V European Conference on Computational Fluid Dynamics
ECCOMAS CFD 2010
J. C. F. Pereira and A. Sequeira (Eds)
Lisbon, Portugal,14-17 June 2010

CONSISTENCY OF SIMPLEC SCHEME IN COLLOCATED GRIDS

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Key words: Collocated grid, Pressure-velocity coupling, SIMPLEC scheme

Abstract. *The most common procedures to deal with the pressure velocity coupling within a pressure-based, segregated approach are those pertaining to the SIMPLE family of schemes. These approaches connect velocity corrections to pressure gradient corrections by appropriately neglecting some terms in the correcting velocity equation derived from the momentum equations. In collocated grids there exist two types of velocity fields: the convecting velocity, a continuity-satisfying (CS) field and the convected velocity, a momentum-satisfying (MS) variable. SIMPLE-family schemes were originally derived for staggered grids where there is only one type of velocity and no interpolation is required to obtain the CS field from the MS one. In collocated grids the convecting face velocity, that is the only one required to satisfy continuity, is calculated in a special manner usually following what is called the Pressure Weighted Interpolation Method (PWIM) (C. Rhie, W. Chow. AIAA J. vol 21(11), pp 1525-1532, 1983). To carry over the SIMPLE-related staggered approaches to a collocated grid the velocity corrections should be linked to the pressure gradient corrections by algebraic manipulations of the MS field equations and then transferred via PWIM to the CS expressions. In this paper it will be argued that some of previously employed implementations of SIMPLEC are inconsistent with this procedure.*

1 INTRODUCTION

Within a pressure-based method for numerically solving the Navier-Stokes equations, SIMPLE-type schemes are the most popular approaches in handling the pressure velocity coupling. Generally speaking, in these methods the velocity is originally predicted with the momentum equations containing the pressure field at the previous step and later corrected with a velocity/pressure-gradient correction whose expression is derived from the original momentum equation. The continuity equation is transformed into another one for the pressure correction that is next utilized to drive the velocity field towards satisfying mass conservation and in some of the schemes also serves to update the pressure field. In a collocated variable arrangement there are peculiarities associated to the use of these schemes due to the dual velocity field at nodes and faces. SIMPLE schemes were originally devised in a staggered variable arrangement where there is only a relation between velocity corrections and pressure gradient corrections. This is no longer the case in a collocated grid where the momentum-satisfying field follows a relation at the nodes whereas at the faces it is dependent on the actual approach employed in the collocated grid (PWIM/MWIM and/or their transient extensions). This paper focusses on the correct way to derive the relation at the faces for SIMPLE [1] and SIMPLEC [2], especially for the latter where an inconsistency in its expression as used by some previous researchers will be commented upon.

2 SIMPLE AND SIMPLEC SCHEMES FOR AN UNSTEADY FLOW

In a collocated arrangement there are two types of velocity fields: the convected velocity located at the nodes that satisfies the momentum equation (Momentum-Satisfying field) and the convecting velocity at the faces that is required to satisfy continuity (Continuity-Satisfying field). The MS field is computed following the traditional procedure of first discretizing the momentum equation and then solving the resulting algebraic system. Each convected velocity equation in the system represents a balance in a certain finite volume between the net convective and diffusive fluxes and the source terms. Contrary to the convected velocity the CS field is not governed by a convection-diffusion equation of its own. At any arbitrary face location e, let us say along the x coordinate, its value is obtained with an algebraic expression derived, with some assumptions, from the e-averaged nodal equations. The e-average of a variable ϕ is an arithmetic average defined as

$$\overline{\phi}^e = f_x\phi_P + (1 - f_x)\phi_E \quad i = P, E \tag{1}$$

f_x is a weighting factor and P and E are the nodes that share the face e. This average provides the value of ϕ at e in terms of known values at P and E. Depending on the way f_x is defined the average can be strictly geometric or be weighted by some other variable. Details are given in Pascau [3]. Because of this dual velocity field the collocated arrangement requires two SIMPLE relations between the correcting velocity and the correcting pressure gradient.

