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ASSIMILATION OF METEOROLOGICAL OBSERVATIONS USING LARGE-SCALE OPTIMIZATION

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Abstract. *Atmospheric data assimilation is formulated as a large-scale nonlinear optimization problem. A large-scale optimization method is used to solve the problem. An efficient method is developed for computing the gradient of the objective function. Numerical experiments are presented*

1 INTRODUCTION

Numerical simulation of the atmospheric flow is an initial/boundary value problem: given an initial condition of the atmosphere, and appropriate surface and lateral boundary conditions, the model simulates the atmospheric evolution. Assimilation of meteorological observations is the process through which all the available information is used in order to estimate as accurately as possible the state of the atmospheric flow (an initial condition)[1]. The available information consists of the observation proper, and of the physical laws that govern the evolution of flow. In this paper, the assimilation is formulated as a nonlinear optimization problem with tens of thousands of variables. A large-scale optimization method is employed to solve the problem. At every step, it requires the gradient of the objective function. A method is developed for computing the gradient efficiently. Finally, numerical experiments are presented.

2 FORMULATION

The model equation for the atmospheric flow is written as

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x},\mathbf{y}) + \mathbf{D}(\mathbf{x},\mathbf{y}) \tag{1}$$

$$\frac{d\mathbf{y}}{dt} = \mathbf{C}(\mathbf{x},\mathbf{y}) \tag{2}$$

where

$$\mathbf{x} = (\theta, q_v, q_c, q_r, K_m) \qquad \mathbf{y} = (u, v, w, \pi) \tag{3}$$

Here q_v, q_c, and q_r are the mixing ratios of water vapor, cloud water, and rainwater, respectively; θ and K_m are the potential temperature and the SGS eddy coefficient for momentum, respectively; u, v, and w are the x , y, and z components of the wind velocity, respectively; and π is the deviation of the nondimensional pressure.

Let $\mathbf{w}^0 = (\mathbf{x}^0, \mathbf{y}^0) = (\mathbf{x}(t_0), \mathbf{y}(t_0))$ denote the initial condition from which the model equation is numerically integrated. The initial condition defines a unique solution $\mathbf{x}(t)$, $\mathbf{y}(t)$ to the model equation.

The distance function is taken as

$$J = \int_{t_0}^{t_1} H[q_r(t), u(t)]\, dt \tag{4}$$

where $H[q_r(t), u(t)]$ is a scalar measuring the distance between q_r, u and their observations available at time t. The available observations are assumed to be distributed over a time interval $[t_0, t_1]$. The constraint is the model equation. For a given initial condition and for the corresponding solution $\mathbf{x}(t)$, $\mathbf{y}(t)$ of the model equation, the distance function is evaluated; the finite difference scheme integrates the model equation by marching outward from the initial condition[2]. Thus, the distance function is regarded as a function of $\mathbf{w}^0$.

Hence, assimilation of meteorological observations is formulated as a bound-constrained optimization problem:

$$\max J(\mathbf{w}^0)$$
$$\text{subject to}$$
$$\theta^0, q_v^0, q_c^0, q_r^0, K_m^0 \geq 0$$

An optimization method is used to fond the value $\mathbf{w}^0_{min}$ which minimizes J, starting from an initial guess of $\mathbf{w}^0_{min}$.

3 NUMERICAL OPTIMIZATION

The optimization problem has thousands or millions of variables. Hence, the L-BFGS-B method is used to solve this large-scale problem. L-BFGS-B is a limited memory algorithm for solving large nonlinear optimization problems subject to simple bounds on the variables.

The method proceeds roughly as follows[3]. At each iteration a limited memory BFGS approximation to the Hessian is updated. This limited memory matrix is used to define a quadratic model of the objective function. A search direction is then computed using the gradient projection method. Finally a line search is performed along the search direction.

4 GRADIENT COMPUTATION

Each iteration of the L-BFGS-B method requires the gradient of the objective function. If a finite-difference derivative approximation is used to calculate the gradient, then it requires executing as many integrations of the model equation as the number of the variables. Hence, huge CPU time is consumed.

