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NUMERICAL SOLUTION OF 2D AND 3D STRATIFIED FLOWS IN ATMOSPHERIC BOUNDARY LAYER

Jiří Šimonek[*] [†], Karel Kozel[*] [†] and Zbyněk Jaňour[†]

[*]Faculty of Mechanical Engineering - Czech Technical University
(Department of Technical Mathematics)
Karlovo nám. 13, 121 35 Praha 2, Czech Rep.
e-mail: jiri.simonek@fs.cvut.cz

[†]Institute of Thermomechanics - Academy of Sciences of the Czech Republic
Dolejškova 1402/5, 182 00 Praha 8, Czech Rep.
e-mail: karel.kozel@fs.cvut.cz, janour@it.cas.cz

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Abstract. *The work deals with the numerical solution of the 2D and 3D turbulent stratified flows in atmospheric boundary layer over the "sinus hills". Mathematical model for the turbulent stratified flows in atmospheric boundary layer is the Boussinesq model - Reynolds averaged Navier-Stokes equations (RANS) for incompressible turbulent flows with addition of the density change equation. The artificial compressibility method and the finite volume method have been used in all computed steady cases. Lax-Wendroff scheme (MacCormack form) has been used to find the numerical solution and turbulence was modeled by the Cebecci-Smith algebraic turbulence model. Computations have been performed with Reynold's number* 10^8 *that corresponds approximatelly to the upstream velocity* $u_\infty = 1.5\,\frac{m}{s}$ *and with density range* $\rho \in [1.2;\, 1.1]\,\frac{kg}{m^3}$.

1 INTRODUCTION

The numerical solution of the 2D and 3D turbulent stratified flows in atmospheric boundary layer over the "sinus hills" is introduced. Mathematical model for the turbulent stratified flows in atmospheric boundary layer is the Boussinesq model - Reynolds averaged Navier-Stokes equations (RANS) for incompressible turbulent flows with addition of the density change equation.

2 MATHEMATICAL MODEL

Reynolds averaged Navier-Stokes equations for 3D incompressible flows with addition of the equation of density change (Boussinesq model) have been used as a mathematical model for flows in atmospheric boundary layer:

$$\begin{aligned} u_x + v_y + w_z &= 0 && (1)\\ u_t + (u^2+p)_x + (u\cdot v)_y + (u\cdot w)_z &= \nu\cdot[(\nu_t u_x)_x + (\nu_t u_y)_y + (\nu_t u_z)_z] && (2)\\ v_t + (u\cdot v)_x + (v^2+p)_y + (w\cdot v)_z &= \nu\cdot[(\nu_t v_x)_x + (\nu_t v_y)_y + (\nu_t v_z)_z] && (3)\\ w_t + (u\cdot w)_x + (v\cdot w)_y + (w^2+p)_z &= \nu\cdot[(\nu_t w_x)_x + (\nu_t w_y)_y + (\nu_t w_z)_z] - \frac{\rho}{\rho_0}g && (4)\\ \rho_t + u\cdot\rho_x + v\cdot\rho_y + w\cdot\rho_z &= 0, && (5) \end{aligned}$$

where $(u,\, v,\, w)$ is a velocity vector, $p = \frac{P}{\rho_0}$ (P- static pressure, ρ_0 - initial maximal density), ρ - density, ν - laminar kinematic viscosity, ν_T - turbulent kinematic viscosity computed by the Cebecci-Smith algebraic turbulence model and g - gravity acceleration. Using artificial compressibility method, continuity equation is completed by term $\frac{p_t}{\beta^2}$,
$\beta^2 \in \mathbf{R}^+$.
Density and pressure are changing depending on height (z-axis) as follows:

$$\begin{aligned} \rho_\infty(z) &= -\frac{\rho_0-\rho_h}{h}\cdot z + \rho_0 && (6)\\ \frac{\partial p_\infty}{\partial z} &= -\frac{\rho_\infty(z)}{\rho_0}\cdot g && (7) \end{aligned}$$

