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ENHANCEMENT OF PISO SCHEME IN COLLOCATED GRIDS

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Abstract. *In collocated grids there exist two types of velocity fields: the convecting velocity, a continuity-satisfying (CS) field and the convected velocity, a momentum-satisfying (MS) variable. The connection between both is usually established via the PWIM interpolation (C. Rhie, W. Chow. AIAA J. vol 21(11), pp 1525-1532, 1983) before using a pressure velocity coupling. In these paper two schemes for the PV coupling in collocated grids that combine robustness and speed are put forward along with a discussion on the similitudes between SIMPLER and PISO.*

1 INTRODUCTION

It has been more than three decades since the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) [1] was proposed for the efficient treatment of the pressure-velocity coupling in the computation of fluid flow. Due to its ample current use it is considered a significant breakthrough in the development of Computational Fluid Dynamics (CFD) and its applications. During these thirty plus years since its inception some variants of SIMPLE, generally named as SIMPLE-type approaches, have been put forward to increase the robustness and convergence speed of the original scheme. They came up as a response to some weaknesses of this approach evidenced in the computational experiments where SIMPLE was put to the test. One of the drawbacks experienced by SIMPLE is that the pressure correction was found to be reasonably good at correcting velocities (with underrelaxation) but the updated pressure field was far from the correct pressure field, so another strong pressure underrelaxation was required. SIMPLE Revised (SIMPLER) [2] was devised to obtain a better pressure estimation by deriving a new pressure equation without relying on the pressure correction. This strategy also allows underrelaxation factors close to one, favorably contributing to convergence speed-up. Later a closely related scheme, PISO (Pressure Implicit with Splitting of Operators) [3], was proposed by incorporating a second correction step in both velocity and pressure by including some factors that were not considered in the first step.

The idea that these two schemes of the SIMPLE family, namely SIMPLER and PISO, are very close to one another or even that they are the same scheme has been around for some time. The conditions under which both schemes are identical were established by Braaten [4] and later rederived by Chow and Cheung [5], both in a staggered grid. They concluded that SIMPLER and PISO with $\alpha_p = 1$ are in fact the same scheme if all equations are solved exactly in each iteration (i.e., down to machine accuracy). However, owing to the dual velocity field present in collocated grids these two conditions are not enough for ensuring that both will converge in an equal number of iterations. We will show that in a collocated grid these two requirements have to be supplemented with a third one concerning the second velocity nodal correction in PISO. The three conditions are necessary and sufficient for the equivalence of the schemes.

The paper first describes PISO scheme and then the conditions under which it is equivalent to SIMPLER. Finally, an enhancement of the PISO scheme based on the use of SIMPLEC as its first step will be presented.

2 PISO scheme

PISO was originally derived with SIMPLE as its first step but there is no reason why another more robust and faster pressure velocity coupling such as SIMPLEC be used. As the second step is conditioned by the election of SIMPLE or SIMPLEC, we will first describe PISO/SIMPLE and in a later section the expression of PISO/SIMPLEC will be derived. There is a companion paper in these Proceedings where the notation is explained.

PISO second step for a generic node P starts with

$$\widetilde{A}^u_{P|P} u''_P \;=\; \alpha_u \sum_{j|P} A^u_j u'_j - \alpha_u \Delta V_P \left.\frac{\partial p}{\partial x}\right|''_P \quad ; \quad \widetilde{A}^u_{P|P} = A^u_{P|P} + \frac{\rho_P \Delta V_P}{\Delta t} \tag{1}$$

in which the first term in the RHS has been estimated explicitly with the values of the correction field obtained at the first step. Double prime has been used to denote a second correction. Equivalently,

$$(1+\delta_P) u''_P \;=\; \alpha_u \frac{\sum_{j|P} A^u_j u'_j}{A^u_{P|P}} - \alpha_u \Delta t \frac{\delta_P}{\rho_P} \left.\frac{\partial p}{\partial x}\right|''_P \quad ; \quad \delta_P = \frac{\rho_P \Delta V_P}{\Delta t A^u_{P|P}} \tag{2}$$

If the correction equation at the faces is written for the double prime correction

$$(1+\delta_e) u''_e \;=\; \overline{(1+\delta_i) u''_i}^{\,e} + \alpha_u \Delta t \left[\overline{\frac{\delta_i}{\rho_i} \left.\frac{\partial p}{\partial x}\right|''_i}^{\,e} - \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|''_e \right] \tag{3}$$

Substituting Eqn. 2 and that for u''_E in Eqn. 3 we obtain

$$(1+\delta_e) u''_e \;=\; \alpha_u \overline{\left(\frac{\sum_{j|i} A^u_j u'_j}{A^u_{P|i}} \right)}^{\,e} - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|''_e \tag{4}$$

and again the expression is identical to that at the nodes as in SIMPLE first step.

