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OPTIMAL LOCATION OF SUCTION OR BLOWING JETS USING THE CONTINUOUS ADJOINT APPROACH

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Abstract. *This paper presents the use of the continuous adjoint method as a low-cost tool to derive useful information regarding the optimal location and type of steady suction/blowing jets, used to control flow separation. An objective function that expresses the total pressure losses between the inlet and outlet of the flow domain is devised. The derivatives of this objective function with respect to hypothetical jet velocities at the wall boundaries are then computed using the continuous adjoint method. Emphasis is laid on the computation of the exact sensitivity derivatives and, for this reason, the adjoint to the turbulence (Spalart-Allmaras) model is also used, as proposed in a recent publication by the same authors. The proposed method is demonstrated by controlling the separated flow in a S-shaped duct.*

1 INTRODUCTION

In several applications, given a function F measuring the performance of an aerodynamic body (the shape of which is described using N parameters, $b_n,\ n = 1, \ldots, N$), the sensitivity derivatives of F with respect to b_n ($\frac{\delta F}{\delta b_n},\ n = 1, \ldots, N$) must be computed. Computing $\frac{\delta F}{\delta b_n}$ is a prerequisite for the use of any gradient–based optimization method or for plotting sensitivity maps to reveal the areas mainly contributing to performance loss.

Thus far, adjoint methods have widely been used to compute the gradient of F. They are inspired by the control theory and their major advantage is that the CPU cost of computing the gradient is approximately equal to the cost of numerically solving the flow equations, in contrast to other more expensive, rival methods such as finite differences, the complex variable method, etc. The adjoint approach to aerodynamic design based on the potential flow model was proposed by Pironneau, [1]. Later on, Jameson [2, 3] extended it scid and viscous transonic flows. Adjoint methods appear in both continuous and discrete form. In the continuous approach, [1, 2, 4, 5], the adjoint pde's and their boundary conditions are derived by developing the variation in F augmented by the state (i.e. flow) equations weighted by the adjoint variables. The so–derived continuous adjoint equations are, then, discretized (similarly to the flow equations) and numerically solved. Discrete adjoints are not considered in this paper; a comparison between continuous and discrete adjoint can be found in [6]. In case of turbulent flows, to compute the exact sensitivity derivatives, the linearization (in discrete adjoint, [7, 8, 9, 10, 11, 12, 13]), or differentiation (in continuous adjoint, [14]), of the turbulence variable equation(s) must be taken into account. The present paper is concerned with the continuous adjoint method adapted to turbulent flows in the presence of flow control mechanisms and is based on the mathematical formulation presented in [14]. Developed by the same authors, [14] proposed the adjoint to the Spalart-Allmaras turbulence model and demonstrated the need for solving this extra equation if accurate derivatives are to be computed.

The role of flow control, based on suction/blowing/synthetic jets, is to prevent or delay separation, control transition to turbulent flow, suppress or enhance turbulence, control shock waves and the associated boundary layer development, etc. These are all related to the drag (in external aerodynamics) or losses (internal aerodynamics). Passive flow control, based on surface modifications, is beyond the scope of this paper and will not be discussed further. The present paper deals exclusively with active flow control, namely steady suction and blowing.

Early enough, Prandtl, [15], proposed the use of suction through a slot to control the boundary layer separation. Since then, the progress was important, [16]. Suction/blowing can be applied by means of porous media, multiple small surface slots and perforations, [17]. Flow control applications in ducted flows, similar to those presented herein, can be found in [18, 19]. The Reynolds–Averaged Navier–Stokes (RANS) are frequently used to simulate active flow control systems, [20, 21]. In [22], numerical results from RANS solvers predicted the effect of steady and unsteady flow control over a bump; the (steady suction) control device were modeled by a boundary condition imposed directly on the body surface, making thus easier the mesh generation. In [23], a computational study was conducted on two S–diffusers with blowing; [24] applied RANS models to simulate the effect of periodic suction/blowing on the flow past the NACA0012 airfoil.

Optimization methods have also been employed to flow control configurations. A discrete adjoint technique based on the unsteady Navier–Stokes equations for the optimal control of vortex shedding behind a cylinder, by means of suction and/or blowing, can be found in [25]. In [26], a continuous adjoint method using backward-in-time integration of the unsteady adjoint equations was used to find the type of control (blowing or suction at the wall) that prevents the development of streamwise vortices causing transition to turbulence. [26] is dealing with the flow control past a flat plate and a curved surface, by minimizing the mean streamwise energy of the perturbation and the downstream energy of the longitudinal velocity perturbation.

In the present paper, CFD techniques for numerically solving the flow and adjoint equations are employed and, based on the so-computed fields, the sensitivity derivatives of the total pressure losses functional are computed. Sensitivity derivatives with respect to the flow control parameters, i.e. the suction/blowing velocities, instead of the fixed geometrical quantities parameterizing the shape, are computed. These guide the engineer to choose the optimal position and type (suction or blowing) of control. Since the flow separation reflects on the total pressure losses between the inlet and outlet, high absolute sensitivity values indicate the recommended positions of flow control whereas their signs determine whether suction or blowing must be used.