The $\rho_{infty}(z)$ (6) is the linear decreasing function of density and the relation (7) is the hydrostatic equilibrium relation.
It is possible to consider $p = p_\infty + p'(x,y,z,t)$ and $\rho = \rho_\infty + \rho'(x,y,z,t)$, where the term p_∞ is the initial state of pressure, the term p' is the pressure disturbance. The term ρ_∞ is the initial state of density and the term ρ' is the density disturbance. If one substitutes these terms and adds the artificial compressibility term to the RANS system: eqs. (1) - (5), one obtains following system:

$$\frac{p'_t}{\beta^2} + u_x + v_y + w_z = 0 \tag{8}$$

$$u_t + (u^2 + p')_x + (u \cdot v)_y + (u \cdot w)_z = \nu \cdot [(\nu_t u_x)_x + (\nu_t u_y)_y + (\nu_t u_z)_z] \tag{9}$$

$$v_t + (u \cdot v)_x + (v^2 + p')_y + (w \cdot v)_z = \nu \cdot [(\nu_t v_x)_x + (\nu_t v_y)_y + (\nu_t v_z)_z] \tag{10}$$

$$w_t + (u \cdot w)_x + (v \cdot w)_y + (w^2 + p')_z = \nu \cdot [(\nu_t w_x)_x + (\nu_t w_y)_y + (\nu_t w_z)_z] - \frac{\rho'}{\rho_0} g \tag{11}$$

$$\rho_t + u \cdot \rho_x + v \cdot \rho_y + w \cdot \rho_z = 0, \tag{12}$$

All solved cases have been solved using these substitutions.

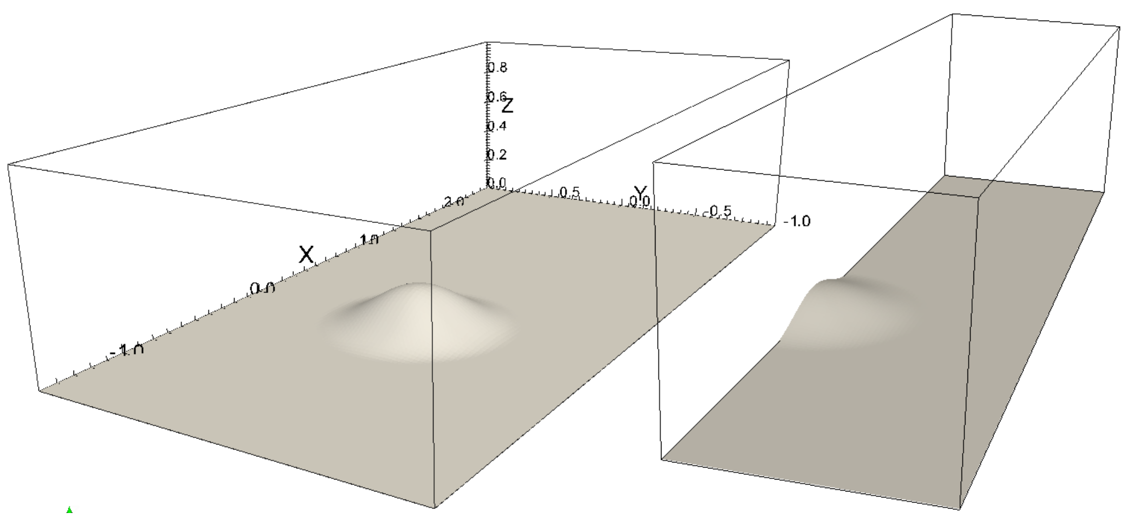

Figure 1: 3D Computational domains

3 BOUNDARY CONDITIONS

Inlet boundary conditions has been set as follows:
$u = u_\infty = 1.0$, $v = v_\infty = 0$, $w = w_\infty = 0$, $\rho = \rho_\infty(z)$, where $\rho_\infty(z)$ is a linear function which is decreasing with increasing z:

$$\rho_\infty(z) = -\frac{\rho_0 - \rho_h}{h} \cdot z + \rho_0, \;\; z \in [0;\, h]$$

where ρ_0 is a lower (maximal) density and ρ_h is a upper (minimal) density (both are constants). Pressure change term p' has been extrapolated.
Outlet boundary conditions: $p' = 0$ and $(u,\, v,\, w)$ and density ρ have been extrapolated.