There are additional possibilities that provide a slightly slower convergence. For instance, let us write Eqn. 2 in a different (but still correct) way

$$u''_P \;=\; \alpha_u \frac{\sum_{j|P} A^u_j u'_j}{\widetilde{A}^u_{P|P}} - \alpha_u \frac{\Delta t}{\rho_P} \frac{\delta_P}{1+\delta_P} \left.\frac{\partial p}{\partial x}\right|''_P \tag{5}$$

If Choi's approach [6] for unsteady problems is followed to derive the face expression we obtain

$$u''_e = \alpha_u \overline{\left(\frac{\sum_{j|i} A^u_j u'_j}{\widetilde{A}^u_{P|i}} \right)}^{\,e} - \alpha_u \frac{\Delta t}{\rho_e} \frac{\delta_e}{1+\delta_e} \left.\frac{\partial p}{\partial x}\right|''_e \tag{6}$$

The average utilized in this expression follows Choi's proposal that has proved to give rise to a solution dependent on the time step. Yet, this approach for PISO produces a steady solution that is time step independent although it causes an increase in the number of iterations required to reach steady state. The reason is that the inconsistent average is employed in the expressions that correct the velocities and not in the face velocity expression where PICTURE [7] consistent average is used. The outcome is that the corrected face velocity does not satisfy its equation at the start of next iteration and as a result the convergence is slowed down. This problem is more important in, but not exclusive of, inconsistent schemes as the momentum residuals and pressure underrelaxation also contribute to the lack of exact satisfaction of the face velocity expression at the start of a new iteration.

The effect of the inconsistency is more noticeable when the time step is such that $\delta_{i,e} \sim 1$. If $\delta_{i,e} >> 1$ the first term in the RHS of both approaches is

$$\overline{\left(\frac{\sum_{j|i} A^u_j u'_j}{A^u_{P|i}}\right)}^e \Bigg/ \overline{\left(\frac{\Delta V_i}{A^u_{P|i}}\right)}^e \quad \text{and} \quad \overline{\left(\frac{\sum_{j|i} A^u_j u'_j}{\Delta V_i}\right)}^e$$

and we found negligible differences between these terms in the two computational cases tested. The result of not using a correct implementation is always to increase unnecesarily the number of iterations required to convergence but the advantages of using the appropriate relation stand out for a particular interval of time step values. For a steady case ($\Delta t \to \infty, \delta \to 0$) there is no difference between both formulations. PISO in a structured collocated grid was employed by Kobayashi and Pereira [8] for a steady problem. There is no difference with our formulation as they employed Eqn. 4 for the second PISO step particularized for a steady case ($\delta_e \to 0$).

3 SIMPLER AND PISO, ARE THEY THE SAME SCHEME?

In the context of collocated grids it is very easy to show that under certain circumstances to be specified SIMPLER and PISO are in fact the same scheme. To clarify this issue we will study the changes underwent by the velocity and pressure fields in one iteration. If starting with the same field values in both schemes and performing one iterative step the values have changed the same amount we will conclude that both schemes are identical. The expression of the face velocity at the start of the continuity-based correction of the velocities is

$$\begin{aligned} (1+\delta_e)u^*_e &= \alpha_u \overline{\left(\frac{\sum_{j|i} A^u_j u^*_j + S^u_i \Delta V_i}{A^u_{P|i}}\right)}^e - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|^l_e + \\ &+ (1-\alpha_u)(1+\delta_e)u^l_e + \alpha_u \delta_e u^n_e \end{aligned} \tag{7}$$

The first step of both schemes is a SIMPLE step where the correcting velocities are linked to the pressure correction gradient with

$$(1+\delta_e)u'_e = -\alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|'_e \tag{8}$$

We will follow PISO procedure and eventually check if the equation for the pressure at iteration $l+1$ of PISO is the same as that for SIMPLER, which is

$$
\begin{aligned}
(1+\delta_e)u_e^{l+1/2} &= \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j^{l+1/2} + S_i^u \Delta V_i}{A_{P|i}^u}\right)}^e - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{l+1} + \\
&+ (1-\alpha_u)(1+\delta_e)u_e^l + \alpha_u \delta_e u_e^n \qquad (9)
\end{aligned}
$$