To discuss the procedure a general unsteady case (real or pseudo) with underrelaxation will be considered. The relaxed unsteady discretized momentum equation for the u velocity is

$$\begin{aligned}
\widetilde{A}^u_{P|P} u^*_P &= \alpha_u \left(\sum_{j|P} A^u_j u^*_j + S^u_P \Delta V_P \right) - \alpha_u \Delta V_P \left. \frac{\partial p}{\partial x} \right|^l_P + (1-\alpha_u) \widetilde{A}^u_{P|P} u^l_P + \\
&+ \alpha_u \frac{\rho_P \Delta V_P}{\Delta t} u^n_P \\
\widetilde{A}^u_{P|P} &= \sum_{j|P} A^u_j + \frac{\rho_P \Delta V_P}{\Delta t} = A^u_{P|P} + \frac{\rho_P \Delta V_P}{\Delta t}
\end{aligned} \tag{2}$$

α_u being the relaxation factor. Two indices are used where needed. For instance the sum is carried over all j-neighbours of P, $(j|P)$, and $A^u_{P|P}$ is the diagonal, (P), coefficient of node P. The term in brackets contains two addends: the contribution of the neighbour nodes and all sources but the pressure gradient which is considered separately. There is also a contribution from the previous iteration u^l_P and the preceding time step u^n_P. The equivalent fictitious equation for the east face velocity is

$$\begin{aligned}
\widetilde{A}^u_{P|e} u^*_e &= \alpha_u \left(\sum_{j|e} A^u_j u^*_j + S^u_e \Delta V_e \right) - \alpha_u \Delta V_e \left. \frac{\partial p}{\partial x} \right|^l_e + (1-\alpha_u) \widetilde{A}^u_{P|e} u^l_e + \\
&+ \alpha_u \frac{\rho_e \Delta V_e}{\Delta t} u^n_e \\
\widetilde{A}^u_{P|e} &= \sum_{j|e} A^u_j + \frac{\rho_e \Delta V_e}{\Delta t} = A^u_{P|e} + \frac{\rho_e \Delta V_e}{\Delta t}
\end{aligned} \tag{3}$$

and written in an alternative way

$$\begin{aligned}
u^*_e &= \alpha_u \left(\frac{\sum_{j|e} A^u_j u^*_j + S^u_e \Delta V_e}{\widetilde{A}^u_{P|e}} \right) - \alpha_u \frac{\Delta V_e}{\widetilde{A}^u_{P|e}} \left. \frac{\partial p}{\partial x} \right|^l_e + (1-\alpha_u) u^l_e + \\
&+ \alpha_u \frac{\rho_e \Delta V_e}{\Delta t \widetilde{A}^u_{P|e}} u^n_e
\end{aligned} \tag{4}$$

Let us note that this expression is never computed as it stands, it is only employed to derive a workable u^*_e expression. There are several terms in the previous equation that are not directly computable, namely, the factor $\Delta V_e / \widetilde{A}^u_{P|e}$ and the term in brackets. The diagonal coefficient $\widetilde{A}^u_{P|e}$ is never assembled because the discretized transport equation at e is not computed. It has to be obtained as a function of the diagonal coefficients of the nodes. On the other hand, how to estimate the term in brackets (the contribution from neighbour faces plus the source term) is at the root of the different approaches to calculate the face velocity. PICTURE [3], the consistent extension to unsteady problems of the well known Rhie-Chow procedure, assumes that a factor contained in this term can be calculated as an arithmetic mean of its counterparts in the equations of the nodes that share the face, just as Rhie-Chow proposed for a steady problem. The core of this approach is the following expression