Boundary conditions on the wall: $u = 0$, $v = 0$, $w = 0$, pressure perturbations and density have been extrapolated on the wall.
Boundary conditions on the upper domain boundary: $p' = 0$, $\frac{\partial u}{\partial n} = 0$, $\frac{\partial v}{\partial n} = 0$, $\frac{\partial w}{\partial n} = 0$, $\rho = \rho_h$
Boundary conditions on side-walls of the domain: symmetry boundary conditions $\frac{\partial p'}{\partial n} = 0$, $(u,\, v,\, w) \cdot \vec{n} = 0$, $\frac{\partial \rho}{\partial n} = 0$

4 NUMERICAL SOLUTION

In all cases the artificial compressibility method and the finite volume method have been used on structured grid of quadrilateral (2D) and hexahedral (3D) cells (uniform in x and y direction, refined near walls in z direction, 200x100x80 cells). Consider RANS system: eq. (8) - (12) in a vector form:

$$W_t + F_x + G_y + H_z = (R_x + S_y + T_z) + K \tag{13}$$

where:

$$W = \left\| \begin{matrix} \frac{p'}{\beta^2} \\ u \\ v \\ w \\ \rho \end{matrix} \right\|, \quad F = \left\| \begin{matrix} u \\ u^2 + p' \\ u \cdot v \\ u \cdot w \\ u \cdot \rho \end{matrix} \right\|, \quad G = \left\| \begin{matrix} v \\ v \cdot u \\ v^2 + p' \\ v \cdot w \\ v \cdot \rho \end{matrix} \right\|, \quad H = \left\| \begin{matrix} w \\ w \cdot u \\ w \cdot v \\ w^2 + p' \\ w \cdot \rho \end{matrix} \right\|,$$

$$R = \nu \left\| \begin{matrix} 0 \\ \nu_t u_x \\ \nu_t v_x \\ \nu_t w_x \\ 0 \end{matrix} \right\|, \quad S = \nu \left\| \begin{matrix} 0 \\ \nu_t u_y \\ \nu_t v_y \\ \nu_t w_y \\ 0 \end{matrix} \right\|, \quad S = \nu \left\| \begin{matrix} 0 \\ \nu_t u_z \\ \nu_t v_z \\ \nu_t w_z \\ 0 \end{matrix} \right\|, \quad K = \left\| \begin{matrix} 0 \\ 0 \\ 0 \\ -\frac{\rho'}{\rho_0} g \\ 0 \end{matrix} \right\| \tag{14}$$

Lax-Wendroff scheme (MacCormack form) has been used in following form:
Predictor step:

$$W_i^{n+\frac{1}{2}} = W_i^n - \frac{\Delta t}{\mu(D_i)} \left(\sum_{k=1}^{6} (\tilde{F} - \tilde{R},\ \tilde{G} - \tilde{S},\ \tilde{H} - \tilde{T})_{i,k}^n \cdot \vec{n}_{i,k}^0 \cdot \Delta S_{i,k} \right) + \Delta t \cdot K_i^n, \tag{15}$$

if D_k is from D_i in forward direction, then: $\tilde{F} = F_k^n$, $\tilde{G} = G_k^n$, $\tilde{H} = H_k^n$,
else: $\tilde{F} = F_i^n$, $\tilde{G} = G_i^n$, $\tilde{H} = H_i^n$
Corrector step:

$$W_i^{n+1} = \frac{1}{2}(W_i^{n+\frac{1}{2}} + W_i^n) - \frac{\Delta t}{2\mu(D_i)} \left(\sum_{k=1}^{6} (\tilde{F} - \tilde{R},\ \tilde{G} - \tilde{S},\ \tilde{H} - \tilde{T})_{i,k}^{n+\frac{1}{2}} \cdot \vec{n}_{i,k}^0 \cdot \Delta S_{i,k} \right) + \frac{\Delta t}{2} \cdot K_i^{n+\frac{1}{2}} \tag{16}$$

if D_k is from D_i in backward direction, then: $\tilde{F} = F_k^{n+\frac{1}{2}}$, $\tilde{G} = G_k^{n+\frac{1}{2}}$, $\tilde{H} = H_k^{n+\frac{1}{2}}$, else: $\tilde{F} = F_i^{n+\frac{1}{2}}$, $\tilde{G} = G_i^{n+\frac{1}{2}}$, $\tilde{H} = H_i^{n+\frac{1}{2}}$. Viscous fluxes have been computed centrally. The Jameson's artificial dissipation has been used to stabilize numerical solution. The Cebecci-Smith algebraic turbulence model [9] has been used to compute the turbulent viscosity ν_t.

5 NUMERICAL RESULTS

Following cases of stratified turbulent flows in atmospheric boundary layer have been computed. Authors consider flows over a geometry with the "sinus hill" (with the height 10% of its basis length) and with $Re = 10^8$ - figures show results with density change $\rho_\infty \in [1.2;\ 1.1]$.

