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MODEL-REDUCED VARIATIONAL DATA ASSIMILATION FOR SHALLOW WATER FLOW MODELING

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Abstract. *Accurate forecasting of the storm surges is very important in the Netherlands since large areas of the land lie below sea level. Timely water level forecasts are necessary to support the decision of the proper closure of the movable storm surge barriers. Dutch continental shelf model (DCSM), is a shallow sea model of the entire European continental shelf, which is used in the Netherlands to forecast the storm surges in the North Sea. Performance of the DCSM regarding the storm surges is influenced by its performance in forecasting the astronomical tides. The Adjoint method has often been used for the calibration of the large scale numerical flow models. The drawback of the adjoint method is the programming effort required for the implementation of the adjoint model code. The presented work is based on model reduction using Proper Orthogonal Decomposition (POD), which shifts the minimization into lower dimensional space and avoids the implementation of the adjoint of the tangent linear approximation of the original nonlinear model. The DCSM domain consists of* 201×173 *grid blocks with 19809 computational grid points. The POD-based model-reduced approach has been applied successfully to the DCSM for the estimation of the 13 uncertain parameters of water depth and space varying bottom friction coefficient. The main findings are: (1) A low dimensional model of order 37 can be constructed that captures 95% relative energy. (2) An overall improvement of 10% is obtained as compared to the operational DCSM. (3) The approach with relatively little computational cost and without the burden of implementation of the adjoint model can be used in variational data assimilation.*

1 INTRODUCTION

Accurate sea water level forecasting is crucial in the Netherlands. This is mainly because large areas of the land lie below sea level. Forecasts are made to support the storm surge flood warning system. Timely water level forecasts are necessary to support the decision for closure of the movable storm surge barriers in the Eastern Scheldt and the New Waterway. The surge is predicted by using a numerical hydrodynamic model, the Dutch continental shelf model (DCSM) (see [1, 2]). The performance of the DCSM regarding the storm surges is influenced by its performance in forecasting the astronomical tides. Using inverse modeling techniques, these tidal data can be used to improve the model results.

The adjoint method is a well-known approach to inverse modeling. The method aims at adjusting a number of unknown control parameters on the basis of given data. The adjoint approach is computationally very efficient because one gradient calculation requires just a single simulation of the forward model and a single simulation of the adjoint model backward in time, irrespective of the number of parameters. The adjoint method has been used and applied successfully to many types of inverse problems in ground water flow studies (e.g., [3]), in meteorology (e.g., [4]), in oceanography (e.g., [5]) and in shallow water flow models (e.g., [6, 7, 8, 9]).

One of the drawbacks of the adjoint method is the programming effort required for the implementation of the adjoint model. Research has recently been carried out on automatic generation of computer codes for the adjoint, and adjoint compilers have now become available (see [10]). Even with the use of these adjoint compilers, developing an adjoint model is often a significant programming effort, that hampers new applications of the method.

Proper orthogonal decomposition (POD) is a model reduction method considered as an application of the singular value decomposition (SVD) to the approximation of general dynamical systems ([11]). It is a data driven projection based method originally developed by Karl Pearson ([12]). The POD method has been applied in many fields like image processing, signal processing, data compression, oceanography, chemical engineering and fluid mechanics (see [13]). In the POD method the projection subspace is determined by processing data obtained from numerical simulations of the high dimensional model, which is expected to provide information about the dynamical behavior of the system. The high dimensional equations are projected on to the low dimensional subspace resulting in a low dimensional model. In this way, the POD method reduces the CPU time of model simulation. The method has recently been investigated for state and parameter estimation by (e.g., [14, 15, 16, 17]).