$$
\begin{aligned}
H_e^u = \frac{\sum_{j|e} A_j^u u_j^* + S_e^u \Delta V_e}{\widetilde{A}_{P|e}^u} &= \frac{\sum_{j|e} A_j^u u_j^* + S_e^u \Delta V_e}{(1+\delta_e) A_{P|e}^u} = \\
&= \frac{1}{1+\delta_e} \overline{\left(\frac{\sum_{j|i} A_j^u u_j^* + S_i^u \Delta V_i}{A_{P|i}^u} \right)}^e \qquad i = P, E
\end{aligned} \tag{5}
$$

where the factor calculated as an average is apparent and δ_e is defined as

$$
\delta_e = \frac{\rho_e \Delta V_e}{\Delta t A_{P|e}^u} \quad \Rightarrow \quad \widetilde{A}_{P|e}^u = A_{P|e}^u + \frac{\rho_e \Delta V_e}{\Delta t} = (1+\delta_e) A_{P|e}^u \tag{6}
$$

Averaging this factor and not the whole term in brackets in Eqn. 4 is necessary in order to obtain a consistent scheme, that is, one that provides steady solutions independent of the time step [3]. At the same time, we cannot include the previous iteration or the preceding time step contribution in H_e^u, the factor to be calculated as an average, otherwise the solution would depend on the time step and the relaxation factor. Once a means to estimate $\Delta V_e / \widetilde{A}_{P|e}^u$ is proposed the procedure could be considered complete as all terms in Eqn. 4 will then be computable. In terms of δ_e this final expression is

$$
\begin{aligned}
u_e^* &= \frac{\alpha_u}{1+\delta_e} \overline{\left(\frac{\sum_{j|i} A_j^u u_j^* + S_i^u \Delta V_i}{A_{P|i}^u} \right)}^e - \alpha_u \frac{\Delta t}{\rho_e} \frac{\delta_e}{1+\delta_e} \left. \frac{\partial p}{\partial x} \right|_e^l + (1-\alpha_u) u_e^l + \\
&+ \alpha_u \frac{\delta_e}{1+\delta_e} u_e^n
\end{aligned} \tag{7}
$$

Although using Eqn. 7 to obtain u_e^* is completely acceptable, it is preferable to derive an alternative expression by noticing that the e-average in Eq. 5 can also be obtained by e-averaging the two nodal equations at P and E. The reason for seeking this alternative is that we have in fact two options to evaluate the i-terms in Eqn. 7. The first option is to calculate the average with factors that have been estimated before solving the momentum equation, that is, using the following approximation

$$
\frac{\sum_{j|e} A_j^u u_j^* + S_e^u \Delta V_e}{A_{P|e}^u} = \overline{\left(\frac{\sum_{j|i} A_j^{u(l)} u_j^l + S_i^{u(l)} \Delta V_i}{A_{P|i}^{u(l)}} \right)}^e \tag{8}
$$

where the (previous) iteration l at which the factors are calculated is explicitly indicated. The second option is to employ in the average the nodal velocities coming from the momentum equation solution

$$\frac{\sum_{j|e} A_j^u u_j^* + S_e^u \Delta V_e}{A_{P|e}^u} = \overline{\left(\frac{\sum_{j|i} A_j^{u(l)} u_j^* + S_i^{u(l)} \Delta V_i}{A_{P|i}^{u(l)}}\right)}^e \tag{9}$$

Xu and Zhang [4] showed that this second version was faster to converge because it uses velocity values as soon as they become available. The actual implementation does not use the average in Eqn. 9 as it is, instead it operates with the nodal equations to cast this average as a function of face velocity values and averages. The procedure involves writing an equation for the node E similar to Eqn. 2 and e-average the equations at nodes P and E. The final expression is

$$\begin{aligned} \overline{(1+\delta_i)u_i^*}^e &= \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j^* + \Delta V_i S_i^u}{A_{P|i}^u}\right)}^e + \alpha_u \Delta t \overline{\left[\frac{\delta_i}{\rho_i}\left(-\left.\frac{\partial p}{\partial x}\right|_i^l\right)\right]}^e \\ &+ (1-\alpha_u)\overline{(1+\delta_i)u_i^l}^e + \alpha_u \overline{\delta_i u_i^n}^e \quad i = P, E \end{aligned} \tag{10}$$

