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SIMULATION OF TWO-PHASE FLOWS WITH FREE SURFACE IN A TANK USING ARBITRARY LAGRANGIAN-EULERIAN AND LEVEL SET COUPLED METHOD

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Abstract. *The motion of two-phase flows with free surface in an oscillating tank has been simulated numerically as a fluid-structure interaction problem using the Arbitrary Lagrangian-Eulerian and level set coupled method. It was shown by comparing with an existing experiment that the sloshing behavior of the free surface was predicted well. The time evolution of the liquid level obtained by the present method was similar to that by the method with oscillating body force when the wave height was small. The difference of the two methods was shown to be notable as the wave height increased. It was found that the flow field was different even when the wave height was small, and the effect of the moving wall was not seen in the case with the oscillating body force.*

1 INTRODUCTION

Two-phase flows with free surface are seen widely in engineering fields, and predictions of fluid phenomena with complicated surface motion are of practical importance, since such fluid motion may result in large pressure impact on structures. Thermal conditions such as heat transfer between fluid and structure are also affected by surface motion. Sloshing in an oscillating tank has been studied both experimentally and analytically[1,2,3] in relation to sea transport of oil, fuel behavior of spacecraft, seismic response of liquid metal reactors, and so on.

Free surface flows are generally solved in two ways: one is a single-phase treatment where a single-phase liquid flow is calculated by imposing a free surface boundary condition[1,2], and the other is a two-phase treatment where a stratified two-phase flow with interface is calculated by tracking the interface or the volume fraction of one phase[3]. The effect of gas-phase flow field is not included in the single-phase treatment. The effect of oscillating tank on fluid motion is also taken into account in two ways: one is to include a body force induced by the tank motion in the momentum equation of fluid, and the other is to move directly the computational grid according to the tank motion. Although the method with body force is easy from the view point of numerical simulation, the method with moving grid is apparently corresponding to the real phenomena.

In this study, the motion of the free surface and the two-phase flow field in an oscillating tank are simulated numerically as a fluid-structure interaction problem. A stratified two-phase flow is contained in a rectangular tank, and the tank is set in oscillatory motion. Incompressible Navier-Stokes equations are solved using the level set method[4]. In the level set method, the level set function, which is the distance function from the free surface, is calculated by solving the transport equation using the flow velocities. The motion of the tank is modeled using the Arbitrary Lagrangian-Eulerian (ALE) method[5], where the computational grid points are moved with the velocity of the tank. Both the liquid-phase and the gas-phase flow fields with the free surface motion induced by the oscillating tank are thus obtained in this study. It is shown by comparing the simulation results with the existing experimental results that the sloshing behavior of the free surface is predicted well by the present method. The simulation results are also compared with the case with the body force, and it is shown for small wave heights that the results with the body force are similar to the results with the moving grid. The effects of amplitude and frequency on the simulation results are also discussed.

2 NUMERICAL SIMULATION

2.1 Governing equations and numerical method

Governing equations for the two-phase flow field are the equation of continuity and the incompressible Navier-Stokes equations:

$$\nabla \cdot u = 0 \tag{1}$$

and

$$\rho \frac{\mathrm{D}u}{\mathrm{D}t} = -\nabla p + \nabla \cdot (2\mu D) + F_s \ , \tag{2}$$

where ρ, u, p and μ, respectively, are the density, the velocity, the pressure and the viscosity, D is the viscous stress tensor, and F_s is a body force due to the surface tension. The surface tension force is given by

$$F_s = \sigma\kappa\delta\nabla\phi \quad , \tag{3}$$

where σ, κ, δ and ϕ are the surface tension, the curvature of the interface, the Dirac delta function and the level set function[4], respectively. The level set function is a distance function defined as $\phi=0$ at the free surface, $\phi<0$ in the liquid region, and $\phi>0$ in the gas region. The curvature is expressed in terms of ϕ:

$$\kappa = \nabla \cdot \left(\frac{\nabla\phi}{|\nabla\phi|}\right) \quad . \tag{4}$$