[18] proposed a method based on POD, which shifts the minimization into lower dimensional space and avoids the implementation of the adjoint of the tangent linear approximation of the original nonlinear model. In their approach, an ensemble of snapshot vectors of forward model simulations is used to determine an approximation of the covari-

ance matrix of the model variability and a small number of leading eigenvectors of this matrix are used to define a model subspace. By projecting the original model onto this subspace, an approximate linear reduced model is obtained. Due to the linear character of the reduced model, its adjoint can be implemented easily and the minimizing problem is solved completely in reduced space with very low computational cost. The method has recently been tested successfully to estimate water depth in the tidal model DCSM with a coarse grid ([19]). Several synthetic cases were used to show that the depth parameters were correctly identified in the selected regions of the model domain. The generation of an ensemble in the POD method involves running the forward model several times. The computational cost of the method is dominated by the generation of this ensemble. It was also found in the study that if the dynamics of the system does not change significantly then a smaller simulation period can be chosen to generate an ensemble of forward model simulations for an optimization problem over larger period ([19]).

DCSM is a spherical grid based water level model for the Northwest European Continental Shelf (with 19809 computational grid points). The first version of DCSM was developed in the 1980ś and has been through numerous improvements since then. The POD based model reduced approach described in this paper is used for the estimation of the depth values and space varying bottom friction coefficient in this model DCSM. Here, the POD method has been used to improve the operational model DCSM with real data. The computational costs of the method is dominated by the generation of an ensemble of forward model simulations. The simulation period of the ensemble is equivalent to the timescale of the original model. Here an accurate reduced model is obtained from an ensemble with a relatively short simulation period as compared to the calibration period.

The paper is organized as follows. Section 2 explains classical method for parameter estimation, the methodology of POD projection based reduced method for the calibration is explained in Section 3. Section 4 discussed the experiments with the model DCSM to estimate the water depth and the bottom friction coefficient. Section 5 presents the conclusions.

2 PARAMETER ESTIMATION USING ADJOINT METHOD

Consider the data assimilation problem as general nonlinear dynamical system. The discrete system equation for the state vectors $\mathbf{X}(t_i) \in \Re^n$ is given by:

$$\mathbf{X}(t_{i+1}) = M_i[\mathbf{X}(t_i), \gamma] \tag{1}$$

where M_i is a nonlinear and deterministic dynamics operator that includes inputs and propagates the state from time t_i to time t_{i+1}, γ be vector of uncertain parameters which needs to be determined . Suppose now that we have imperfect observations $\mathbf{Y}(t_i) \in \Re^q$ of the dynamical system (1), that are related to model state at time t_i through

$$\mathbf{Y}(t_i) = H\mathbf{X}(t_i) + \eta(t_i) \tag{2}$$

with $H : \Re^n \rightarrow \Re^q$ is a linear observation operator that maps the model fields on observation space and $\eta(t_i)$ is an unbiased, random Gaussian error vector with covariance matrix R_i.

The idea of parameter estimation is to identify the values of uncertain model parameters γ. We assume that the difference between data and simulation results is only due to measurement errors and incorrectly prescribed model parameters. A most commonly used measure that determines this difference is the weighted sum of squared residuals. The problem of estimation is then solved by directly minimizing the cost function J

$$J(\gamma) = \sum_i [\mathbf{Y}(t_i) - H(\mathbf{X}(t_i))]^T R_i^{-1} [\mathbf{Y}(t_i) - H(\mathbf{X}(t_i))] \tag{3}$$

with respect to the parameters γ, satisfying the discrete nonlinear forecast model (1).

The minimization of the cost function (J) is often based on quasi-Newton methods. These methods require the computation of the gradient of the cost function. The gradient vector (∇J) gives information about the direction (positive or negative) and the size of adjustments for each individual parameter. The adjoint method computes the exact gradient efficiently. The principle of the adjoint method is based on the systematic use of the chain rule of differentiation. Regardless of the number of parameters, the time required to compute the gradient using adjoint technique is more or less identical and is comparable with the computational time needed for a single simulation run of the nonlinear model (1). It requires one forward simulation with original the nonlinear model (1) and a second additional simulation backward in time with the adjoint model:

$$\nu(t_i) = (\frac{\partial M_i}{\partial \mathbf{X}(t_i)})^T \nu(t_{i+1}) - 2HR_i^{-1}[\mathbf{Y}(t_i) - H(\mathbf{X}(t_i))] \tag{4}$$

where $\nu(t_i)$ represents the solution of the adjoint model. The gradient ