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NUMERICAL SIMULATION OF WAVE OVERTOPPING IN EXTERNALLY EXCITED TANKS

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Abstract. *In the present paper, overtopping of free surface waves is simulated in a rectangular tank subjected to harmonic and seismic excitations. A two dimensional mathematical model is used for the numerical computations. The computational procedure is based on the finite volume discretization of Navier-Stokes equations. Free surface position is tracked using volume of fluid (VOF) method using piecewise linear interface construction (PLIC) technique. Mathematical model and computer code are validated by comparing simulation results with existing experimental data and good agreement is achieved indicating that the present model is capable of predicting the safe freeboard height required in liquid storage tanks. . Amount of liquid exchange by spilling over the sharp crest boundary walls between two neighboring domains is also evaluated to test the volume conserving capability of the numerical scheme.*

1 INTRODUCTION

Hydrodynamic pressure impacts occur and sloshing waves splash on the tank wall during earthquake. Liquid sloshing in a partially filled tank has been extensively researched in recent years for the hydrodynamic design of large storage tanks subjected to earthquake excitation. There are numerous analytical, experimental and numerical studies in the literature for the analyses of liquid sloshing in tanks. Faltinsen et al. [1] derived an analytical solution for the liquid motion in a rectangular tank which is oscillated horizontally. Akyildiz and Ünal [2] analyzed the pressure variations inside an oscillating tank experimentally in three dimensions (3D). They investigated the nonlinear liquid sloshing inside a rectangular tank using a finite difference model based on the solution of Navier-Stokes equations and VOF method to track the free surface [3]. Frandsen [4] analyzed the liquid sloshing in excited tanks using a non-linear finite difference model based on the potential flow equations. Liu and Lin [5] developed a numerical model called NEWTANK to analyze the 3D non-linear liquid sloshing with broken free surface and they analyzed the baffle effects on the liquid sloshing in a tank using virtual boundary force method [6].

In this study a two dimensional computational model is developed based on the finite volume discretization of Navier-Stokes equations. VOF method based on piecewise linear interface construction technique (PLIC) is used to simulate the violent free surface configurations in tank. Numerical model and computer code are validated by comparing the computed result with existing experimental results. A test case is proposed to test the volume conservation property of the free surface tracking algorithm including wave breaking and wave overtopping effects.

2 GOVERNING EQUATIONS

The equations of motion for two dimensional incompressible flows in a vertical plane are given in integral form as

$$\frac{\partial}{\partial t}\int_{cv} u d\forall + \int_{cs} u\vec{V}\cdot\overrightarrow{dA} = -\frac{1}{\rho}\int_{cv}\frac{\partial p}{\partial x}d\forall + \nu\int_{cs}\vec{\nabla}u\cdot\overrightarrow{dA} - \int_{cv} a_x d\forall \tag{1}$$

$$\frac{\partial}{\partial t}\int_{cv} w d\forall + \int_{cs} w\vec{V}\cdot\overrightarrow{dA} = -\frac{1}{\rho}\int_{cv}\frac{\partial p}{\partial z}d\forall + \nu\int_{cs}\vec{\nabla}w\cdot\overrightarrow{dA} - \int_{cv}(g + a_z)d\forall \tag{2}$$

$$\int_{cs}\vec{V}\cdot\overrightarrow{dA} = 0. \tag{3}$$

where x and z are coordinate axes in horizontal and vertical directions respectively, a_x and a_z are ground accelerations, u and w are velocity components, $\vec{V}$ is velocity vector relative to the moving ground, p is pressure, t is time, g is gravitational acceleration, ν is kinematic viscosity, ρ is fluid density, $\vec{\nabla}$ is the del operator, CV indicates control volume, CS indicates control surface and $\overrightarrow{dA}$ is the area element normal to the control surface pointing out of the control volume. Ground accelerations are included to represent earthquake excitations or accelerations due to shaking of experimental tanks. The computational domain and the grid system are assumed to move with the ground. The integral equations are discretized on a staggered grid arrangement [7].

