A STABILIZED FORMULATION FOR THE INCOMPRESSIBLE NAVIER–STOKES EQUATIONS USING FINITE CALCULUS

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ABSTRACT

We present a stabilized finite element formulation for incompressible fluids based on a Finite Calculus (FIC) [1] framework. The focus of the paper is twofold - derivation of stabilized forms of the momentum and mass balance (incompressibility) equations via FIC procedures. Using the Galerkin method approximate weak forms of the FIC equations are written wherein the terms involving the derivatives of the characteristic lengths are dropped. Nevertheless the consistency of the formulation is not violated. The characteristic lengths that appear in the FIC form of the momentum equations are designed to introduce a combination of an upwind plus discontinuity capturing effect. The distinction is that in general the upwinding is not streamline and the discontinuity capturing is neither isotropic nor purely crosswind. The point of departure in the later derivation is the higher order FIC form of the mass balance (incompressibility) equation. The approximate weak form of this FIC equation involves a boundary integral term which is similar to the boundary integral modification advocated for the Galerkin least squares method [2]. If the characteristic length is defined parallel to the pressure gradient, then the stabilization term for the mass balance equation may be expressed in a Laplacian form with a nonlinear stabilization coefficient. Likewise a composite expression for the stabilization coefficient involving the residual of the momentum equation may be written. This composite expression is well behaved in the limit of either the viscosity coefficient or the Reynolds number tending to zero.

REFERENCES

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