2 OBJECTIVE FUNCTION AND DESIGN VARIABLES

In a flow control simulation, the boundary velocity components at the "wall" nodes where the jet applies must be set equal to the jet velocities. In this paper, the latter are assumed not to vary with time, since only steady suction or blowing is considered. Let Q be the number of grid nodes along the wall boundary (marked with superscript b) and $q = 1, ..., Q$ the index corresponding to them. Then, v^b_{pq} denotes the jet velocity component along the p direction, at node q; $p=1,2$. For any node p, $v^b_{pq}=0$ corresponds to the standard no-slip condition and indicates the absence of either suction or blowing at this node. In contrast, non-zero v^b_{pq} values correspond to suction or blowing. The two options can be distinguished by the sign of $v^b_{pq}n^b_p$, where n^b_p are the components of the outward unit vector normal to the boundary.

This paper presents an adjoint–based method to provide useful recommendations on the optimal location and type (suction or blowing) of flow control jets. For this purpose, the sensitivity derivatives of the integral of viscous losses with respect to $v^b_{pq}, q = 1, ..., Q$ are computed. The difference in total pressure between the inlet (S_I) to and the outlet

(S_O) from the flow domain is defined by

$$F = -\int_{S_I}\left(p+\frac{1}{2}v_k^2\right)v_i n_i dS - \int_{S_O}\left(p+\frac{1}{2}v_k^2\right)v_i n_i dS \tag{1}$$

In eq. 1, v_k, p stand for the velocity components and the static pressure and n_i are the components of the unit, normal to the boundary vector.

3 FLOW MODEL

The incompressible fluid flow is modeled through the RANS equations and the Spalart–Allmaras one–equation turbulence model [27]. The mean–flow equations are written as

$$R_{U,i} = v_j\frac{\partial v_i}{\partial x_j} + \frac{\partial p}{\partial x_i} - \frac{\partial}{\partial x_j}\left[(\nu+\nu_t)\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)\right] = 0\ , \qquad i=1,2 \tag{2a}$$

$$R_{U,3} = \frac{\partial v_j}{\partial x_j} = 0 \tag{2b}$$

where ν and ν_t are the bulk and turbulent viscosity coefficients. The turbulence model equation is the following

$$R_{\widetilde{\nu}} = \frac{\partial(v_j\widetilde{\nu})}{\partial x_j} - \frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\widetilde{\nu}}{\sigma}\right)\frac{\partial\widetilde{\nu}}{\partial x_j}\right] - \frac{c_{b2}}{\sigma}\left(\frac{\partial\widetilde{\nu}}{\partial x_j}\right)^2 - \widetilde{\nu}\,P\left(\widetilde{\nu}\right) + \widetilde{\nu}\,D\left(\widetilde{\nu}\right) = 0 \tag{3}$$

and should be solved for $\widetilde{\nu}$ which, along with v_i and p, constitute the state variables. The required eddy viscosity coefficient ν_t is derived from $\nu_t = \widetilde{\nu} f_{v_1}$. The production and destruction terms in eq. 3 are given by $P(\widetilde{\nu}) = c_{b1}\widetilde{S}$ and $D(\widetilde{\nu}) = c_{w1} f_w(\widetilde{S})\frac{\widetilde{\nu}}{d^2}$, where $\widetilde{S} = S + \frac{\widetilde{\nu}}{d^2\kappa^2} f_{v_2}$, $S = |e_{ijk}\frac{\partial v_k}{\partial x_j}\delta i1|$ is the vorticity magnitude, e_{ijk} is the permutation symbol, d is the distance from the wall, $f_{v_1} = \frac{\chi^3}{\chi^3+c_{v_1}^3}$, $f_{v_2} = 1-\frac{\chi}{1+\chi f_{v_1}}$, $\chi = \frac{\widetilde{\nu}}{\nu}$, $f_w = g\left(\frac{1+c_{w_3}^6}{g^+c_{w_3}^6}\right)^{1/6}$, $g = r + c_{w_2}(r^6 - r)$ and $r = \frac{\widetilde{\nu}}{\widetilde{S}\kappa^2 d^2}$. The model constants are: $c_{b1} = 0.1355$, $c_{b2} = 0.622$, $\kappa = 0.4187$, $\sigma = \frac{2}{3}$, $c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{(1+c_{b2})}{\sigma}$, $c_{w2} = 0.3$, $c_{w3} = 2$, $c_{v1} = 7.1$ and $c_{v2} = 5$, [27].

The proposed method was developed for structured grids. The mean–flow equations are solved in a segregated manner using the SIMPLE algorithm [28], with collocated, cell–centered storage of flow variables. Odd–even decoupling is avoided by standard numerical dissipation schemes associated with the computation of pressure gradients. Second–order upwind schemes are used for the convection terms. The system of the discretized equations is solved using the biconjugate gradient stabilized, CGSTAB, algorithm, [29].