The density and viscosity are given, respectively, by

$$\rho = \rho_l + (\rho_g - \rho_l)H \tag{5}$$

and

$$\mu = \mu_l + (\mu_g - \mu_l)H \quad , \tag{6}$$

where the subscripts g and l denote gas and liquid phases, respectively, and H is the smeared Heaviside function defined by

$$H = \begin{cases} 0 & (\phi < -\varepsilon) \\ \frac{1}{2}[1 + \frac{\phi}{\varepsilon} + \frac{1}{\pi}\sin(\frac{\pi\phi}{\varepsilon})] & (-\varepsilon \le \phi \le \varepsilon) \\ 1 & (\varepsilon < \phi) \end{cases} \quad , \tag{7}$$

where ε is a small positive constant for which $\nabla\phi = 1$ for $|\phi| \le \varepsilon$. The time evolution of ϕ is given by

$$\frac{D\phi}{Dt} = 0 \quad . \tag{8}$$

In this study, the computational grid is moving with the same velocity as the velocity of the oscillating tank, and the ALE method[5] is applied. The substantial derivative terms in Eqs. (2) and (8) are thus defined by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (u - U)\cdot\nabla \quad , \tag{8}$$

where U is the velocity of the computational grid.

In order to maintain the level set function as a distance function, an additional equation is solved:

$$\frac{\partial\phi}{\partial\tau} = (1 - |\nabla\phi|)\frac{\phi}{\sqrt{\phi^2 + \alpha^2}} \quad , \tag{9}$$

where τ and α are an artificial time and a small constant, respectively. The level set function becomes a distance function in the steady-state solution of the above equation. The following equation is also solved to preserve the total mass of liquid and gas phases in time[6]:

$$\frac{\partial\phi}{\partial\tau} = (A_o - A)(1 - \kappa)|\nabla\phi| \quad , \tag{10}$$

where A denotes the mass corresponding to the level set function and A_0 denotes the mass for the initial condition. The total mass of the twp-phase flow in a tank is conserved in the steady-state solution of the above equation.

The finite difference method is used to solve the governing equations. The staggered mesh is used for spatial discretization of velocities. The convection terms are discretized using the second order upwind scheme and other terms by the central difference scheme. Time integration is performed by the second order Adams-Bashforth method. The SMAC method is used to obtain pressure and velocities.

2.2 Simulation conditions

The sloshing of water in a rectangular tank is simulated in the following. The simulation conditions are almost the same as the conditions of the sloshing experiment[1]. The size of the tank is 1.0 m x 1.2 m x 0.1 m, and the initial water level is 0.5 m as shown in Fig. 1. The tank is set in oscillating motion in one horizontal direction. The oscillation of the tank location in the horizontal direction is given by

$$x = A\sin(\omega t) \quad , \tag{11}$$

where A = 0.0093 m and ω = 5.311 rad/s are, respectively, the amplitude and the angular frequency of the oscillation. The velocity of the computational grid is used in the present simulation and is given as the differential of the tank location,

$$U = A\omega\cos(\omega t) \quad . \tag{12}$$

In this study, the case with the oscillatory body force is compared with the case with the oscillatory tank. The oscillatory body force is given as the differential of the tank velocity,

$$f = -A\omega^2\sin(\omega t) \; . \tag{13}$$

The above body force is applied as the external force term in the momentum equation, and the tank is not moved and U=0 in this case.

Two-dimensional calculations are performed, since the thickness of the tank is much smaller than other sides as shown in Fig. 1. The number of calculation grid is determined as 50 x 60 after several sensitivity calculations, and the grid size is 0.02 m in the following. The time step size is 0.002 s, and the Courant number is much smaller than unity throughout calculations. The slip boundary conditions are applied at all the tank walls.

3 RESULTS AND DISCUSSION

The time evolution of the liquid level at the left-side wall is shown in Fig. 2 along with the experimental results[7]. It is shown that the agreement between the simulation and the experiment is good even for large liquid level and the growth of the free surface is simulated well by the present numerical method. The shapes of the free surface at 3.54 s and 7.08 s during the transients[1] are shown in Fig. 3. The simulated free surfaces on the left are compared with the free surface observed in the experiment on the right. It is shown again that the sloshing phenomena are simulated well by the ALE and level set coupled method. It is confirmed through these comparisons that the present numerical method simulates the interaction between the moving tank and the two-phase flows with free surface very well.