and then

$$\begin{aligned} \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j^* + \Delta V_i S_i^u}{A_{P|i}^u}\right)}^e &= \overline{(1+\delta_i)u_i^*}^e + \alpha_u \Delta t \overline{\left[\frac{\delta_i}{\rho_i}\left.\frac{\partial p}{\partial x}\right|_i^l\right]}^e \\ &- (1-\alpha_u)\overline{(1+\delta_i)u_i^l}^e - \alpha_u \overline{\delta_i u_i^n}^e \quad i = P, E \end{aligned} \tag{11}$$

Substituting this in Eqns. 4 by means of Eqn. 5 gives the final expression in terms of δ_i and δ_e

$$\begin{aligned} (1+\delta_e)u_e^* &= \overline{(1+\delta_i)u_i^*}^e + \alpha_u \Delta t \left[\overline{\frac{\delta_i}{\rho_i}\left.\frac{\partial p}{\partial x}\right|_i^l}^e - \frac{\delta_e}{\rho_e}\left.\frac{\partial p}{\partial x}\right|_e^l\right] + \\ &+ (1-\alpha_u)\left[(1+\delta_e)u_e^l - \overline{(1+\delta_i)u_i^l}^e\right] + \alpha_u\left[\delta_e u_e^n - \overline{\delta_i u_i^n}^e\right] \end{aligned} \tag{12}$$

Thus the CS field at the current iteration depends on the MS and pressure fields at the same iteration, as well as on values from previous inner iteration and time step. Eqn. 7 and Eqn. 12 are totally equivalent only in the case that Eqn. 10 is satisfied, that is, when the momentum equation has zero residual at iteration l. In any other case both are different approximations to the same face velocity. From a computational point of view it is more convenient this last expression as we do not have to recalculate the summation in Eqn. 7 with the newly available nodal velocities. In the final expression there is only one factor, δ_e, to be estimated as a function of its nodal values so we only require one assumption for the evaluation of the complete expression. Although other averages are not excluded, in all computational cases to be presented the traditional arithmetic average has been adopted, $\delta_e = \overline{\delta_i}^e$.

We calculate the face velocity with Eqn. 12 employing the most recent values of the nodal velocity u^*, right after solving the momentum equation for the MS field. Let us assume that Eqn. 2 is rewritten for a sought MS field that, with an updated pressure field to be determined and via Eqn. 12, provides a CS field that satisfies continuity. All these fields will be denoted with the superindex $l+1$.

$$\begin{aligned}\widetilde{A}^u_{P|P} u^{l+1}_P &= \alpha_u \left(\sum_{j|P} A^u_j u^{l+1}_j + S^u_P \Delta V_P \right) - \alpha_u \Delta V_P \left. \frac{\partial p}{\partial x} \right|^{l+1}_P + \\ &+ (1-\alpha_u) \widetilde{A}^u_{P|P} u^l_P + \alpha_u \frac{\rho_P \Delta V_P}{\Delta t} u^n_P \end{aligned} \tag{13}$$

When the relaxed iterations have converged the $l+1$ iteration value becomes the $n+1$ time step value. The SIMPLE procedure starts out by deriving the expression for the nodal velocity correction in terms of the pressure gradient correction using the momentum equation and then transferring this expression to that of the face velocity correction. The first task is accomplished subtracting Eqn. 2 from Eqn. 13, considering $u^{l+1} = u^* + u'$, u' being the velocity correction

$$\widetilde{A}^u_{P|P} u'_P = \alpha_u \sum_{j|P} A^u_j u'_j - \alpha_u \Delta V_P \left. \frac{\partial p}{\partial x} \right|'_P \tag{14}$$

where the change in the source has been neglected. SIMPLE also neglects the contribution from the neighbour corrections giving a final algebraic relation

$$u'_P = -\alpha_u \frac{\Delta V_P}{\widetilde{A}^u_{P|P}} \left. \frac{\partial p}{\partial x} \right|'_P \quad \Rightarrow \quad (1+\delta_P) u'_P = -\alpha_u \Delta t \frac{\delta_P}{\rho_P} \left. \frac{\partial p}{\partial x} \right|'_P \tag{15}$$