A method is developed for gradient computation as follows[4]. If the initial condition is perturbed, the solution of the model equation is also perturbed. From the discretized model equation, the equation is derived for the perturbations. On the other hand, the distance function is discretized. If the initial condition is perturbed, the discretised distance function is also perturbed. From the perturbation equation and the perturbed distance function, the algorithm for gradient computation is derived.

5 NUMERICAL EXPERIMENTS

A numerical experiment is presented for a three-dimensional downburst. A downburst is a strong downdraft which induces an outburst of damaging winds on or near the ground. A downburst is initiated by placing a distribution of rainwater, and is simulated by integrating the model equation. The simulation domain is 20km in both horizontal directions and 10km in the vertical. The origin of the coordinate system is located at the center of the base. In Figures 1, 2, 3 and 4 are the rainwater, water vapor, flow and potential temperature, respectively, in the $x - z$ cross section at $y = 0$.

Out of the simulation result, the x component of the wind velocity and the rainwater are assumed to be observations from a Doppler radar. In order to recover the initial fields used for the simulation, the L-BFGS-B method solves the large-scale nonlinear optimization problem. It starts from an initial guess for the initial fields. Figure 5 shows

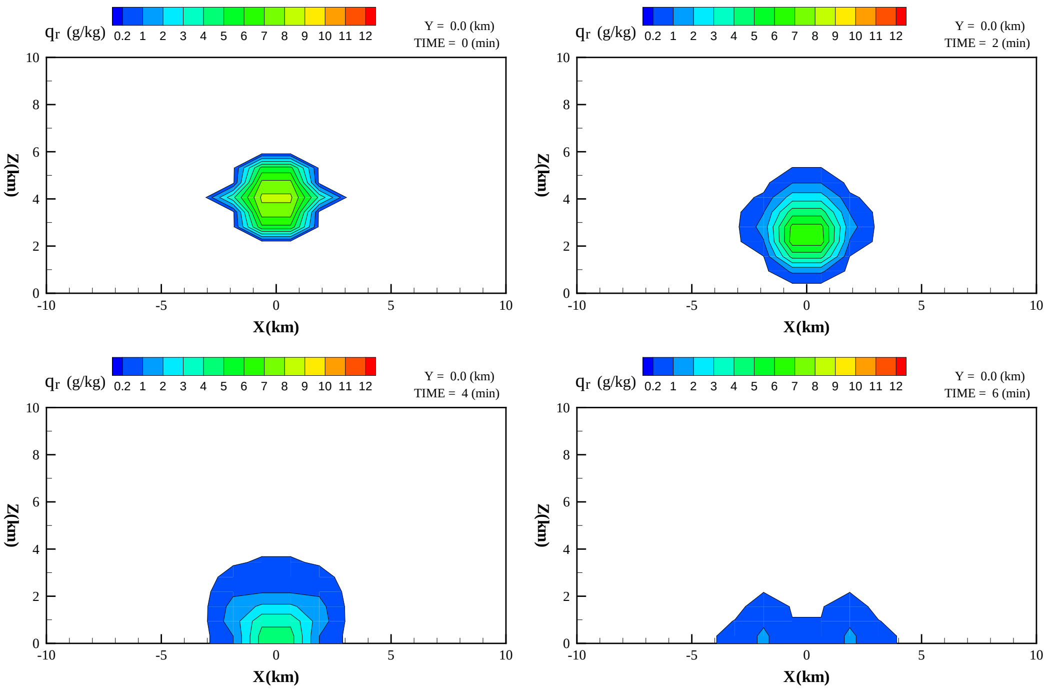

Figure 1: Evolution of the rainwater field (g/kg) in the $x - z$ cross section at $y = 0$ after the downburst initiation

the initial guess for the water vapor, flow, potential temperature, and pressure fields at $t = 0$. Figure 6 shows the convergence history of the objective function.

Figures 7, 8, 9 and 10 compare the water vapor, potential temperature, flow, and pressure recovered by the optimization with the corresponding ones of the initial fields in the $x - z$ cross section at $y = 0$.

6 CONCLUSIONS

- The developed method for the gradient computation reduces CPU time by a factor of about 100 compared with the fnite difference mrethod.
- The large-scale nonlinear optimization problem is solved by the L-BFGS-B algorithm. for submission that can be found in the Conference webpage.
- The recovered fields agree reasonasbly with the initial fields.