By summing up Eqns. 7 and 8 we obtain the equation satisfied by the intermediate velocity, $u^{l+1/2} = u^* + u'$, and the intermediate pressure, $p^{l+1/2} = p^l + \alpha_p p'$. We shall assume $\alpha_p = 1$ in both schemes, in fact we will see that this is one of the requirements for the two schemes to be equivalent[1]. Note that the underrelaxation for the velocity has already been introduced implicitly in the momentum equation.

$$
\begin{aligned}
(1+\delta_e)u_e^{l+1/2} &= \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j^* + S_i^u \Delta V_i}{A_{P|i}^u}\right)}^e - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{l+1/2} + \\
&+ (1-\alpha_u)(1+\delta_e)u_e^l + \alpha_u \delta_e u_e^n \qquad (10)
\end{aligned}
$$

The second PISO correction is given by the following expression

$$
(1+\delta_e)u_e'' = \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j'}{A_{P|i}^u}\right)}^e - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{''} \qquad (11)
$$

where $u' = u^{l+1/2} - u^*$. Summing these last two equations we obtain

$$
\begin{aligned}
(1+\delta_e)u_e^{l+1} &= \alpha_u \overline{\left(\frac{\sum_{j|i} A_j^u u_j^{l+1/2} + S_i^u \Delta V_i}{A_{P|i}^u}\right)}^e - \alpha_u \Delta t \frac{\delta_e}{\rho_e} \left.\frac{\partial p}{\partial x}\right|_e^{l+1} + \\
&+ (1-\alpha_u)(1+\delta_e)u_e^l + \alpha_u \delta_e u_e^n \qquad (12)
\end{aligned}
$$

that is the same as Eqn. 9 if $u_e^{l+1} = u_e^{l+1/2}$, that is, if $u_e'' = 0$, issue that is discussed further on.

Now, it is worth commenting on the various factors that contribute to making the actual results of the two schemes different even after only one iteration. The practical implementation clearly separates from that discussed above and this has disputed the idea that the two schemes are even close to each other [9]. Consider for instance that for both schemes to be identical the momentum coefficients in SIMPLER do not have to be updated (nor δ) after the first SIMPLE step because PISO uses the original factors in order to calculate the first term in the RHS of Eqn 12. In our computational tests the effect of using newly calculated values was most often negligible, it reduced the number

[1]PISO and SIMPLER usually employ α_p near one.

of iterations but always slightly. Since the computing time also rises it is not clear if this update before assembling the pressure equation is benefitial at all. In our code we can switch it on and off as part of the input options, so when it is in place SIMPLER and PISO do not end an iteration with the same expression for u_e^{l+1}. Another differentiating factor is the value of the coefficient α_p. It has to be one because in any other case $p^{l+1} \neq p^l + p' + p''$ and there will be an extra term in Eqn. 12 given by

$$-(1-\alpha_p)\,\alpha_u \Delta t \frac{\delta_e}{\rho_e}\left(\left.\frac{\partial p}{\partial x}\right|_e' + \left.\frac{\partial p}{\partial x}\right|_e''\right)$$

that will make it different from Eqn. 9. This term is also responsible for causing the corrected u_e value to separate from that obtained with the new pressure field in the u_e expression.

We also found that there may exist differences between both schemes in collocated grids even if the conditions above for staggered grids are satisfied. The reason is that due to the dual velocity field a relation that is true for the face velocities may not be so for the nodes. This comment specially refers to the second velocity correction in PISO. It should be realized that if the continuity equation is solved exactly in the first SIMPLE step (it is always so in 1D), the second step of PISO only serves to improve the pressure field. The velocity coming up of the first SIMPLE step already satisfies the cell mass balance so the second correction produces a zero u'' field at the faces, that is, $u_e^{l+1} = u_e^{l+1/2}$. That means

$$\overline{\left(\frac{\sum_{j|i} A_j^u u_j'}{A_{P|i}^u}\right)}^e = \Delta t \frac{\delta_e}{\rho_e}\left.\frac{\partial p}{\partial x}\right|_e'' \tag{13}$$

and the face velocity is not corrected with PISO. However the relation at node P used to correct its velocity is

$$\alpha_u\left(\frac{\sum_{j|P} A_j^u u_j'}{A_{P|P}^u} - \Delta t \frac{\delta_P}{\rho_P}\left.\frac{\partial p}{\partial x}\right|_P''\right) \tag{14}$$

and this term in brackets is not zero even though Eqn. 13 is true. The velocity at the node will then be corrected by PISO and the starting values at next iteration will be different from those of SIMPLER which does not possess a second correction step. The effect of this second nodal correction is not negligible, we encountered cases that converged in a similar fashion to SIMPLER when this correction was not applied and they diverged otherwise.