The time evolution of the liquid level for the case with the oscillating body force is compared with that for the case with the oscillating tank in Fig. 4. The growth of the free surface is shown to be not much different between the two cases, though the growth rate becomes slightly smaller for the case with the body force as the wave height increases. The shapes of the free surface and the velocity fields at 3.54 s and 7.08 s are shown in Fig. 5. It is noted that the velocity vector at 3.54 s is 5 times larger than that at 7.08 s. It is found that the surface shape is not much different between the two cases, but the velocity field is affected. The gas-phase velocity is much smaller for the case with the body force at 3.54 s, and the flow direction is different in the liquid phase. Although the difference of velocity field is small at 7.08 s, the gas-phase velocity is also smaller for the case with the body force, and the location of the vortex center is slightly shifted. These differences are resulted from the difference of sloshing mechanism in two methods: in the case with the oscillating tank, the liquid and gas phases are pushed by the tank wall and the sloshing motion starts, while in the case with the body force, the liquid and gas phases are accelerated directly and push the tank wall, and the sloshing motion starts. The effect of wall motion is clearly seen in the case with the moving tank at 3.54 s in Fig. 5, though the liquid level and the surface shape are not much affected.

In order to see the effect of oscillation amplitude and frequency on the two methods, the amplitude A and the frequency ω are increased in Figs. 6 and 7, respectively. The results with doubled and tripled amplitudes are shown in Fig. 6. It is shown in the beginning stage that the time evolution of the liquid level is almost the same for both the method. In other words, the difference of the two methods becomes notable as the wave height increases. This is also the case with the base amplitude shown in Fig. 4. The results with doubled and tripled oscillation frequency are shown in Fig. 7. The sloshing phenomena with the growth of surface wave are not seen, since the increased frequency is much different from the resonant frequency. Large surface oscillation does not appear, but complicated surface fluctuations occur. It is of interest that the wave height becomes larger as the oscillation frequency increases. The wave height is relatively small throughout the simulation in comparison with that in Fig. 6, and the difference between the two methods is small.

4 CONCLUSIONS

The motion of the free surface and the two-phase flow field in the oscillating tank have been simulated numerically as a fluid-structure interaction problem using the ALE and level set coupled method. It was shown by the comparison with the experiment that the sloshing behavior of the free surface was predicted well by the present numerical method. The simulation results were also compared with the case with the oscillating body force, which is an easier numerical method for simulating oscillatory phenomena. It was shown that the time evolution of the liquid level for the case with the body force was not much different from that for the case with the oscillating tank. The difference of the two methods became notable as the wave height increased. The flow field was shown to be different even when the wave height was not so large, and the effect of the moving wall was seen only in the case with the oscillating tank. The effects of oscillation amplitude and frequency on the simulation results were also shown to be small.

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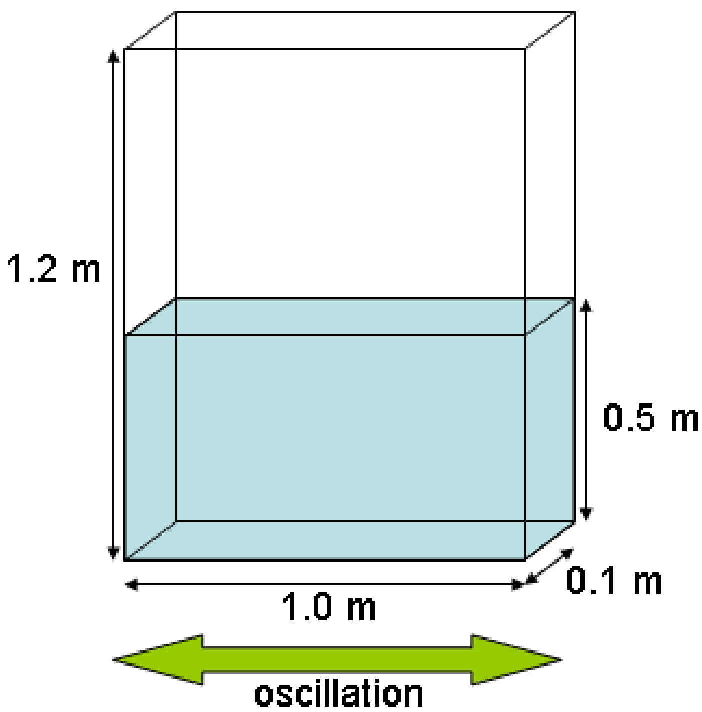