$$u_{i,j}^{n+1} = F_{i,j}^{n} + \Delta t\left\{\left[\left(p_{i,j}^{n+1} - p_{i+1,j}^{n+1}\right)/\rho\right]/\Delta x_{i+1/2}\right\} \tag{4}$$

$$w_{i,j}^{n+1} = G_{i,j}^{n} + \Delta t \left\{ \left[\left(p_{i,j}^{n+1} - p_{i,j+1}^{n+1} \right) / \rho \right] / \Delta z_{j+1/2} - g \right\} \tag{5}$$

$$F_{i,j}^{n} = u_{i,j}^{n} + \Delta t \left\{ \left[\nu (Difu)_{i,j}^{n} - (Conu)_{i,j}^{n} \right] / \left(\Delta x_{i+1/2}\, \Delta z_{j} \right) \right\} \tag{6}$$

$$G_{i,j}^{n} = w_{i,j}^{n} + \Delta t \left\{ \left[\nu (Difw)_{i,j}^{n} - (Conw)_{i,j}^{n} \right] / \left(\Delta x_{i}\, \Delta z_{j+1/2} \right) \right\} \tag{7}$$

$$(u_{i,j}^{n+1} - u_{i-1,j}^{n+1}) / \Delta x_i + (w_{i,j}^{n+1} - w_{i,j-1}^{n+1}) / \Delta z_j = 0. \tag{8}$$

where Δt is the time step, Δx and Δz are mesh sizes, *Dif* and *Con* represent diffusive and convective fluxes. To utilize the advantage of staggered grid system, convenient control volumes are selected for each equation. First order derivatives in diffusive fluxes are discretized using second order polynomial approximation on a variable mesh. Convective fluxes are evaluated by first order upwind (FOU). Pressure solution is obtained from the Poisson equation for pressure. Discretized form of the Poisson equation for pressure is obtained by substituting Eqs. 4 and 5 into 8

$$\begin{aligned}
&\left[\left(p_{i,j}^{n+1} - p_{i+1,j}^{n+1} \right) / \Delta x_{i+1/2} - \left(p_{i-1,j}^{n+1} - p_{i,j}^{n+1} \right) / \Delta x_{i-1/2} \right] / \Delta x_i \\
&+ \left[\left(p_{i,j}^{n+1} - p_{i,j+1}^{n+1} \right) / \Delta z_{j+1/2} - \left(p_{i,j-1}^{n+1} - p_{i,j}^{n+1} \right) / \Delta z_{j-1/2} \right] \\
&/ \Delta z_j = - \left[\frac{F_{i,j}^{n} - F_{i-1,j}^{n}}{\Delta x_i} + \frac{G_{i,j}^{n} - G_{i,j-1}^{n}}{\Delta z_j} \right] \frac{\rho}{\Delta t}
\end{aligned} \tag{9}$$

The momentum equations and the pressure Poisson equation are solved by sequential iterations. The detailed description of the computational algorithm is given in Ref. [7]. A computer code in FORTRAN language is developed by the authors to perform the computations.

3 FREE SURFACE TRACKING: VOF /PLIC METHOD

Free surface position is captured using the VOF method which was originally developed by Hirt and Nichols [8]. In the VOF method, a function F is introduced

$$\frac{\partial F}{\partial t} + \vec{\nabla}\left(\vec{V} F \right) = 0 \tag{10}$$

where F is a flow variable with values between zero and one . In particular, $F = 1$ corresponds to full cell, $F = 0$ to an empty cell, and $0 < F < 1$ to a surface cell. In the VOF/PLIC method an interface line is constructed in free surface cells using the gradient of F function. In the reconstruction of the interface, normal direction to the interface is calculated by

$$\vec{n} = \frac{\vec{\nabla} F}{\left| \vec{\nabla} F \right|} \tag{11}$$

The interface is moved with the velocity field and volume fluxes can be calculated using the procedure described in Ref. [9].

4 TEST CASES AND APPLICATIONS

4.1 Liquid sloshing in tank subjected to harmonic oscillations

Liquid sloshing in a partially filled rectangular tank excited horizontally is selected to test the computational model and computer code. The parameters of the problem are selected the same as in the experiments of Okamoto and Kawahara [10] to compare the numerical results with the experimental data. The tank width b = 1.0 m, initial water depth in the tank h = 0.5 m, period of oscillation T = 1.183 s and the amplitude of displacement is A = 0.93 cm. This test case was defined such that the frequency of oscillation was equal to the natural frequency of the tank to observe possible resonating free surface oscillations.

The computed free surface profile at t = 3.55 s is compared to experimental data in Figure 1. The spatial and temporal agreement between the computation and experiment clearly show that the present computational model can capture the free surface deformations accurately.