At the inlet, the v_i and $\widetilde{\nu}$ distributions are specified and zero Neumann conditions are imposed to p. The exit static pressure is arbitrarily set to zero and zero Neumann conditions are imposed to all remaining flow variables at the exit.

4 THE ADJOINT APPROACH TO THE FLOW CONTROL PROBLEM

The sensitivity derivatives $\frac{\delta F}{\delta v^b_{pq}}$ are deduced from the differentiation of eq. 1 with respect to v^b_{pq}, as follows

$$\begin{aligned}\frac{\delta F}{\delta v^b_{pq}} &= = -\int_{S_I}\left(\frac{\delta p}{\delta v^b_{pq}} + v_k\frac{\delta v_k}{\delta v^b_{pq}}\right) v_i n_i dS - \int_{S_I}\left(p+\frac{1}{2}v_k^2\right)\frac{\delta v_i}{\delta v^b_{pq}} n_i dS \\ &- \int_{S_O}\left(\frac{\delta p}{\delta v^b_{pq}} + v_k\frac{\delta v_k}{\delta v^b_{pq}}\right) v_i n_i dS - \int_{S_O}\left(p+\frac{1}{2}v_k^2\right)\frac{\delta v_i}{\delta v^b_{pq}} n_i dS \end{aligned} \tag{4}$$

Note that the sensitivities of all geometrical quantities with respect to v^b_{pq} are zero. Since $\frac{\delta F}{\delta v^b_{pq}}$ depend on variations $\frac{\delta()}{\delta v^b_{pq}}$ in the state variables, the state equations (eqs. 2a to 3) must be differentiated with respect to v^b_{pq}. Based on these differentiations, $\frac{\delta v_i}{\delta v^b_{pq}}$, $\frac{\delta p}{\delta v^b_{pq}}$ and $\frac{\delta \nu_t}{\delta v^b_{pq}}$ must satisfy the following equations:

$$\begin{aligned}\frac{\delta R_{U,i}}{\delta v^b_{pq}} =& \frac{\delta v_j}{\partial v^b_{pq}}\frac{\partial v_i}{\partial x_j} + v_j\frac{\partial}{\partial x_j}\left(\frac{\delta v_i}{\delta v^b_{pq}}\right) + \frac{\partial}{\partial x_i}\left(\frac{\delta p}{\delta v^b_{pq}}\right) - \frac{\partial}{\partial x_j}\left\{(\nu+\nu_t)\left[\frac{\partial}{\partial x_j}\left(\frac{\delta v_i}{\delta v^b_{pq}}\right) + \frac{\partial}{\partial x_i}\left(\frac{\delta v_j}{\delta v^b_{pq}}\right)\right]\right\} \\ &-\frac{\partial}{\partial x_j}\left[\frac{\delta \nu_t}{\delta v^b_{pq}}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] = 0\ , \qquad i=1,2\end{aligned} \tag{5}$$

$$\frac{\delta R_{U,3}}{\delta v^b_{pq}} = \frac{\partial}{\partial x_j}\left(\frac{\delta v_j}{\delta v^b_{pq}}\right) = 0 \tag{6}$$

$$\begin{aligned}\frac{\delta R_{\widetilde{\nu}}}{\delta v^b_{pq}} =& \frac{\partial}{\partial x_j}\left(\frac{\delta v_j}{\delta v^b_{pq}}\widetilde{\nu}\right) + \frac{\partial}{\partial x_j}\left(v_j\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}}\right) - \frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\widetilde{\nu}}{\sigma}\right)\frac{\partial}{\partial x_j}\left(\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}}\right)\right] - \frac{1}{\sigma}\frac{\partial}{\partial x_j}\left(\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}}\frac{\partial \widetilde{\nu}}{\partial x_j}\right) \\ &-2\frac{c_{b2}}{\sigma}\frac{\partial \widetilde{\nu}}{\partial x_j}\frac{\partial}{\partial x_j}\left(\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}}\right) + \widetilde{\nu}\left(-\frac{\delta P}{\delta v^b_{pq}} + \frac{\delta D}{\delta v^b_{pq}}\right) + (-P+D)\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}} = 0\end{aligned} \tag{7}$$

where, for the bulk viscosity, the assumption $\frac{\delta \nu}{\delta v^b_{pq}} = 0$ was made. Eqs. 5, 6 and 7 are often referred to as the direct differentiation of eqs. 2a, 2b and 3 with respect to v^b_{pq}. To complete the derived system of pde's,

$$\frac{\delta \nu_t}{\delta v^b_{pq}} = \frac{\delta \nu_t}{\delta \widetilde{\nu}}\frac{\delta \widetilde{\nu}}{\delta v^b_{pq}} \tag{8}$$

must also be used.