Figure 1: Schematic of sloshing experiment

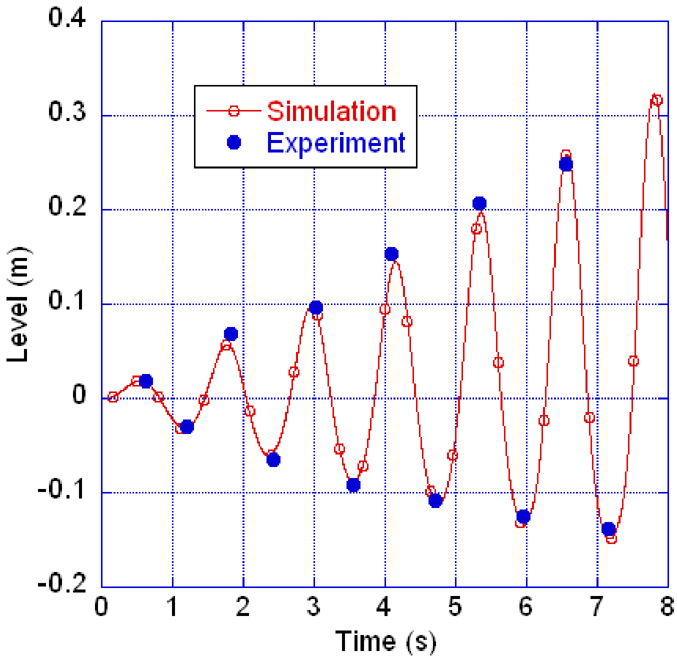
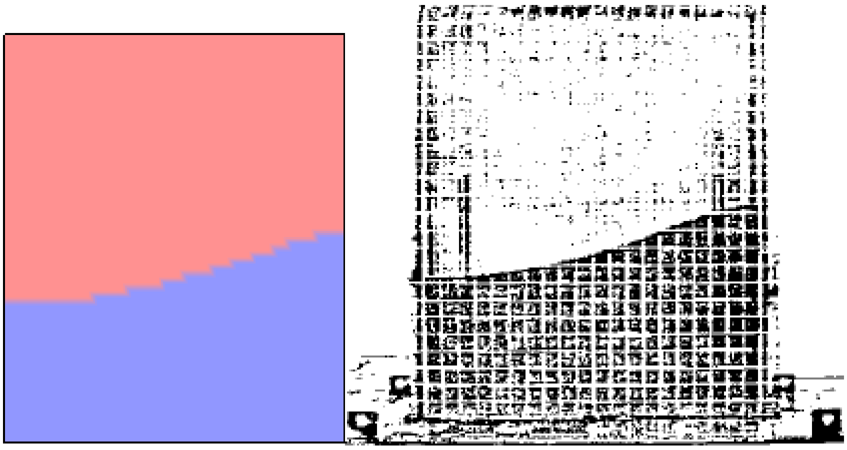
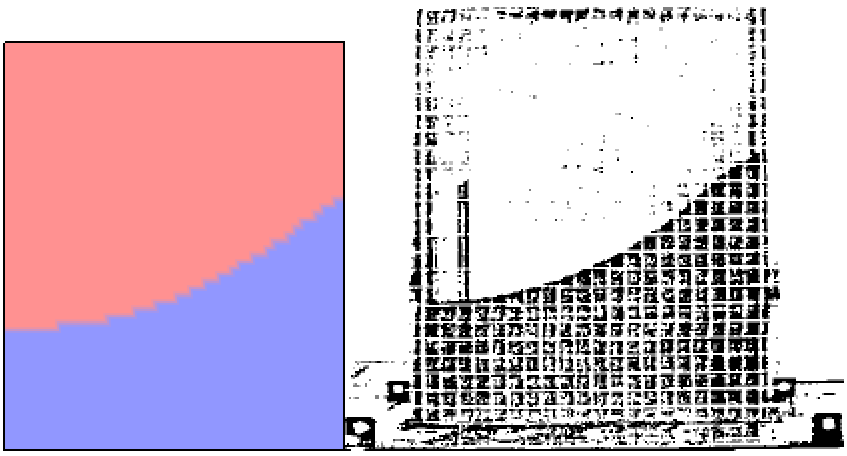