The expression at the face at iteration $l+1$ is

$$\begin{aligned}(1+\delta_e) u^{l+1}_e &= \overline{(1+\delta_i) u^{l+1}_i}^e + \alpha_u \Delta t \left[\overline{\frac{\delta_i}{\rho_i} \left. \frac{\partial p}{\partial x} \right|^{l+1}_i}^e - \frac{\delta_e}{\rho_e} \left. \frac{\partial p}{\partial x} \right|^{l+1}_e \right] + \\ &+ (1-\alpha_u) \left[(1+\delta_e) u^l_e - \overline{(1+\delta_i) u^l_i}^e \right] + \alpha_u \left[\delta_e u^n_e - \overline{\delta_i u^n_i}^e \right] \end{aligned} \tag{16}$$

Subtracting Eqn. 12 from Eqn. 16 we obtain

$$(1+\delta_e) u'_e = \overline{(1+\delta_i) u'_i}^e + \alpha_u \Delta t \left[\overline{\frac{\delta_i}{\rho_i} \left. \frac{\partial p}{\partial x} \right|'_i}^e - \frac{\delta_e}{\rho_e} \left. \frac{\partial p}{\partial x} \right|'_e \right] \tag{17}$$

but Eqn. 15 and a similar one for u'_E show that

$$\overline{(1+\delta_i) u'_i}^e = -\alpha_u \Delta t \overline{\frac{\delta_i}{\rho_i} \left. \frac{\partial p}{\partial x} \right|'_i}^e \tag{18}$$

then the face correction is

$$(1+\delta_e)u'_e = -\alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{\prime} \tag{19}$$

formally identical to the nodal correction. This expression is well known but we wanted to specify all steps taken in the proper derivation. The steady state expressions are recovered when $\delta_{i,e} \to 0$ and $\delta_{i,e}\Delta t/\rho_{i,e} \to \Delta V_{i,e}/A^u_{P|i,e}$, that is

$$u'_e = -\alpha_u \frac{\Delta V_e}{A^u_{P|e}} \left.\frac{\partial p}{\partial x}\right|_e^{\prime} \qquad u'_i = -\alpha_u \frac{\Delta V_i}{A^u_{P|i}} \left.\frac{\partial p}{\partial x}\right|_i^{\prime} \quad i = E, P \tag{20}$$

and similar equations for the other faces ($w$est, $n$orth and $s$outh in 2D).

We have just shown that if SIMPLE is used both nodal and face corrections have the same expression. We must stress that the face relation has not been assumed a priori nor has it been obtained with the fictitious 'momentum equation' at the face, rather it has been extracted from the face velocity expression and the momentum corrections.

Now let us show that when SIMPLEC is employed the face expression is different from the nodal one. SIMPLEC proposal does not neglect the neighbour contribution in Eqn. 14, instead it subtracts from both sides $\alpha_u(\sum_{j|P} A^u_j)u'_P$

$$\left(\widetilde{A}^u_{P|P} - \alpha_u \sum_{j|P} A^u_j\right) u'_P \;\; = \;\; \alpha_u \sum_{j|P} A^u_j (u'_j - u'_P) - \alpha_u \Delta V_P \left.\frac{\partial p}{\partial x}\right|_P^{\prime} \tag{21}$$