The first step of the adjoint approach is to define the augmented functional by summing up F and the product of the state equations and the Lagrange multipliers or adjoint variables u_i, q and $\widetilde{\nu}_a$ (adjoint velocity components, pressure and turbulent viscosity, respectively) integrated over the flow domain Ω. The sensitivities of the augmented functional are, then, given by

$$\frac{\delta L}{\delta b_n}=\frac{\delta J}{\delta b_n}+\int_{\Omega} u_i\,\frac{\partial R_{U,i}}{\partial b_n}\,d\Omega+\int_{\Omega} q\,\frac{\partial R_{U,3}}{\partial b_n}\,d\Omega+\int_{\Omega}\widetilde{\nu}_a\,\frac{\partial R_{\widetilde{\nu}}}{\partial b_n}\,d\Omega \tag{9}$$

The Gauss divergence theorem is used and the adjoint variables are computed by eliminating all field and boundary integrals of $\frac{\delta v_i}{\delta v^b_{pq}}$, $\frac{\delta p}{\delta v^b_{pq}}$ and $\frac{\delta \widetilde{\nu_a}}{\delta v^b_{pq}}$. The remaining terms, which depend on the computed u_i, q and $\widetilde{\nu}_{aa}$ fields, stand for the derivatives of F with respect to v^b_{pq}, [14]. The adjoint to the mean flow equations are given by

$$-v_j\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{\partial}{\partial x_j}\left[(\nu+\nu_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right]+\frac{\partial q}{\partial x_i}-\widetilde{\nu}\frac{\partial \widetilde{\nu_a}}{\partial x_i}-\frac{\partial}{\partial x_l}\left(e_{jli}e_{jmq}\frac{\mathcal{C}_S}{S}\frac{\partial v_q}{\partial x_m}\widetilde{\nu}\widetilde{\nu_a}\right)=0 \qquad i=1,2 \tag{10a}$$

$$\frac{\partial u_j}{\partial x_j}=0 \tag{10b}$$

The adjoint to the Spalart-Allmaras equation, [14], yields

$$\frac{\partial \widetilde{\nu_a}}{\partial x_j}v_j+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\widetilde{\nu}}{\sigma}\right)\frac{\partial \widetilde{\nu_a}}{\partial x_j}\right]=\frac{1}{\sigma}\frac{\partial \widetilde{\nu_a}}{\partial x_j}\frac{\partial \widetilde{\nu}}{\partial x_j}+2\frac{c_{b2}}{\sigma}\frac{\partial}{\partial x_j}\left(\widetilde{\nu_a}\frac{\partial \widetilde{\nu}}{\partial x_j}\right)+\widetilde{\nu_a}\widetilde{\nu}\,\mathcal{C}_{\widetilde{\nu}}(\widetilde{\nu},\vec{v}) +\frac{\delta \nu_t}{\delta \widetilde{\nu}}\frac{\partial u_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+(-P+D)\,\widetilde{\nu_a} \tag{11}$$

The adjoint state equations (eqs. 10a, 10b and 11) are discretized and solved similarly to the state equations.

The adjoint boundary conditions are consistent with the state ones. At the inlet, the adjoint velocity is fully determined by the conditions $u_in_i=v_in_i$ and $u_it_i=0$ (where t_i are the components of the unit, tangent to the boundary vector). Zero Neumann condition is imposed to q and zero Dirichlet condition to $\widetilde{\nu_a}$. At the outlet grid nodes, the system of the following two equations, [14],

$$q=u_jv_j+u_in_iv_jn_j+(\nu+\nu_t)\frac{\partial u_i}{\partial x_j}n_in_j+\widetilde{\nu_a}\widetilde{\nu}+\widetilde{\nu_a}\widetilde{\nu}\frac{\mathcal{C}_S}{S}e_{jmq}e_{jli}\frac{\partial v_q}{\partial x_m}n_ln_i-\frac{1}{2}v_i^2-v_i^2n_j^2 \tag{12}$$

$$0=u_it_iv_jn_j+(\nu+\nu_t)\frac{\partial u_i}{\partial x_j}t_in_j+\widetilde{\nu_a}\widetilde{\nu}\frac{\mathcal{C}_S}{S}e_{jmq}e_{jli}\frac{\partial v_q}{\partial x_m}n_lt_i \tag{13}$$

must be solved for the three adjoint variables, by setting one of them equal to an arbitrary value and solving eqs. 12 and 13 for the other. At the "wall", zero Dirichlet conditions are imposed to u_i and $\widetilde{\nu_a}$ and zero Neumann to q.