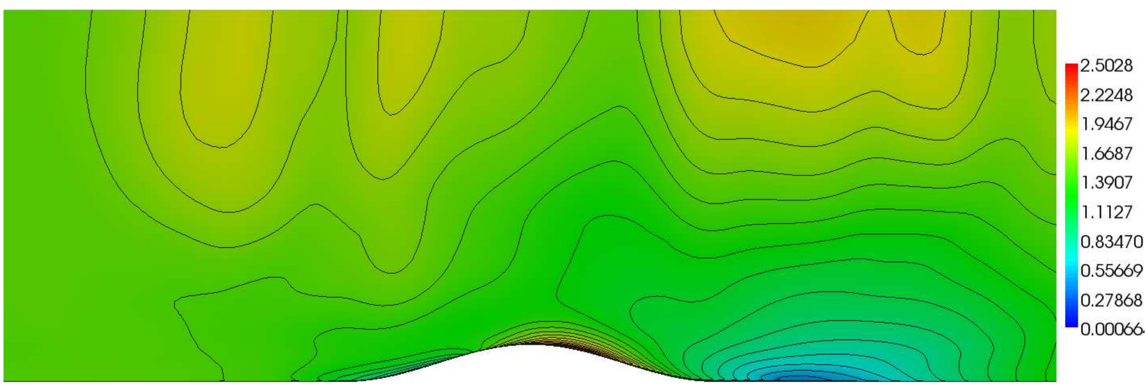

Figure 2: 2D Sin 10%; Velocity isolines $[\frac{m}{s}]$

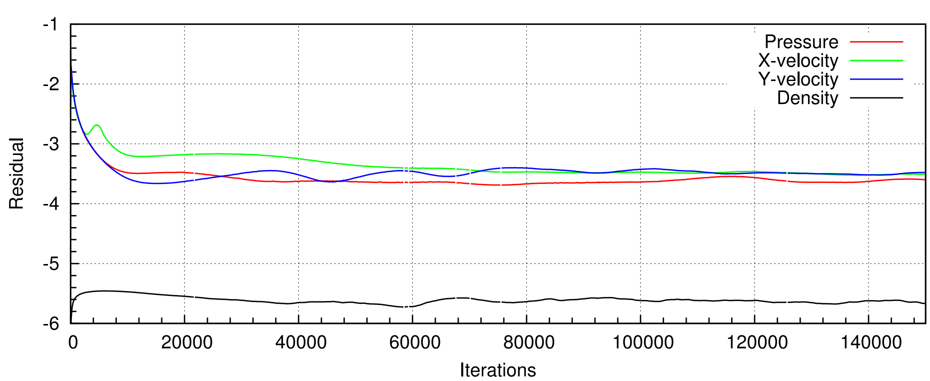

Figure 3: 2D Sin 10%; Residuals

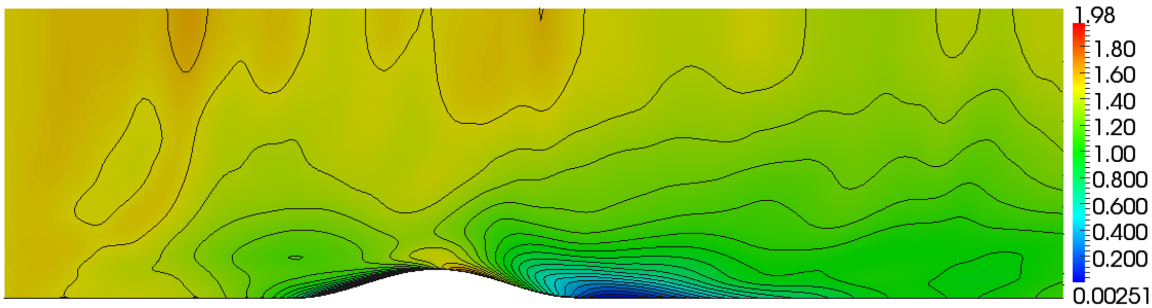

Figure 4: 3D Sin 10%; Half domain symmetrical solution - y-slice in the middle of the hill; Velocity isolines $[\frac{m}{s}]$

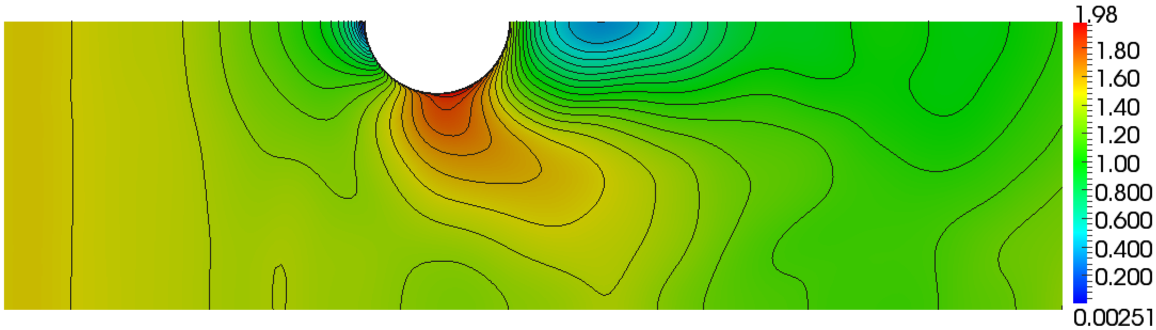

Figure 5: 3D Sin 10%; Half domain symmetrical solution - z-slice in the middle of the hill; Velocity isolines $[\frac{m}{s}]$

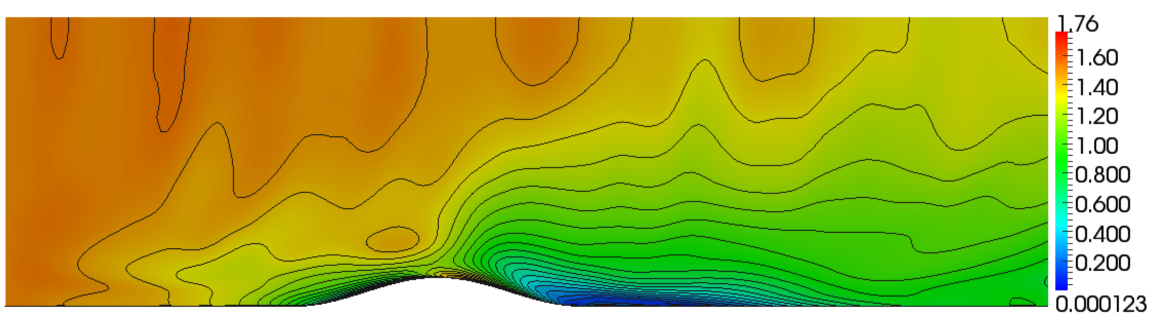

Figure 6: 3D Sin 10%; Full domain solution - y-slice in the middle of the hill; Velocity isolines $[\frac{m}{s}]$

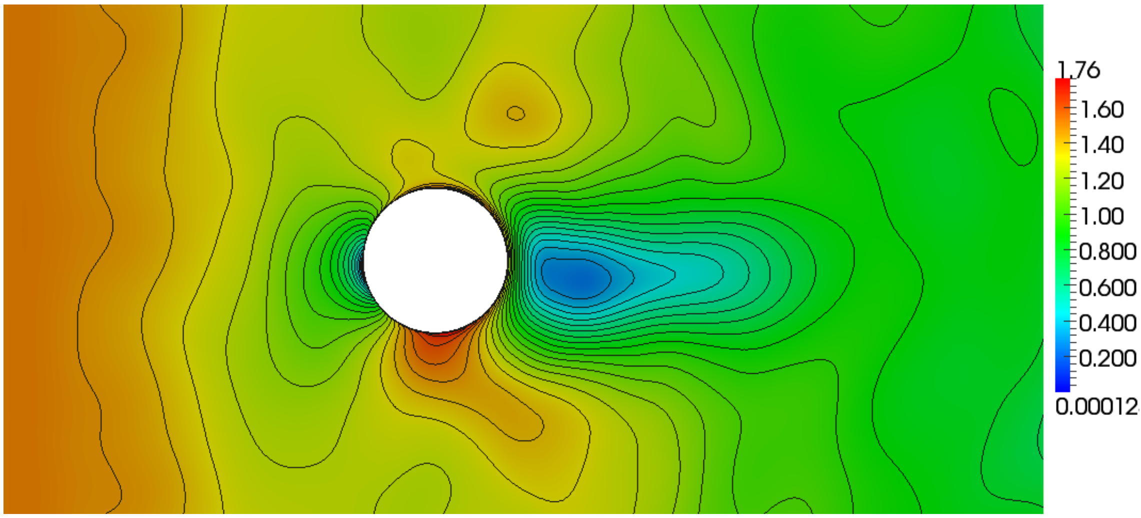

Figure 7: 3D Sin 10%; Full domain solution - z-slice in the middle of the hill; Velocity isolines $[\frac{m}{s}]$