Another factor that makes both schemes move apart is the residual of the continuity equation at the end of each iteration. In two- and three-dimensional flows the continuity-linked pressure (correction) equation is usually solved performing sweeps over the domain, splitting the two-dimensional surfaces or three-dimensional volumes in a series of 1D lines. The number of sweeps employed is always much less than that required to reduce the residual to the level of machine accuracy. Thus, there is a non vanishing residual at the end of a given iteration for both schemes. As the effect of this residual is different (and cumulative) in the two schemes their field values may separate as the procedure progresses.

Summing up, there are various reasons why the equivalence between PISO and SIMPLER should be understood as a mere theoretical result. Even though all equations within an iteration were solved to machine accuracy there are three other requirements in a collocated grid for PISO and SIMPLER to be the same scheme: the momentum coefficients in SIMPLER do not have to be updated after the SIMPLE step, both schemes must use $\alpha_p = 1$ in the pressure update and the nodal velocity does not have to be corrected in the second PISO step. We carried out several computational experiments to check the validity of this theoretical result and found that under the conditions mentioned the residuals associated to PISO and SIMPLER differentiate from each other from the tenth decimal place onwards (in 1D) in a series of ten iterations for a double-precision calculation in all cases tested[2]. In fact, by doing these experiments we realized the non-vanishing nodal correction of the second step of PISO even with a zero u'' field at the faces. In staggered grids last condition has no sense but the first two requirements are the same as well as the neccesity of solving to machine accuracy.

4 SIMPLEC AS THE PREDICTOR-CORRECTOR STEP

The two schemes compared in the previous section are usually employed with SIMPLE as the first predictor-corrector step. As SIMPLEC is much better behaved when the underrelaxation factor is near one, just the region where SIMPLER and PISO are commonly employed, it is adequate to wonder if using SIMPLEC in the first step will improve the efficiency of either. The second step of SIMPLER is the same whatever scheme is used in its first step but it is not so in PISO.

The starting point is the SIMPLEC expression, derived in the companion paper, rearranged to incorporate the second step.

$$(1+\delta_P-\alpha_u r_P)A^u_{P|P}u''_P \;=\; \alpha_u\sum_{j|P}A^u_j(u'_j-u'_P)-\alpha_u\Delta V_P\,\frac{\partial p}{\partial x}\bigg|''_P \tag{15}$$

or alternatively

$$(1+\delta_P)u''_P \;=\; \alpha_u(1+\widetilde{k}_P)\frac{\sum_{j|P}A^u_j(u'_j-u'_P)}{A^u_{P|P}}-\alpha_u\Delta t(1+\widetilde{k}_P)\frac{\delta_P}{\rho_P}\,\frac{\partial p}{\partial x}\bigg|''_P \tag{16}$$

Following the procedure explained before the face expression is obtained as

$$(1+\delta_e)u''_e \;=\; \alpha_u\overline{(1+\widetilde{k}_i)\frac{\sum_{j|i}A^u_j(u'_j-u'_i)}{A^u_{P|i}}}^{e}-\alpha_u\Delta t\frac{\delta_e}{\rho_e}\,\frac{\partial p}{\partial x}\bigg|''_e-\alpha_u\Delta t\,\overline{\widetilde{k}_i\frac{\delta_i}{\rho_i}\,\frac{\partial p}{\partial x}\bigg|''_i}^{e} \tag{17}$$

[2]In 1D all equations are solved exactly at each iteration.

or

$$\begin{aligned}
\text{First step} \quad (1+\delta_e)u'_e &= -\alpha_u \Delta t \frac{\delta_e}{\rho_e} \frac{\partial p}{\partial x}\bigg|_e' - \alpha_u \Delta t\, \overline{\widetilde{k}_i \frac{\delta_i}{\rho_i} \frac{\partial p}{\partial x}\bigg|_i'}^{e} \\
\text{Second step} \quad (1+\delta_e)u''_e &= \alpha_u (1+\widetilde{k}_i) \overline{\frac{\sum_{j|i} A^u_j (u'_j - u'_i)}{A^u_{P|i}}}^{e} - \\
&- \alpha_u \Delta t \frac{\delta_e}{\rho_e} \frac{\partial p}{\partial x}\bigg|_e'' - \alpha_u \Delta t\, \overline{\widetilde{k}_i \frac{\delta_i}{\rho_i} \frac{\partial p}{\partial x}\bigg|_i''}^{e}
\end{aligned} \tag{18}$$