After having computed the adjoint fields that satisfy the adjoint equations, the sensitivity derivatives of the total pressure losses functional with respect to the jet velocity components are given by the expression

$$\frac{\delta F}{\delta v_{pq}^b} = \nu \left(\frac{\partial u_{pq}}{\partial x_j} + \frac{\partial u_{jq}}{\partial x_p} \right) n_{jq} - q_q n_{pq} \tag{14}$$

5 APPLICATION - DISCUSSION

In this section, the proposed adjoint method for the computation of the sensitivities of the objective function F with respect to the jet velocity components is applied to the search for the optimal location and type (suction/blowing) of the jet. The problem under consideration is that of the flow through an S-shaped duct. The total pressure losses sensitivities with respect to the normal to the wall velocities at the wall boundary nodes are computed by solving the state and adjoint equations. Without loss in generality, it is assumed that the jet velocities are applied normal to the wall. Thus, the signed jet velocities become

$$v_q^{jet} = v_{pq}^b n_p^b \tag{15}$$

The effect of inclined (with respect to the normal direction) jet velocities to F could also be investigated by the present method, by computing the corresponding sensitivity derivatives with respect to $v_{pq}^b a_p^b$ where a_p^b are the components of any other unit vector aligned with the desired jet direction at the wall node p. In any case, the proposed method computes $\frac{\delta F}{\delta v_{pq}^b}$ and the sensitivity derivatives $\frac{\delta F}{\delta v_q^{jet}}$ result from (with either n_p^b or a_p^b)

$$\frac{\delta F}{\delta v_q^{jet}} = \frac{\delta F}{\delta v_{pq}^b} n_p^b \tag{16}$$

Locations where the absolute gradients take on the highest absolute values are the most promising for the placement of jets. The sign of the sensitivity derivatives at these points indicates the preferred direction of the jet, choosing between suction and blowing.

The duct geometry along with the velocity magnitude isolines computed by solving the state equations is illustrated in fig. 1. The turbulent flow equations are solved assuming that the “wall” boundaries act as perforated walls, allowing suction or blowing (even with zero velocity since the sensitivities of $\frac{\delta F}{\delta v_{pq}^b}$ for $v_{pq}^b = 0$ are sought). Therefore, the duct walls act, practically, as a standard solid walls. A 220×161 structured grid, sufficiently stretched along the normal to the boundary direction, was used.

Isolines of the computed adjoint velocity magnitude, adjoint pressure and the adjoint turbulence variable are shown in fig. 2. The distribution of the sensitivity derivatives $\frac{\delta F}{\delta v_{pq}^b}$ in terms of the wall grid nodes are illustrated in fig. 3. In fig. 4, the same distribution is plotted along the curved duct boundaries. Figs. 3 and 4 clearly suggest two locations for the placement of the suction jets, one on the lower and the other on the upper surface,

	uncontrolled	study (a)	study (b)	study (c)
F	0.01835	0.01662	0.01817	0.01649
F_T	-	0.01723	0.01809	0.01697

Table 1: Total pressure losses value, obtained by applying steady suction at the lower surface of the duct (a), at the top surface (b) and at both of them (c). The computed values (F) are compared to the values obtained through Taylor series (F_T), based on the sensitivities computed using the proposed adjoint formulation, ($F_T = F + \delta F = F + \frac{\delta F}{\delta v_i}\delta v_i, i = 1, 2$).

almost at the end of the S-type bend. The sensitivity derivatives can also be used to provide suggestions for the optimal width of the jet stream.

To demonstrate the significance of the computed sensitivities, the same Navier-Stokes solver was used to compute the flow with control. According to the previously acquired knowledge, the following cases were studied: (a) flow controlled by a suction jet imposed normal to the wall between $x = -0.026m$ and $x = -0.023m$ over the lower wall of the duct, (b) flow controlled by a suction jet imposed normal to the upper wall between $x = 0.240m$ and $x = 0.242m$ and (c) the combination of (a) and (b). These parts of the lower and upper boundaries become suction slots, where the no-slip condition is no more valid. To avoid numerical difficulties, a cubic polynomial suction velocity profile with maximum velocity equal to 10% of the inlet velocity was imposed. In fig.5, the isolines of the velocity magnitude for the controlled study (c) are shown. With flow control (steady suction), the objective function value in all three studies is reduced, the values are given in Table 1 (non-dimensional values). The most significant reduction of F is achieved using jets over both wall boundaries (i.e. study (c)). Regarding the significance of the two jet locations, the F reduction which is greater in study (a) rather than (b) proves that the lower wall jet plays the most important role. This is in accordance with the sensitivity derivatives values which take on higher values on the lower wall.

6 CONCLUSIONS

The continuous adjoint method was used to compute the sensitivity derivatives of the objective function expressing the total pressure losses of duct flows with respect to hypothetical signed velocities at the "wall" nodes. In a flow control problem, the optimization variables are the suction or blowing velocities which are expected to affect the extent of the flow separation zone and, thus, the overall losses. At the CPU cost of a state (flow) and a costate (adjoint) problem solution, a clear indication of the type (suction or blowing) and location of the control jet can be obtained. Though the present formulation computes sensitivity derivatives at zero jet velocities (uncontrolled case), which practically means that this method (in its present form) cannot also provide the optimal jet velocity, it is a very useful tool for the preliminary design of optimal flow control systems.