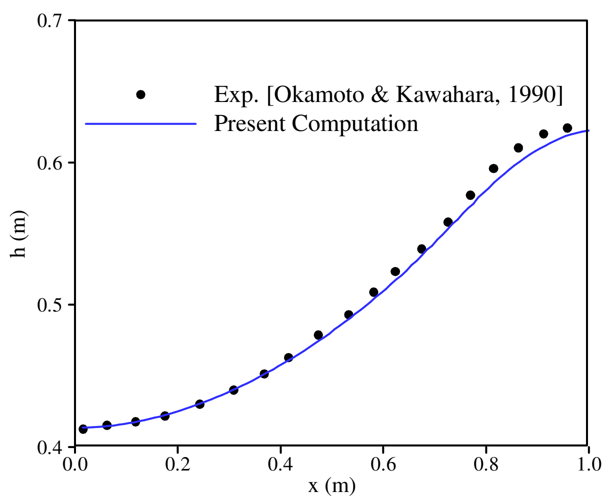

Figure 1. Comparison of free surface profiles at t = 3.55 s with the experimental data

Water level at the left wall of the tank during oscillation is shown as a function of simulation time in Figure 2. During the simulation, the surface waves are amplified, reaching a maximum wave height of approximately 0.4 m, and then the amplitude of sloshing waves decreases gradually with the effect of resonance.

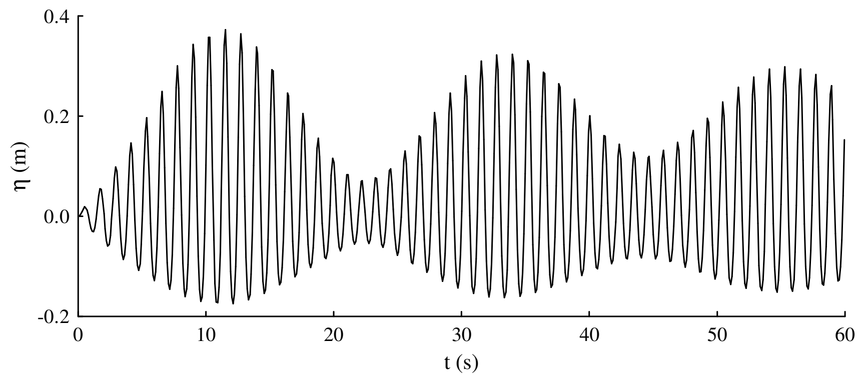

Figure 2. Water level at the left wall of tank as a function of time

It's seen from Figure 2 that the maximum surface level occurs at approximately t = 11.5 s and the free surface profile at this time is shown in Figure 3. It can be seen from the figure that the steep surface waves in the resonant case can be modeled accurately using the present computational model.

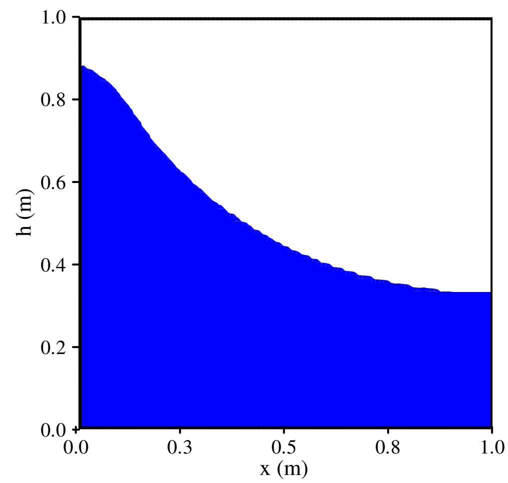

Figure 3. Free surface profile at t = 11.5 s

4.2 Liquid Sloshing in tank subjected to seismic excitation

Liquid sloshing in a rectangular tank subjected to actual ground motion can be simulated using the present computational model. Chen et al. [11] carried out numerical simulations using the potential flow equations and reported numerical result for the liquid sloshing in a wide rectangular tank with the width of 9.14 m and height of 6.1 m subjected to east-west component of El Centro earthquake (Figure 4). Initial water depth in the tank is 4.57 m.

