EXPLICIT RUNGE KUTTA RESIDUAL DISTRIBUTION FOR SHALLOW WATER FLOWS

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ABSTRACT

In this work we apply the explicit Runge-Kutta Residual Distribution (RKRD) schemes of [1] to the solution of the shallow water equations on unstructured triangulations. In the simplest setting of second order RK time integration, high order RKRD can be recast as the following two step update :

$$|S_i|\frac{\mathbf{u}_i^1-\mathbf{u}_i^n}{\Delta t} = -\sum_{T|i\in T}\beta_i^T\phi^T(\mathbf{u}_h^n)$$
$$|S_i|\frac{\mathbf{u}_i^{n+1}-\mathbf{u}_i^1}{\Delta t} = -\sum_{T|i\in T}\beta_i^T\Phi^T(\mathbf{u}_h^1,\mathbf{u}_h^n)$$

having denoted by T and i a generic triangle and node of the mesh, and by $\mathbf{u}_h$ a continuous polynomial approximation of the unknown $\mathbf{u}$ built starting from the knowledge its nodal values. For the shallow water equations, the *element fluctuation* $\phi^T(\mathbf{u}_h^n)$ and *element residual* $\Phi^T(\mathbf{u}_h^1,\mathbf{u}_h^n)$ are obtained as

$$\phi^T(\mathbf{u}_h^n) = \oint\limits_{\partial T}\boldsymbol{\mathcal{F}}_h(\mathbf{u}_h^n)\cdot\hat{n}\,dl + \int\limits_T\boldsymbol{\mathcal{S}}_h(\mathbf{u}_h^n,x,y)\,dx\,dy$$
$$\Phi^T(\mathbf{u}_h^1,\mathbf{u}_h^n) = \int\limits_T\frac{\mathbf{u}_h^1-\mathbf{u}_h^n}{\Delta t}\,dx\,dy + \frac{1}{2}\phi^T(\mathbf{u}_h^n)+\frac{1}{2}\phi^T(\mathbf{u}_h^1)$$

with $\boldsymbol{\mathcal{F}}$ the (physical) conservative flux and $\boldsymbol{\mathcal{S}}$ the topography source term. We will elaborate on the conditions under which the scheme above preserves, on unstructured grids, known steady states such as the lake at rest, and also moving steady states with constant total energy. We will prove that these conditions are *only* related to the choice of the dicrete polynomial approximation $\{\mathbf{u}_i\}_{\forall\,i}\mapsto\mathbf{u}_h$, and on the quadrature used to evaluate the fluctuation. Once these two are properly chosen, *any* RKRD scheme will preserve exactly the afore-mentioned steady states. Numerical experiments will be shown to confirm the theory.

REFERENCES

[1] M. Ricchiuto and R. Abgrall, Explicit Runge-Kutta Residual-Distribution schemes for time dependent problems, *INRIA Report* RR-6998 (2009), accepted on *J.Comput.Phys.*

[2] M. Ricchiuto, R. Abgrall and H. Deconinck, Application of conservative residual distribution schemes to the solution of the shallow water equations on unstructured meshes, *J.Comput.Phys.* **222**, pp. 287-331 (2007)