

# HIGH-ORDER ACCURATE IMPLICIT RUNGE-KUTTA SCHEMES FOR THE SIMULATION OF UNSTEADY FLOW PHENOMENA IN TURBOMACHINERY

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## ABSTRACT

High-order accurate implicit Runge-Kutta schemes for the numerical integration of the compressible, unsteady Navier-Stokes equations within the context of a hybrid (structured and unstructured) flow solver are investigated and compared with more conventional implicit first- and second-order accurate time integration schemes. Three Explicit first stage, Stiffly accurate Diagonally Implicit Runge-Kutta (ESDIRK) schemes of second-, third- and fourth-order accuracy are considered. To solve the individual stages both direct [1] and predictor-corrector [2] type approaches in combination with the pseudo-time-stepping method are investigated. For the solutions of the resulting linear systems the Symmetrical-Gauss-Seidel (SGS), Incomplete-Lower-Upper (ILU) and the Symmetrical Successive Over Relaxation (SSOR) are compared. The efficiency gains of the methods over lower order methods as well as the issues of robustness and complexity are discussed and analyzed in detail.

For the basic validation of the methods the advection of a localized entropy disturbance in an otherwise uniform subsonic flow is employed. The results confirm the order of accuracy of the methods investigated and highlight the gains in the absolute level of accuracy at a given resolution. Analysis shows, based on the L1-error norm, that the high-order methods can more than halve (as compared to the second-order Crank-Nicolson method) the number of time-steps required for a given degree of accuracy. In the final paper, following this basic validation and analysis, the methods will be applied to simulate the time-dependent flow at a single radial section of a low-pressure turbine. Here, the issues of efficiency and robustness will be examined and discussed in detail. Finally, the implemented methods will be applied to simulate noise generation and propagation in a modern, transonic fan-stage.

## REFERENCES

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