{N+L} C_{2m} U_m = \sum_{m=1}^{N} C_{2m} U_m + \sum_{m=N+1}^{N+L} C_{2m} U_m$$
(28)

Without regard to what types of boundary conditions are applied, the values of  $U_m$  are always unknown on interior nodes whereas the summation over boundary nodes is known if the Dirichlet boundary condition applied. Anyhow, the complete discretized Poisson equation can be expressed in matrix presentation.

$$G_{ij}U_n^j - \hat{H}_{ij}U_j - \epsilon_i U_i - \sum_{m=1}^N C_{2m}U_m + \sum_{m=N+1}^{N+L} C_{2m}U_m = M_{im}C_{1m}$$
(29)

Here the counter j indicates the terms due to potentials and m arises from force terms (RHS). The subscript i stands on the number of node which the source point is applying on it. The following schematic matrix relation shows the dimensions of matrices.

$$\begin{bmatrix} G_{11} & \cdots & G_{N1} \\ \vdots & (N+L) \times N & \vdots \\ G_{(N+L),1} & \cdots & G_{(N+L),N} \end{bmatrix} \begin{bmatrix} N \times 1 \end{bmatrix} \\ - \begin{bmatrix} G_{11} & \cdots & G_{N1} \\ \vdots & (N+L) \times N & \vdots \\ G_{(N+L),1} & \cdots & G_{(N+L),N} \end{bmatrix} \begin{bmatrix} N \times 1 \end{bmatrix} - [N \times 1][N \times 1]^T - \cdots$$

## NUMERICAL RESULTS AND DISCUSSION

The domain of the problem is a one by one square which the origin of XY coordinate system coincides with the point O(0,0). Thus in this case study the flow is driven down by means of a constant pressure gradient, where here is in the negative direction of Z axis (axis of the duct). The domain boundary discretized into 20 equal constant elements. There are also 400 (20×20) interior or DRM nodes in the domain to make the problem more accurate. For some post-processing reasons, the interior nodes are placed such that producing a fully structural mesh grid. This grid is shown in Fig. 2. As it mentioned before all boundaries are insulating such that B(x,y)=0 on all of them. Furthermore, hence the fluid is supposed to be viscous, no slip boundary condition should be applied on all walls, say, boundaries. In other words, both B(x,y) and V(x,y) will take the value of zero on all boundaries [10]. So the boundary conditions for (3) become

$$\begin{cases} V = 0 \quad on \ \Gamma \\ B = 0 \quad on \ \Gamma \end{cases} \rightarrow \begin{cases} U_1 = 0 \quad on \ \Gamma \\ U_2 = 0 \quad on \ \Gamma \end{cases}$$
(30)

The solution first obtained for  $U_1$  and  $U_2$  and then transformed back to the original variable by virtue of (5).



Fig. 2: The discretized boundary and domain of the problem

Hence the problem was solved by constant elements, there is no node on the vertices of the square. Here, the needed values on the vertices are evaluated using arithmetic average of corresponding adjacent nodes. These values are just used for post-processing the results. Microsoft Visual Basic<sup>®</sup> has been employed to produce the needed codes and subroutines for solving this problem. This program can give the programmer high ability to write their programs user-friendly and with complex mathematical kernel simultaneously. The problem was solved for different Hartmann number from 5 to 20. The main fluid dynamic effect of changing Hartmann number will be seen in formation of velocity and magnetic boundary layer near the walls. Some selected contours of velocity and induced magnetic fields are depicted in Fig. 3-6.



**Fig. 4:** Contour of induced magnetic field (M=5)

As it shown in Fig. 3 to 6, the formation of both induced magnetic and velocity boundary layers are clearly observed. Comparison of Fig. 3 and Fig. 5 depicts that the duct flow will be more uniform as the Hartmann Number M increases. More uniform flow causes thinner boundary layer. In other words, the boundary layer will be formed and developed faster. This behavior also can be seen for induced magnetic or current boundary layer. This fact is well known in theory of magnetohydrodynamic flows specially in duct flows [2]. The thickness of boundary layer has a reciprocal proportion to the value of Hartmann Number M. The degree of this inverse proportionality depends on the direction of walls position, namely parallel or perpendicular to applied magnetic field [11,2]. Although it is not illustrated here, by increasing the Hartmann Number the problem domain should be discretized finer to have the correct solution. Based on several numerical simulation carried out by the authors, it is needed to increase the number of boundary elements 10 times more than before when the Hartmann Number takes the value of 300.



Fig. 6: Contour of induced magnetic (M=20)

Additionally it is evident that the velocity has a symmetrical behavior with respect to both lines parallel to x and y axes which connect the midpoints of the square sides whereas the contour of induced magnetic field is symmetric only respect to horizontal line. Furthermore, the direction of velocity streams do not change throughout all domain and they are clockwise here whereas Fig. 4 clearly demonstrates that the current stream directions are clockwise and counterclockwise near to left and right walls respectively. According to the Fig. 4 one can distinguish a region in the center of the domain in which there is almost no induced current. Thus it must be noted although there is no current in the mentioned region, hence all walls are insulating no zero velocity or stagnant region can be observed in the domain. Eventually it should be recalled that high value of M means stronger applied magnetic field causing formation of boundary layer in narrower region i.e. the thickness of boundary layer will be decreased.

#### CONCLUSION

Evaluation of both induced velocity and current due to an applying constant magnetic field has been carried out in this study. Some well known and typical specification of magnetohydrodynamic flows such as the formation of boundary layer, symmetric or asymmetric behavior of induced quantities and the influence of Hartmann Number M have been reviewed and investigated. The problem has been solved numerically by use of dual reciprocity boundary element method. The numerical results qualitatively compared by well known magnetic driven flows characteristics and a good agreement observed and reported.

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