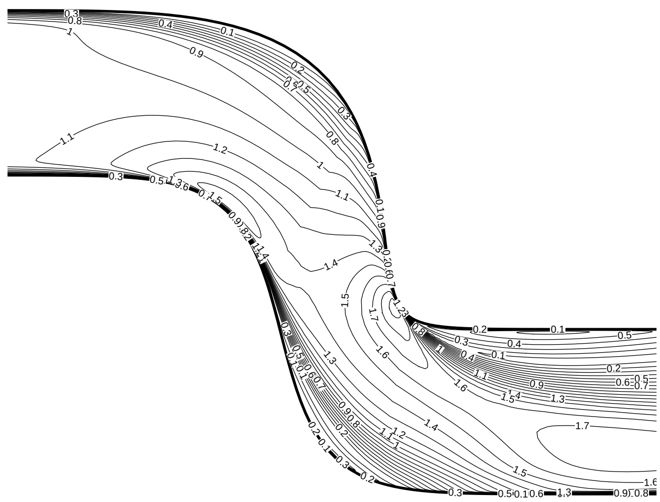

Figure 1: Computed velocity magnitude isolines.

REFERENCES

[1] O. Pironneau, On optimum design in fluid mechanics, *Journal of Fluid Mechanics*, **64**:97–110.

[2] A. Jameson, Aerodynamic design via control theory, *Journal of Scientific Computing*, **3**:233–260, 1988.

[3] A. Jameson, N. Pierce and L. Martinelli, Optimum aerodynamic design using the Navier Stokes equations, *Theoretical and Computational Fluid Dynamics*, **10**:213–237, 1998.

[4] W.K. Anderson and V. Venkatakrishnan, Aerodynamic design optimization on unstructured grids with a continuous adjoint formulation, *AIAA Paper*, 97-0643, 1997.

[5] D.I. Papadimitriou and K.C. Giannakoglou, A continuous adjoint method with objective function derivatives based on boundary integrals for inviscid and viscous flows, *Computers and Fluids*, **36**:325–341, 2007.

[6] S. Nadarajah and A. Jameson, A comparison of the continuous and discrete adjoint approach to automatic aerodynamic optimization, *AIAA Paper*, 2000–0667, 2000.

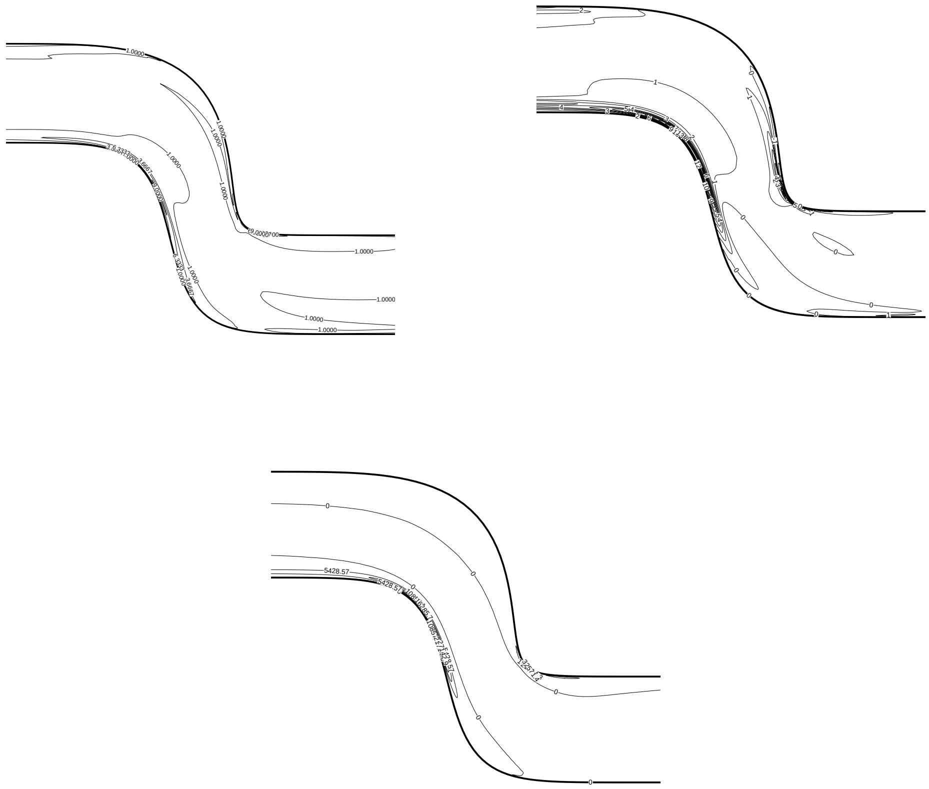

Figure 2: Adjoint velocity magnitude (top-left), adjoint pressure (top-right) and adjoint turbulence model variable $\widetilde{\nu_a}$ (bottom) isolines.

[7] E.J. Nielsen, J. Lu, M.A. Park and D.L. Darmofal, An implicit exact dual adjoint solution method for turbulent flows on unstructured grids, *Computers and Fluids*, **33**:1131-1155, 2004.