5 RESULTS

The comparison of the different schemes, consistent and inconsistent, is carried out in the same two 2D laminar flows examined in the companion paper. The computational setup is repeated here for the sake of clarity. The Reynolds numbers of the two computational experiments are 10^3 and $5.\,10^3$ based on lid velocity. The convergence monitor is a coefficient defined as the ratio of p-norms of the momentum residuals and the left hand side of the discretized momentum equation, the latter considered as a normalizing factor.

$$res_u = \frac{\left(\sum_i \left| A^u_{P|i} u_i - \sum_{j|i} A^u_j u_j - S^u_i \Delta V_i + \Delta V_i \left.\frac{\partial p}{\partial x}\right|_i \right|^p \right)^{1/p}}{\left(\sum_i \left| A^u_{P|i} u_i \right|^p \right)^{1/p}} \quad ; \quad p = 1, 2, \ldots, \infty \tag{19}$$

Likewise, a residual for the υ-velocity can be defined, res_υ. The mass imbalance is calculated as

$$res_m = \frac{\left(\sum_i \left| (\rho_e u^*_e - \rho_w u^*_w) \Delta y + (\rho_n \upsilon^*_n - \rho_s \upsilon^*_s) \Delta x \right|^p \right)^{1/p}}{\left(\sum_i inflow_i^p \right)^{1/p}} \quad ; \quad p = 1, 2, \ldots, \infty \tag{20}$$

where inflow is the mass flow coming into a cell. The monitoring value for the velocities is $res = \max(res_u, res_\upsilon)$ and the calculation stops when $res < 10^{-8}$ and $res_m < 10^{-6}$. The initial condition is 10^{-6} for velocities and pressure, the lid velocity being 1. All cases have been calculated with the residual based on the L_1 norms ($p = 1$) with a grid of 100x100. For $Re = 1000$ the grid is uniform and for $Re = 5000$ it is expanding/contracting in both directions with ratios 1.1 and 1/1.1 respectively.

Figure 1 shows results of PISO with several variants. In the figure PISOnv stands for PISO with no second velocity correction at the nodes. The reason for assessing this alternative is twofold: first, because previous researchers have also considered a partial correction[3] due to empirical evidence that the full correction overestimates the change [10], and second, as discussed formerly, if this correction is not considered the second step of PISO still makes sense, in fact makes it closer to SIMPLER. As seen in the figure

[3]But it was within the first step of SIMPLER

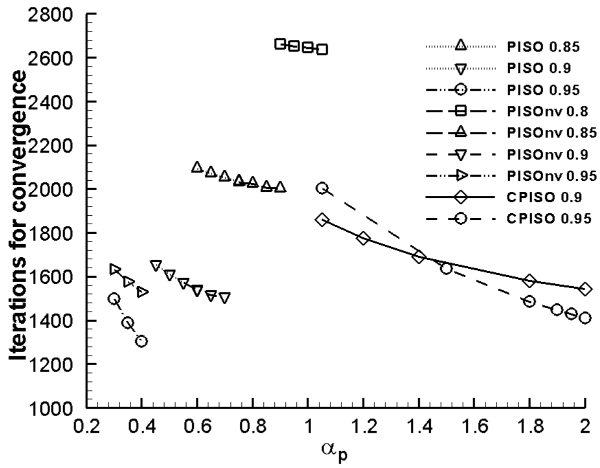

Figure 1: Comparison of the performance of different schemes in the lid-driven cavity case, Re=1000. The numbers in the names refer to the underrelaxation factor for the velocity.

PISOnv allows for higher α_p than PISO although the effect in the convergence speed is very small. The least number of iterations is reached at $\alpha_u = 0.95$ where including the second correction at the nodes improves the performance. In this computational case taking out this correction betters the robustness of the scheme but the convergence is slowed down slightly. CPISO is PISO with SIMPLEC as first step. The proposed name maintains the idea behind the acronym put forward in [11] for SIMPLER. In this computational case standard PISO shows the best performance.