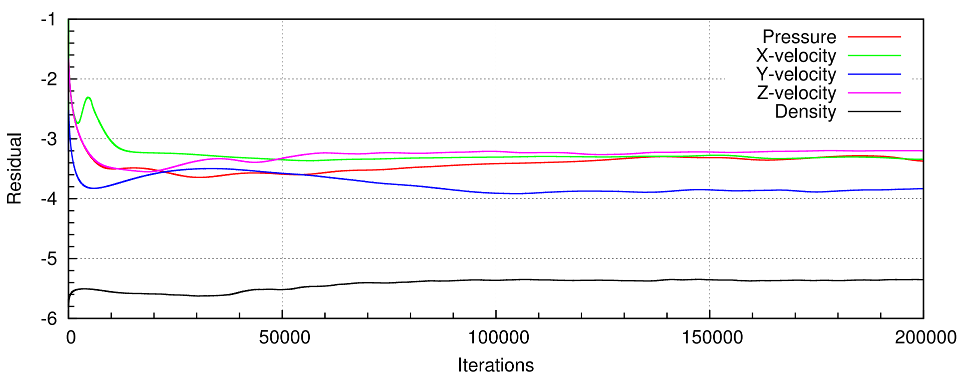

Figure 8: 3D Sin 10%; Full domain solution; Residuals

Figures 2 and 4 show computed field using velocity isolines over 2D and 3D hill (only in the plane $y = 0$) for half domain computational domain. Figure 5 shows results of computation near ground computed in half symmetrical 3D domain. Figure 6 shows results corresponding with figure 4 but computed in full 3D domain.

6 CONCLUSIONS

Results of the 2D and 3D incompressible turbulent stratified flows in atmospheric boundary layer over the "10% sinus hill" with Reynolds number $Re = 10^8$ that corresponds approximately to the upstream velocity $u_\infty = 1.5\,\frac{m}{s}$ and with range of density change $\rho \in [1.2; 1.1]\,\frac{kg}{m^3}$ have been presented. As one can see in the figures 5 and 7 the 3D solution is not symmetrical and therefore it is necessary to perform only the full domain computations in the future. The future work will be to extend this model for more complex geometries in 3D and to make a comparison with other numerical methods and mathematical models for variable density flows.

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REFERENCES

[1] Beneš L., Fürst J., Fraunié Ph.: Numerical Simulation of the Stratified Flow Using High Order Schemes, Engineering Mechanics, 2009, vol. 16, no. 1, p. 39-47

[2] Eidsvik K. J., Utnes T.: Flow separation and hydrostatic transition over hills modeled by the Reynolds equations, Journal of Wind Engineering and Industrial Aerodynamics, Issues 67 - 68 (1997), p. 408 - 413

[3] Feistauer M., Felcman J., Straškraba I.: Mathematical and computational methods for compressible flow, Clarendon Press, Oxford 2003

[4] Hirsh C.: Numerical computation of internal and external flows I. and II., John Willey and Sons, New York 1991

[5] Fletcher C. A. J.: Computational techniques for fluid dynamics I. and II., Springer Verlag, Berlin 1996

[6] Sládek I., Bodnár T., Kozel K.: On a numerical study of atmospheric 2D and 3D flows over a complex topography with forrest including pollution dispersion, Journal of Wind Engineering and Industrial Aerodynamics, Vol. 95 - Issues 9 - 11, 2007, p. 1424 - 1444

[7] Sládek I., Kozel K., Jaňour Z.: On the 2D validation study of the atmospheric boundary layer flow model including pollution dispersion, Engineering Mechanics, Vol 16, No 5, 2009

[8] Šimonek J., Kozel K., Fraunié Ph., Jaňour Z.: Numerical Solution of 2D Stratified Flows in Atmospheric Boundary Layer, Topical problems of fluid mechanics 2008, Prague 2008

[9] Šimonek J., Tauer M., Kozel K., Jaňour Z., Příhoda J.: Numerical Solution of 2D Stratified Flows in Atmospheric Boundary Layer, Colloquium 2008, Prague 2008

[10] Uchida T., Ohya Y.: Numerical study of stably stratified flows over a two dimensional hill in a channel of finite depth, Fluid dynamics research 29/2001 (p. 227 - 250)