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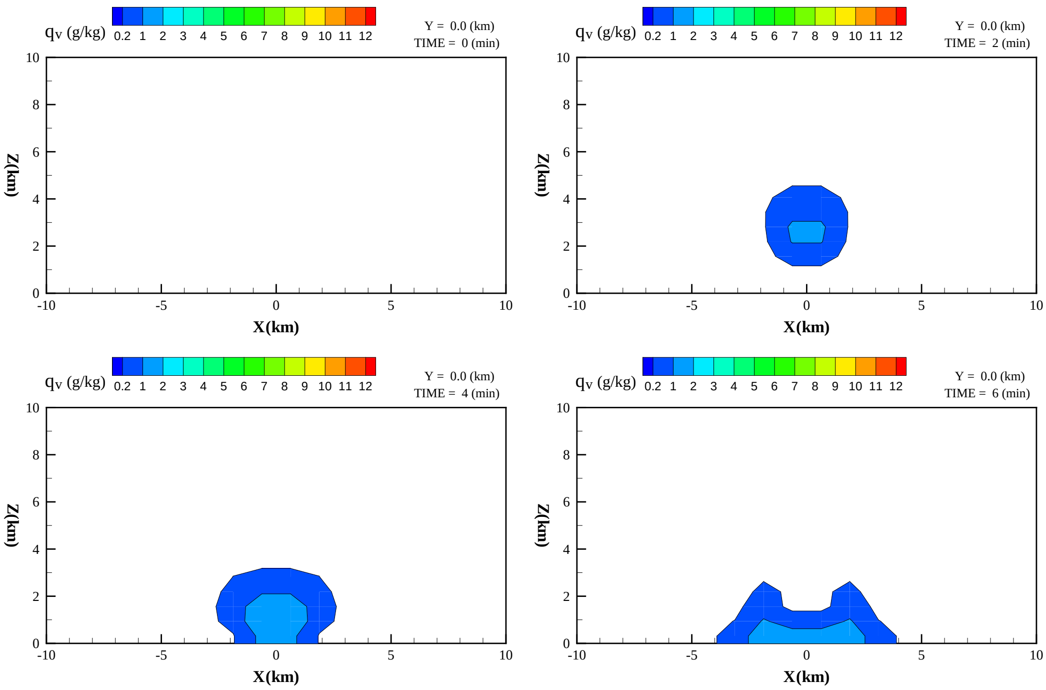

Figure 2: Evolution of the water vapor (g/kg) in the $x - z$ cross section at $y = 0$ after the downburst initiation

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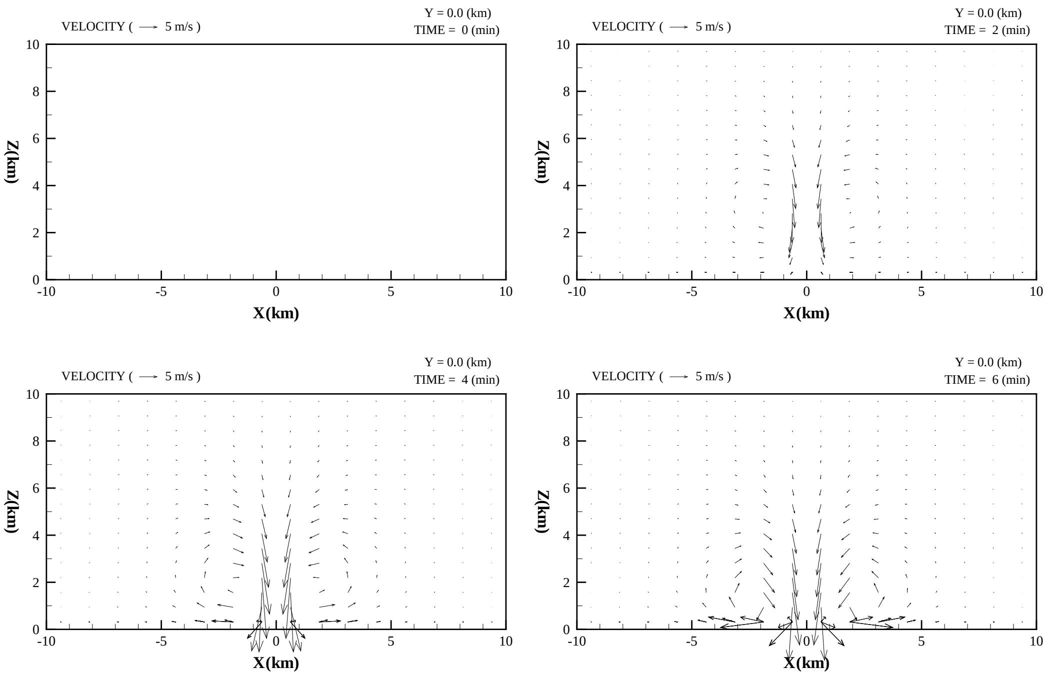

