TIME INTEGRATION SCHEMES FOR INCOMPRESSIBLE TWO-PHASE FLOW PROBLEMS

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ABSTRACT

We consider a standard model for incompressible two-phase flows in which a localized force at the interface describes the effect of surface tension. This surface tension induces discontinuities in the pressure accross the interface. The interface is captured by a level set method. We use a spatial discretization based on multilevel tetrahedral grids and finite elements (Taylor-Hood $\mathcal{P}_2$-$\mathcal{P}_1$) (see [1] for details). If the interface is not aligned to the grid, the discontinuity of the pressure leads to a large pressure discretization error ($\mathcal{O}(\sqrt{h})$ in the L^2-norm), which induces significant spurious velocities. It is shown in [2] that one can get an optimal approximation error bound ($\mathcal{O}(h^2)$ in the L^2-norm) for the pressure, if the pressure ansatz/test spaces ($\mathcal{P}_1$) are enriched with additional basis functions, so called *extended finite elements* (XFEM). Such XFEM techniques have been successfully used in other applications, e.g. fracture mechanics.

Such an XFEM space discretization causes difficulties for the time discretization. The ansatz/test space for the pressure functions is now **time dependent**, since in general they change in every time step. The finite element space for velocity, however, does **not** depend on time. We discuss two techniques for constructing a time discretization scheme. Firstly, a method-of-lines approach is applied: The spatial and time discretization are done consecutively; after the $\mathcal{P}_2$-XFEM spatial discretization a standard time integration method (e.g. θ-scheme) is applied. The second approach is a space-time finite element discretization, in which we use piecewise linear polynomials in time and piecewise quadratic polynomials in space for the velocity. For the pressure, we use piecewise constant polynomials in time and XFEM space functions. The quadrature with respect to time is done by simple quadrature rules, e.g. trapezoidal/rectangle rule. Due to the possible discontinuity of the XFEM basis functions, special quadrature rules in space are used.

We present these methods and show results of a systematic comparative study for a class of incompressible two-phase flow problems.

REFERENCES

[1] "DROPS-Homepage", *http://www.igpm.rwth-aachen.de/DROPS*

[2] Sven Gross and Arnold Reusken, An extended pressure finite element space for two-phase incompressible flows with surface tension, J. Comp. Phys., Vol. **45**, (2007)