Figure 2: Time evolution of liquid level: comparison with experiment



(a) 3.54 s



(b) 7.08 s

Figure 3: Free surface shape: simulation (left) and experiment (right)

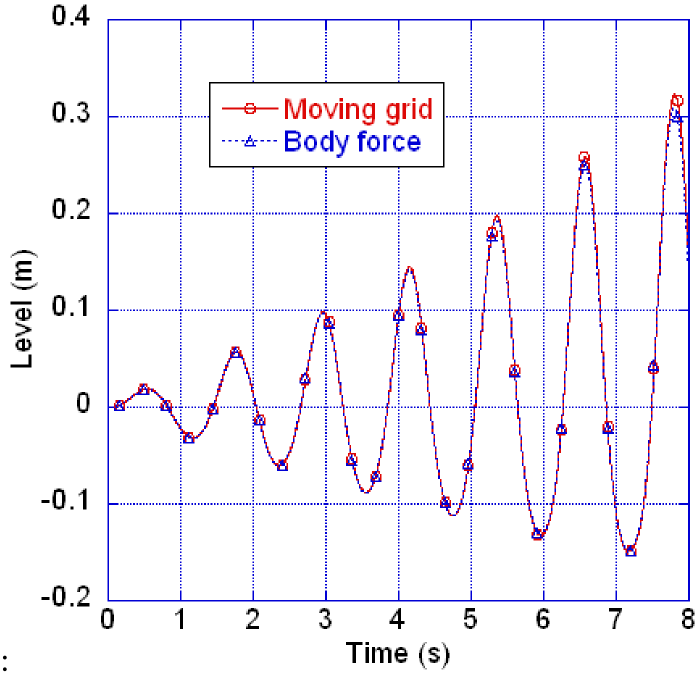
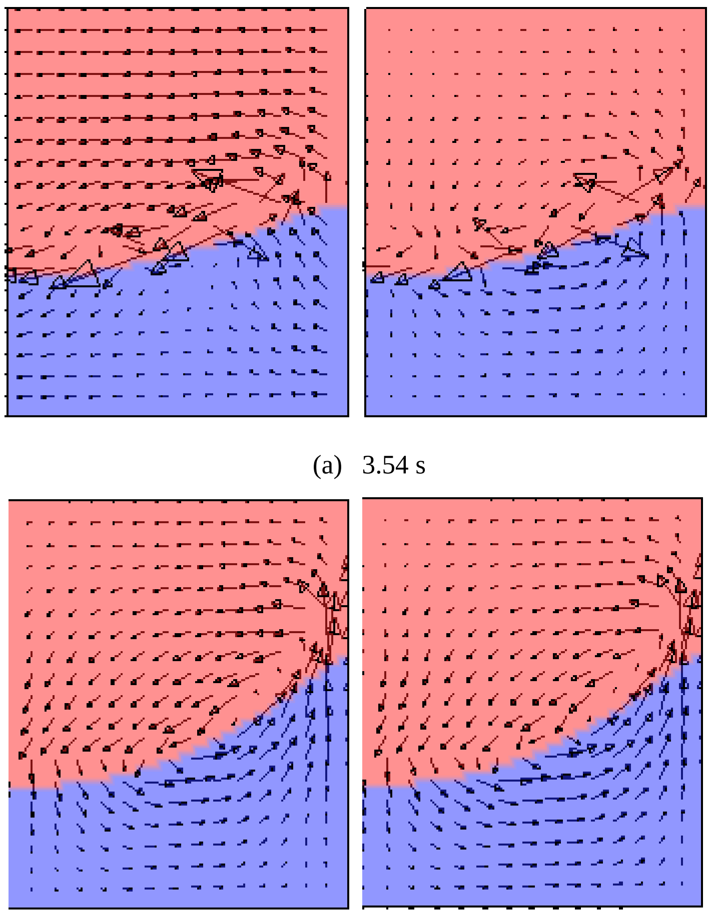