[8] D.W. Zingg, T.M. Leung, L. Diosady, A.H. Truong, S. Elias and M. Nemec, Improvements to a Newton-Krylov adjoint algorithm for aerodynamic optimization, *AIAA Paper*, 2005-4857, 2005.

[9] R.P. Dwight and J. Brezillon, Effect of approximations of the discrete adjoint on gradient–based optimization, *AIAA Journal*, **44(12)**:3022-3031, 2006.

[10] W.K. Anderson and D.L. Bonhaus, Airfoil design on unstructured grids for turbulent flows, *AIAA Journal*, **37(2)**:185-191, 1999.

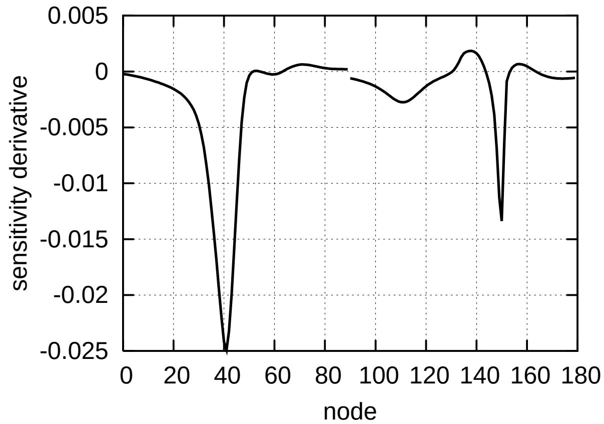

Figure 3: Distribution of nodal sensitivity derivatives $\frac{\delta F}{\delta v_q^{jet}}$ along the lower (first line) and upper (second line) wall.

[11] B.J. Lee and C. Kim, Automated design methodology of turbulent internal flow using discrete adjoint formulation, *Aerospace Science and Technology*, **11**:163-173, 2007.

[12] D.J. Mavriplis, Discrete adjoint–based approach for optimization problems on three–dimensional unstructured meshes, *AIAA Journal*, **45(4)**:740-750, 2007.

[13] C.S. Kim, C. Kim and O.H. Rho, Feasibility study of constant eddy–viscosity assumption in gradient–based design optimization, *Journal of Aircraft*, **40(6)**:1168-1176, 2003.

[14] A.S. Zymaris, D.I. Papadimitriou, K.C. Giannakoglou and C. Othmer, Continuous adjoint approach to the Spalart-Allmaras turbulence model for incompressible flows, *Computers and Fluids*, **38(8)**:1528–1538, 2009.

[15] L. Prandtl, Über Flüssigkeitsbewegung bei sehr kleiner Reibung, *Proceedings of 3rd International Mathematical Congress*, Heidelberg, p. 484–91, 1904.

[16] G.V. Lachmann, Boundary layer and flow control, its principles and application, *Pergamon Press*, NY, vol. 1&2, 1961.

[17] A.L. Braslow, A history of suction-type laminar flow control with emphasis on flight research, *NASA Monographs*, vol. 13, 1999.

[18] V. Kumar and F.S. Alvi, Use of high-speed microjets for active separation control in diffusers, *AIAA Journal*, **44(2)**:273–281, 2006.

[19] B.S. Stratford, The prediction of separation of turbulent boundary layer, *Journal of Fluid Mechanics*, **5**:1–16, 1959.

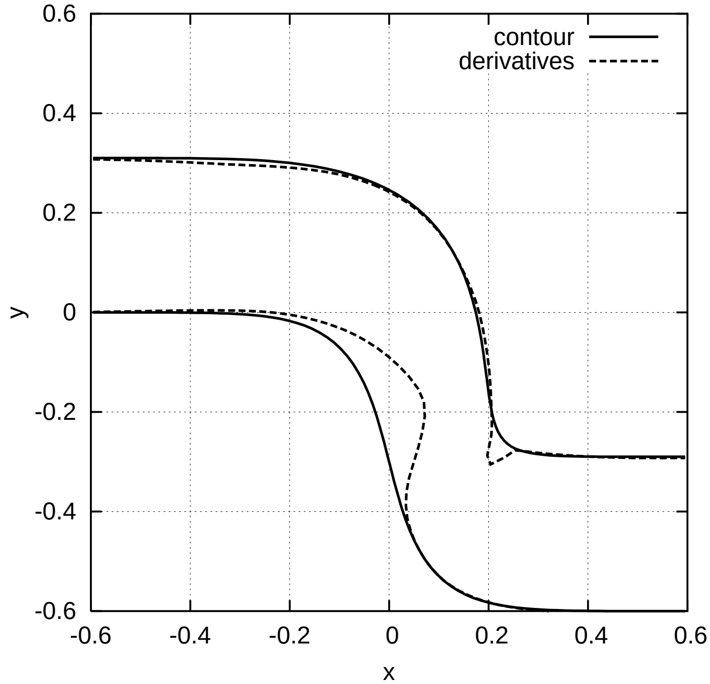

Figure 4: Contour and scaled $\frac{\delta F}{\delta v_q^{jet}}$ distributions along the walls. Derivatives (dashed line) pointing towards the interior of the flow domain correspond to suction.