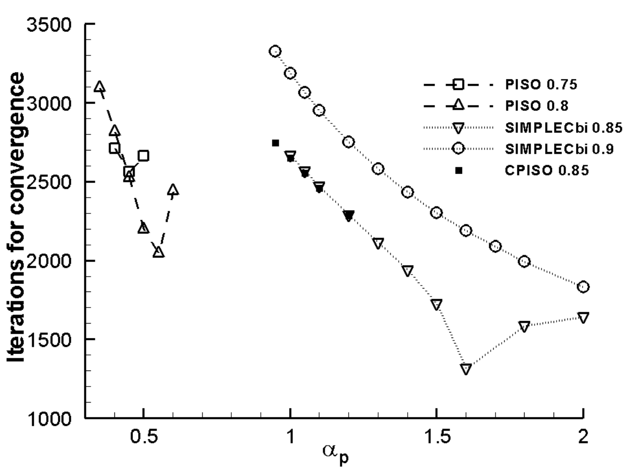

Figure 2: Comparison of the performance of different schemes in the lid-driven cavity case, Re=5000.

Figure 2 depicts the comparison between PISO, SIMPLECbi and CPISO, the latter represented with full symbols. In this case we found that the contribution of the first addend in the RHS of second step, Eqn. 18, was most often negligible and even in a few times counterproductive because it led to blowup. The effect of this term is usually so faint, thus corroborating SIMPLEC assumption, that we took this correction out to assess if it made a difference. In so doing the procedure is equivalent to running SIMPLEC twice with a mass imbalance evaluation in between. We call this scheme SIMPLECbi. The dependence on α_p is much smoother with SIMPLECbi, there being an ample range of α_p for which the convergence is quicker than the best point of PISO. When CPISO and SIMPLECbi converge there is no appreciable difference between them but SIMPLECbi is more robust, i.e., the range of α's with a converged solution is wider. For instance, with the term in, i.e, with CPISO, we could not obtain a converged solution for any case with $\alpha_u = 0.9$, nor for $(0.85, 1.6)$ the point with the least number of required iterations.

6 CONCLUSIONS

In this paper a discussion on the equivalence of SIMPLER and PISO in collocated grids has been presented. For the two schemes to be identical in collocated grids a new requirement, apart from those needed in staggered grids, has been found. As a second contribution two closely related schemes, CPISO and SIMPLECbi, based on PISO and/or SIMPLEC consistent, have been put forward that could be an alternative to the latter, more established schemes. To draw a sounder conclusion additional computational tests are currently underway for new flow geometries.

REFERENCES

[1] Patankar SV, Spalding DB. A calculation procedure for heat, mass and momentum transfer in three dimensional parabolic flows. *International Journal of Heat and Mass Transfer* 1972; **15** : 1787-1806.

[2] Patankar SV. A calculation procedure for two dimensional elliptic simulations. *Numerical Heat Transfer* 1985; **14** : 409-425.

[3] Issa RI. Solution of the implicitly discretised fluid flow equations by operator-splitting. *Journal of Computational Physics* 1986; **62(1)** : 40-65.

[4] Braaten ME. Development and evaluation of iterative and direct methods for the solution of the equations governing recirculating flows. Ph.D. Dissertation 1985. University of Minnesota.

[5] Chow WK, Cheung YL. Comparison of the algorithms PISO and SIMPLER for solving pressure-velocity linked equations in simulating compartmental fire. *Numerical Heat Transfer, Part A* 1997; **31(1)** : 87-112.

[6] Choi SK. Note on the use of momentum interpolation method for unsteady flows. *Numerical Heat Transfer, Part A* 1999; **36** : 545-550.

[7] Pascau A. Cell face velocity alternatives in a structured collocated grid for the unsteady Navier-Stokes equations. *International Journal for Numerical Methods in Fluids*. Accepted for publication. Early view.

[8] Kobayashi MH, Pereira JCF. Numerical comparison of momentum interpolation methods and pressure-velocity algorithms using non-staggered grids. *Communications in Applied Numerical Methods* 1991; **7** : 173-186.

[9] Moukalled F, Darwish M. A unified formulation of the segregated class of algorithms for fluid flow at all speeds. *Numerical Heat Transfer, Part B* 2001; **40(2)** : 99 137.

[10] Cheng YP, Lee TS, Low HT, Tao W-Q. Improvement of SIMPLER algorithm for incompressible flow on collocated grid system. *Numerical Heat Transfer, part B* 2007; **51** : 463-486.

[11] Liu XL, Tao WQ, He YL. A simple method for improving the SIMPLER algorithm for numerical simulations of incompressible fluid flow and heat transfer problems. *Engineering Computations* 2005; **22(8)** ; 921-939.