SIMPLEC assumes that it is the difference between neighbour velocity corrections what is negligible, a distinct assumption from that of SIMPLE where the neighbour velocity corrections were not conserved in the final expression. Hence, it neglects the first term on the right hand side providing the algebraic relation

$$(1+\delta_P)u'_P = -\alpha_u \Delta t \frac{\delta_P}{\rho_P}(1+\widetilde{k}_P) \left.\frac{\partial p}{\partial x}\right|_P^{\prime} \tag{22}$$

where $\widetilde{k}_P$ is defined as

$$\widetilde{k}_P = \frac{\alpha_u \sum_{j|P} A^u_j}{\widetilde{A}^u_{P|P} - \alpha_u \sum_{j|P} A^u_j} = \frac{\alpha_u r_p}{1 - \alpha_u r_p + \delta_P} \quad ; \quad r_p = \frac{\sum_{j|P} A^u_j}{A^u_{P|P}}$$

r_p is equal to one over most part of the domain except at the boundaries due to Dirichlet boundary conditions[1]. Substituting Eqn. 22 and that for node E in Eqn. 17 gives

$$(1+\delta_e)u'_e = -\alpha_u \Delta t \left[\overline{\frac{\delta_i}{\rho_i} \widetilde{k}_i \left.\frac{\partial p}{\partial x}\right|_i^{\prime}}^{e} + \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{\prime} \right] \tag{23}$$

[1]If there is a source term dependent on the velocity that has been linearized, r_p could everywhere be different from one.

which is totally different from that at the nodes. If SIMPLEC idea had been carried over to the face expression the result would have been

$$(1+\delta_e)u_e' = -\alpha_u \Delta t \frac{\delta_e}{\rho_e}(1+\widetilde{k}_e)\left.\frac{\partial p}{\partial x}\right|_e' \tag{24}$$

The two expressions are identical only in the case

$$\overline{\frac{\delta_i}{\rho_i}\widetilde{k}_i \left.\frac{\partial p}{\partial x}\right|_i'}^e = \frac{\delta_e}{\rho_e}\widetilde{k}_e \left.\frac{\partial p}{\partial x}\right|_e' \tag{25}$$

a numerical condition that is rarely satisfied. Note that for a steady problem the expressions are

$$\begin{aligned} u_i' &= -\frac{\alpha_u}{1-\alpha_u}\frac{\Delta V_i}{A^u_{P|i}}\left.\frac{\partial p}{\partial x}\right|_i' \\ u_e' &= -\alpha_u\left(\frac{\Delta V_e}{A^u_{P|e}}\left.\frac{\partial p}{\partial x}\right|_e' + \frac{\alpha_u}{1-\alpha_u}\overline{\left(\frac{\Delta V_i}{A^u_{P|i}}\left.\frac{\partial p}{\partial x}\right|_i'\right)}^e\right) \end{aligned} \tag{26}$$

and the above requirement results in

$$\overline{\frac{\Delta V_i}{A^u_{P|i}}\left.\frac{\partial p}{\partial x}\right|_i'}^e = \frac{\Delta V_e}{A^u_{P|e}}\left.\frac{\partial p}{\partial x}\right|_e' \tag{27}$$

Shen et al. [5] also realized the condition above but they neglected the variation of $\Delta V/A^u_P$ transforming Eqn. 27 in a linearity requirement for the gradient of the pressure correction, that is, they claimed that Eqn. 24 could only be used if the gradient of the pressure correction was linear. Shen et al were interested in devising a scheme that gives steady state solutions free of time step dependence. To do so they had to introduce another relaxation factor that eventually produced an expression with no limitations whereas we derive the face velocity expression based on the nodal correction. For an unsteady incompressible simulation with small time steps such that $\delta_{i,e} >> 1$ the condition given in Eqn. 25 amounts to requiring the pressure gradient to be linear because in that case $\widetilde{k}_{i,e} = \alpha_u/\delta_{i,e}$. A very special case where the previous equality is satisfied for any time step is that of a fully developed flow where $\widetilde{k}_i = \widetilde{k}_e$, $\delta_i = \delta_e$, and the pressure gradient is constant. Apart from the formal limitations there is also a strong case against using Eqn. 24: the corrected velocity would not satisfy the face velocity expression with the updated pressure field. If that approach was followed every inner iteration would start with a newly corrected face velocity incompatible with its equation, thus slowing down the convergence process.