Figure 3: Evolution of the flow field in the $x - z$ cross section at $y = 0$ after the downburst initiation

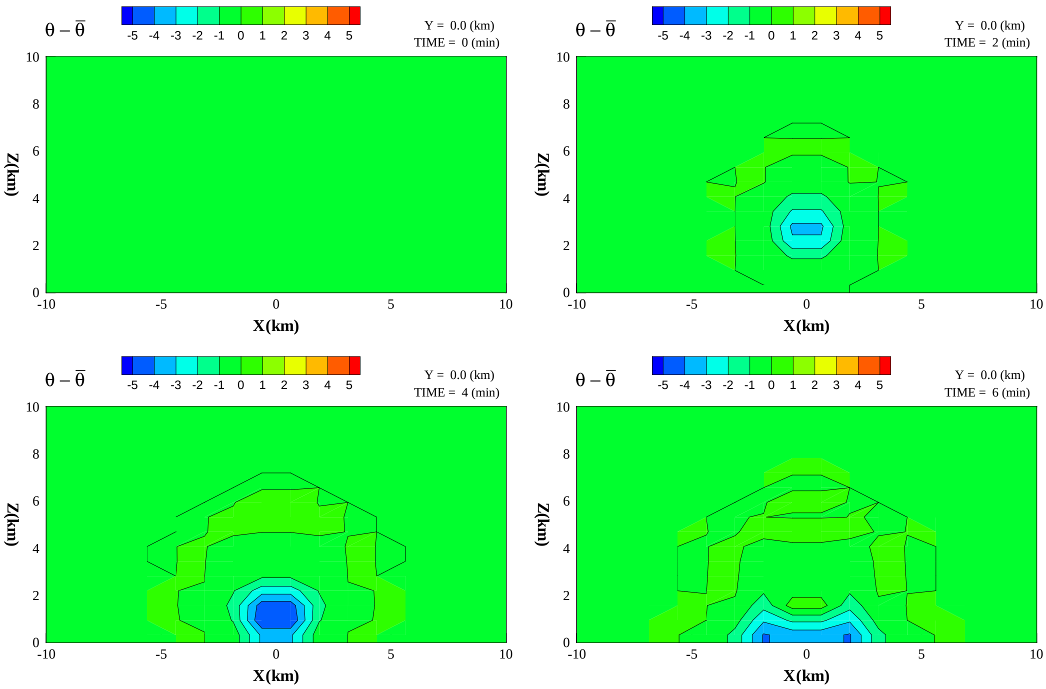

Figure 4: Evolution of the potential temperature field (K) in the $x - z$ cross section at $y = 0$ after the downburst initiation