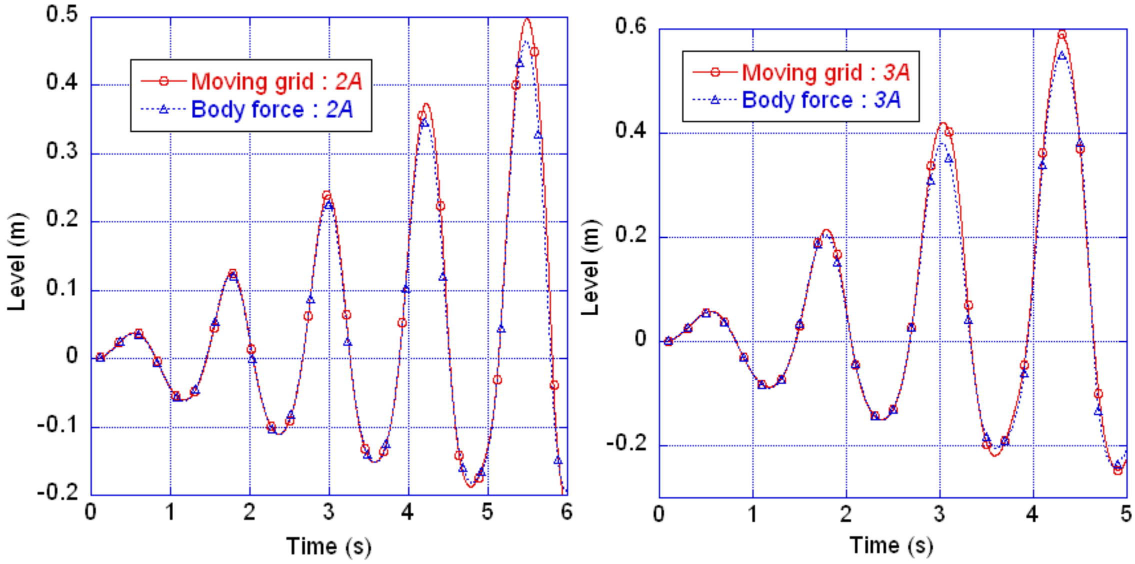
Figure 4: Time evolution of liquid level: comparison with body force method



(a) 3.54 s

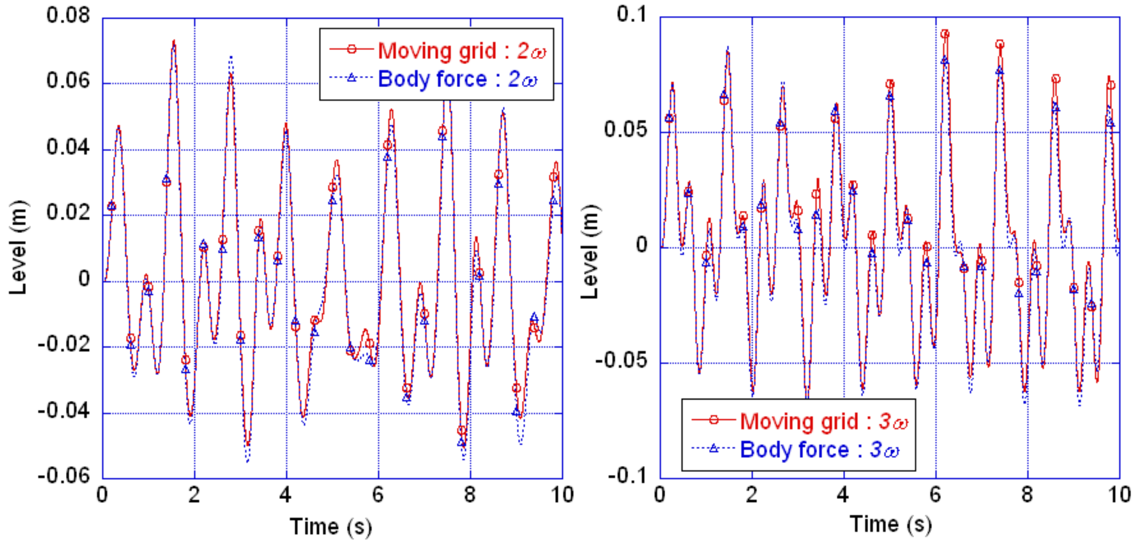
(b) 7.08 s

Figure 5: Free surface shape: moving grid (left) and body force method (right)



(a) Amplitude x 2 (b) Amplitude x 3

Figure 6: Time evolution of liquid level: effect of amplitude



(a) Frequency x 2 (b) Frequency x 3

Figure 7: Time evolution of liquid level: effect of frequency