Apart from Shen et al's formulation SIMPLEC inconsistent in collocated grids was used by Johansson et al. [6] and Oliveira et al [7] in steady problems. In the first paper an ad-hoc amount of dissipation was introduced via a factor between 0.5 and 1 that substitutes the underrelaxation factor in the diffusion coefficient $\alpha_u \Delta V_e / A^u_{P|e}$ of the pressure correction equation after considering the difference between $\Delta V_e / A^u_{P|e}$ and $\overline{(\Delta V_i / A^u_{P|i})}^e$ to be negligible. This latter assumption allows to express the pressure contribution to the face velocity as a dissipative third order derivative which converts into a fourth order derivative when assembling the mass flow balance of the control volume. Their formulation is slightly different from that presented here due to the ad-hoc coefficient but it is still inconsistent, although we must stress again that the inconsistency only affects the number of iterations and not the final solution. A similar coefficient was also introduced by Rahman et al. [8] under a SIMPLE strategy.

The implementation of SIMPLEC consistent via Eqn. 23 and other similar for velocity corrections produces a pentadiagonal matrix (in 1D) that is diagonally dominant with all off-diagonal terms negative. The resulting matrix is solved with the Penta Diagonal Matrix Algorithm (PDMA), an extension of the well known TDMA.

3 RESULTS

The comparison of the different schemes, consistent and inconsistent, is carried out in two 2D laminar flows, both in a lid driven square cavity. The assessment is by no means exhaustive, our intention is just to show the improvement brought about by the consistent implementation of SIMPLEC. The Reynolds numbers of the two computational experiments are 10^3 and $5.\,10^3$ based on lid velocity. The convergence monitor is a coefficient defined as the ratio of p-norms of the momentum residuals and the left hand side of the discretized momentum equation, the latter considered as a normalizing factor.

$$res_u = \frac{\left(\sum_i \left| A^u_{P|i} u_i - \sum_{j|i} A^u_j u_j - S^u_i \Delta V_i + \Delta V_i \left.\frac{\partial p}{\partial x}\right|_i \right|^p \right)^{1/p}}{\left(\sum_i \left| A^u_{P|i} u_i \right|^p\right)^{1/p}} \quad ; \quad p = 1, 2, \ldots, \infty \qquad (28)$$

Likewise, a residual for the υ-velocity can be defined, res_υ. The mass imbalance is calculated as

$$res_m = \frac{\left(\sum_i |(\rho_e u^*_e - \rho_w u^*_w)\Delta y + (\rho_n \upsilon^*_n - \rho_s \upsilon^*_s)\Delta x|^p\right)^{1/p}}{\left(\sum_i inflow^p_i\right)^{1/p}} \quad ; \quad p = 1, 2, \ldots, \infty \qquad (29)$$

where inflow is the mass flow coming into a cell. The monitoring value for the velocities is $res = \max(res_u, res_\upsilon)$ and the calculation stops when $res < 10^{-8}$ and $res_m < 10^{-6}$. The initial condition is 10^{-6} for velocities and pressure, the lid velocity being 1. All cases have been calculated with the residual based on the L_1 norms ($p = 1$) with a grid of 100x100. For $Re = 1000$ the grid is uniform and for $Re = 5000$ it is expanding/contracting in both directions with ratios 1.1 and 1/1.1 respectively. A high-order discretization scheme, NOTABLE [9], with a deferred correction technique has been employed for the convective terms.