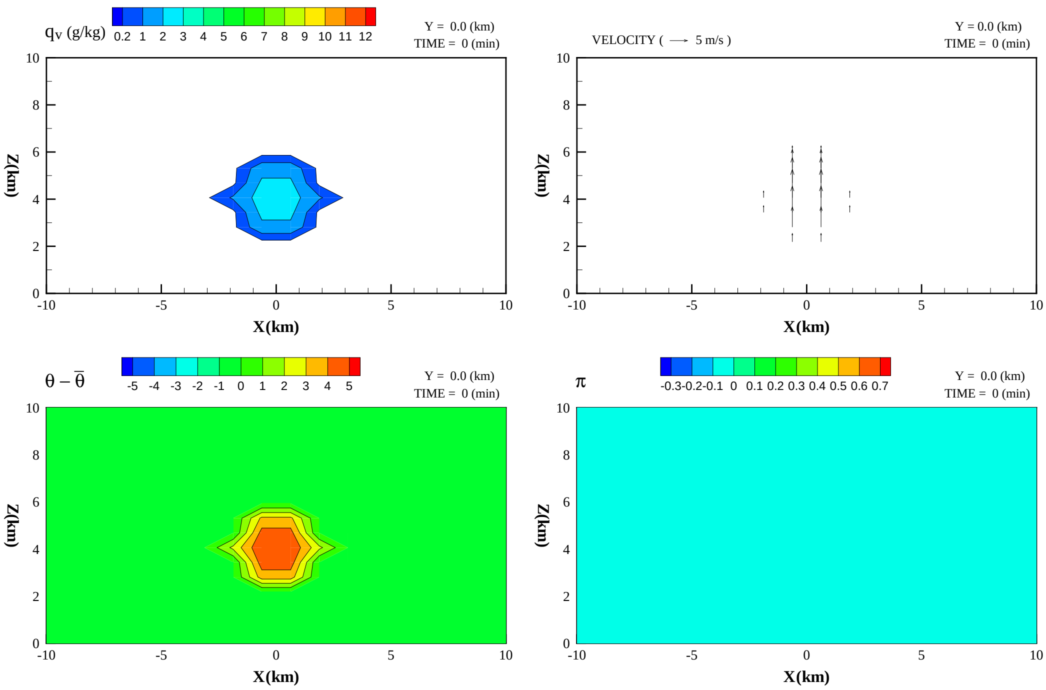

Figure 5: The initial guess for the water vapor (upper left), flow (upper right), potential temperature (lower left), and pressure (lower right) fields in the $x - z$ cross section at $y = 0$

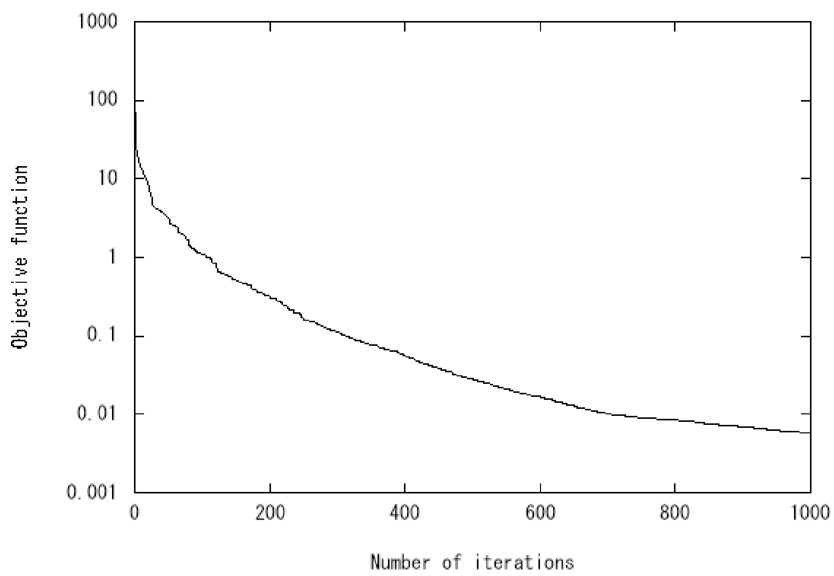

Figure 6: Convergence history of the optimization

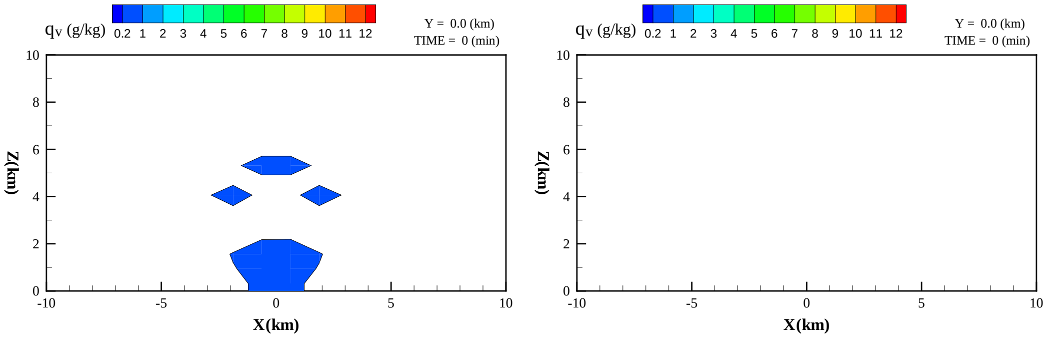

Figure 7: The water vapor field recovered by optimization (left) and the corresponding one of the initial fields (right)

