SPACE-TIME DISCONTINUOUS GALERKIN METHOD FOR NONLINEAR WATER WAVES: VARIATIONAL VS. WEAK FORMULATION

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ABSTRACT

Space-time discontinuous Galerkin (DG) finite element methods are advantageous for mathematical models arising in fluid mechanics that have moving boundaries or interfaces in the flow domain [1,4]. Many water wave problems in marine and offshore engineering are mathematically described by a nonlinear free surface gravity water wave model. This mathematical model arises succinctly from Luke's variational principle [3], which is associated with the conservation of energy and phase space under suitable boundary conditions. Therefore, we present a new variational space-time discontinuous Galerkin (DG) discretization stemming from a discrete variational principle for the free surface waves as opposed to a weak formulation. Thus, we preserve the variational structure, and the associated energy and phase-space volume. To demonstrate its advantage, we compare the space-time DG schemes for the nonlinear free surface waves stemming from a classical weak formulation (see [4]) and variational formulation (see[2]). Advantage of a discrete variational formulation against the weak formulation is clearly seen in Fig. 1 for the linear harmonic waves simulated using both the space-time DG schemes.



Figure 1: Observe the different decay in wave amplitude of free surface height obtained using the spacetime DG scheme based on weak formulation (left) and variational formulation (right) at t = 8T.

REFERENCES

- [1] V.R. Ambati and O. Bokhove. "Space-time discontinuous Galerkin discretization of rotating shallow water equations". *J. Comput. Phys.*, **225**(2), 1233–1261, 2007.
- [2] V.R. Ambati, J.J.W. van der Vegt and O. Bokhove, Variational space-time (dis)continuous Galerkin method for linear free surface waves, In prep., 2009. http://eprints.eemcs.utwente.nl/11714/
- [3] J.C. Luke. "A variational principle for a fluid with a free surface". J. Fluid Mech., 27(2), 395–397, 1967.
- [4] J.J.W. van der Vegt and Y. Xu. "Space-time Discontinuous Galerkin Method for nonlinear water waves". J. Comput. Phys., 224(1) 17–39, 2007.