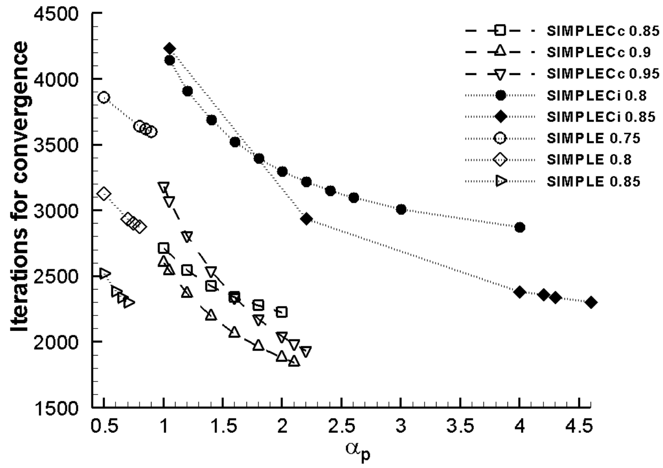

Figure 1: Comparison of the performance of different schemes in the lid-driven cavity case, Re=1000. The numbers in the names refer to the underrelaxation factor for the velocity.

In Figure 1 a comparison of the convergence rate of the inconsistent and consistent SIMPLEC approaches is presented for a steady case, computed by setting $\Delta t = 10^{25}$. Each curve corresponds to a constant value of α_u and it finishes at the last α_p before blowup. For the sake of comparison SIMPLE results are also presented. As can be seen SIMPLE requires α_p to be less than one whereas both consistent and inconsistent SIMPLEC can converge with a much greater factor (up to 4.6 for SIMPLECi (inconsistent)). SIMPLECc (consistent) needs less iterations for convergence than the other two. For a given pair of α's SIMPLECc is much quicker than SIMPLECi, the best case of SIMPLECc being a 25% better than the best case of SIMPLECi in terms of required iterations.

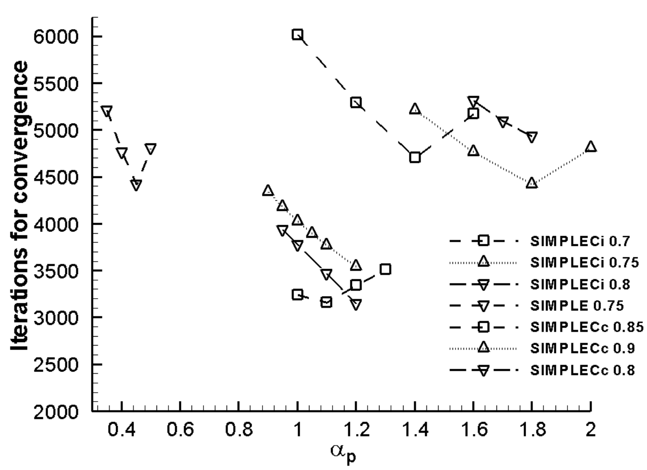

Figure 2: Comparison of the performance of different schemes in the lid-driven cavity case, Re=5000.

Figure 2 shows results for the second case with SIMPLEC and SIMPLE. For $Re = 5000$ the convective term is dominant over a larger part of the domain and the pressure-velocity coupling is more difficult to handle. This leads to an increase in the number of iterations required for convergence with respect to the $Re = 1000$ case. The improvement introduced by SIMPLECc over SIMPLECi is now more noticeable with a reduction of more than 30% in the best case, around 3000 iterations as against 4500. Both SIMPLECi and SIMPLECc are more robust than SIMPLE as the latter requires a fair amount of good luck to hit the optimum underrelaxation factor, being too much penalized if it is missed.

4 CONCLUSIONS

In this paper a SIMPLEC scheme for a collocated grid and consistent with the expression of the face velocity has been proposed. The correct derivation produces a pentadiagonal matrix in each coordinate that is subsequently solved by a PDM algorithm that is slightly more costly in terms of CPU time and memory requirements than the classical TDM algorithm. The assessment is by no means exhaustive but in a flow where the convective term is of the same order as the pressure term in most part of the domain the performance is greatly enhanced.

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