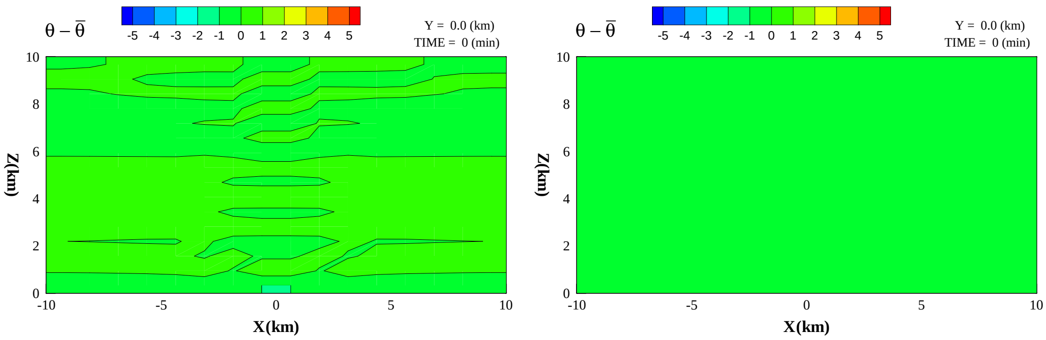

Figure 8: The potential temperature field recovered by optimization (left) and the corresponding one of the initial fields (right)

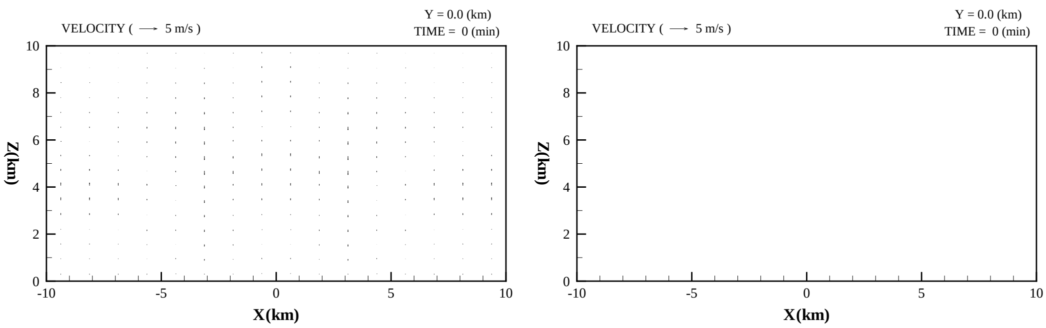

Figure 9: The flow field recovered by optimization (left) and the corresponding one of the initla fields (right)

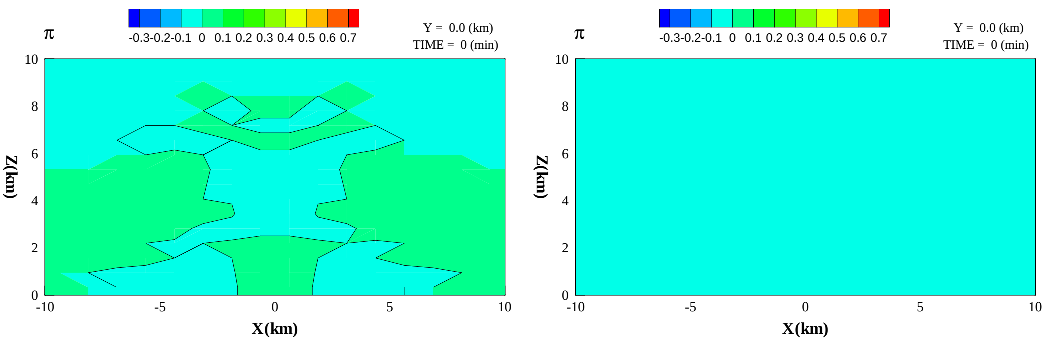

Figure 10: The pressure field recovered by optimization (left) and the corresponding one of the initla fields (right)