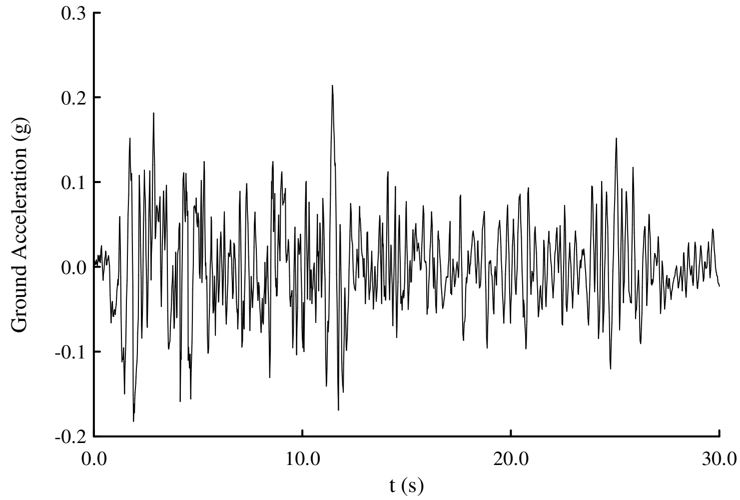

Figure 4. Time history of east-west component of El Centro earthquake

Water level at the left wall of the tank is shown as a function of time in Figure 5. It is seen form Figure 5 that present solutions agree well with the results of [11], but there is a discrepancy at $t = 25$ s. This discrepancy may be due to viscous effects since reference numerical solution is based on the potential flow equations.

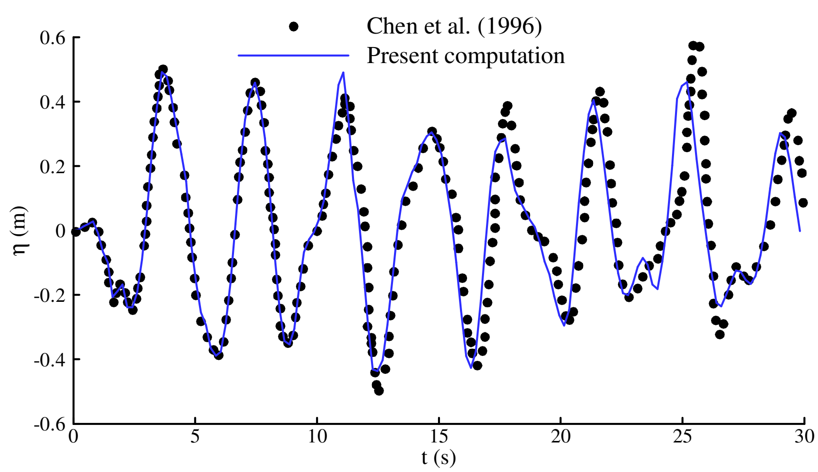

Figure 5. Water level at the left wall of tank as a function of time

4.3 Wave overtopping in excited tank

A rectangular tank is located in a wider tank and both tanks are oscillated horizontally with the same excitation in order to simulate the overtopping of surface waves. Width of the wide tank is set to 1.5 m and the parameters of the small tank are selected the same in section 4.1. Both tanks are excited with the same excitation and excitation parameters are set to the resonant case described in section 4.1. Free surface profiles at different time levels are shown in Figure 6.

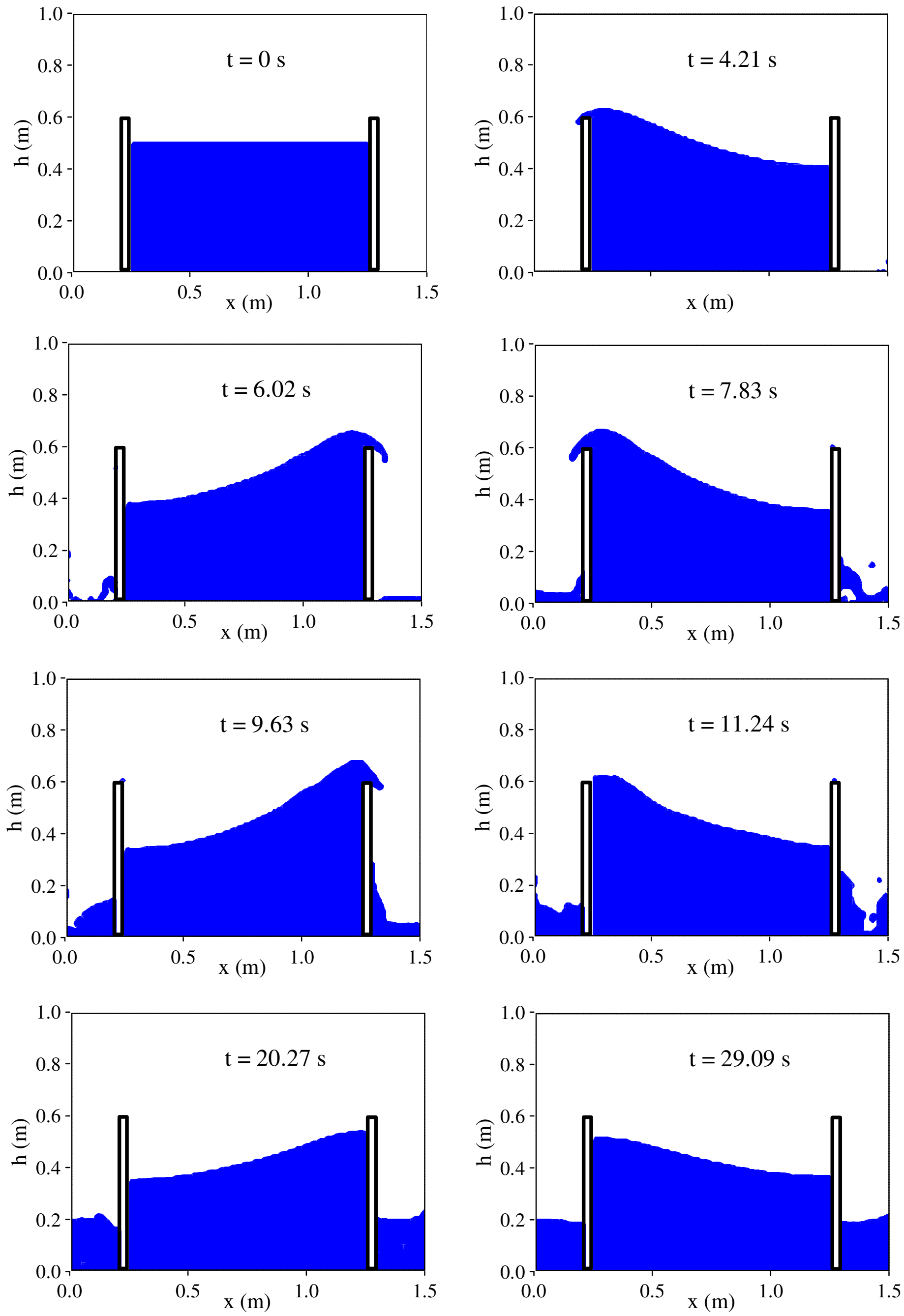

Figure 6. Free surface profiles at different time levels.

Amount of liquid exchange by spilling over the sharp crest boundary between two neighboring domains is evaluated to test volume conserving capability of the numerical scheme. Variation of dimensionless fluid volume V^+ which is the ratio of instantaneous volume of fluid to initial volume of fluid is plotted as a function of time in Figure 7. It's observed from the figure that the volume conservation of VOF/PLIC method violates when the overtopping of free surface waves occurs between 5 *s* and 20 *s* of the simulation.

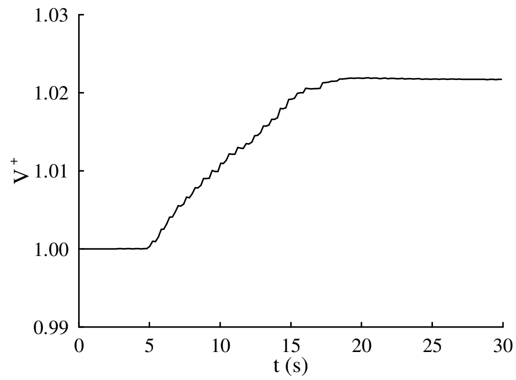

Figure 7. Variation of dimensionless volume of fluid in the tank.

5 CONCLUSIONS

- A two-dimensional computational model is developed for the numerical simulation of unsteady free surface flows in externally excited rectangular tanks.
- Computational model is validated comparing the computed results with existing experimental results for the resonant case. It is concluded that the present computational model can predict the free surface position accurately.
- Proposed model can simulate the nonlinear liquid sloshing in a rectangular tank subjected to actual ground accelerations.
- Wave overtopping in a rectangular tank subjected to horizontal harmonic excitation is simulated and volume conservation property of the VOF/PLIC method is monitored during the simulation duration. It's observed that the volume conservation is violated during the overtopping of surface waves between two neighboring domains.
- Volume conservation property of VOF/PLIC method may be improved coupling VOF method with level set or height function methods.
- The computational model presented is capable of predicting the safe freeboard height required in liquid storage tanks.

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