[20] L.D. Kral, J.F. Donovan, A.B. Cain and A.W. Cary, Numerical simulation of synthetic jet actuators, *AIAA Paper*, 97–1824, 1997.

[21] V.N. Vatsa and E. Turkel, Simulation of synthetic jets using unsteady Reynolds-Averaged Navier Stokes equations, *AIAA Journal*, **44(2)**:217–224, 2006.

[22] G. Iaccarino, C. Marongiu, P. Catalano and Marcello Amato, RANS simulation of the separated flow over a bump with active control, *Center for Turbulence Research Annual Research Briefs*, 2003.

[23] Harishankar and A.M. Pradeep, Numerical simulation of flow separation and its control in S–duct diffusers, *ISABE*, 2009–1350, 2009.

[24] L. Huang, P.G. Huang, R.P. LeBeau and T. Hauser, Numerical study of blowing and suction control mechanism on NACA0012 airfoil, *Journal of Aircraft*, **41(5)**, September–October 2004.

[25] Z. Li, I.M. Navon, M.Y. Hussaini and F.X. Le Dimet, Optimal control of cylinder wakes via suction and blowing, *Computers & Fluids*, **32**:149–171, 2003.

[26] P. Cathalifaud and P. Luchini, Algebraic growth in boundary layers: optimal control by blowing and suction at the wall, *Eur. J. Mech. B–Fluids*, **19**:469–490, 2000.

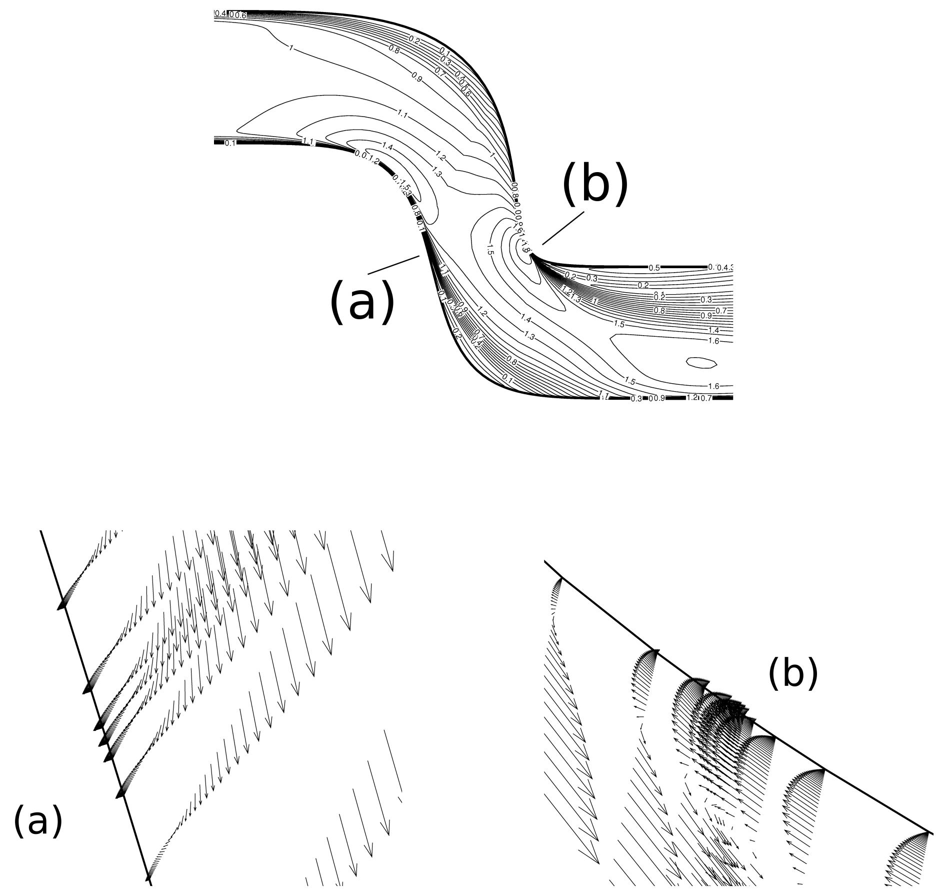

Figure 5: Computed velocity magnitude isolines for the controlled case with one suction jet per surface. Bottom: blow-up views of the flow in the vicinity of the two jets.

[27] P. Spalart and S. Allmaras, A one-equation turbulence model for aerodynamic flows, *AIAA Paper* 92-0439, 1992.

[28] L.S. Caretto, A.D. Gisman, S.V. Patankar and D.B. Spalding, Two calculation procedures for steady, three–dimensional flows with recirculation, *Proc. 3rd Int. Conf. Numerical Methods in Fluid Dynamics*, Paris, 1972.

[29] Y. Saad, Iterative methods for sparse linear systems, *Electronic Edition (copyright by Y. Saad)*, 2000.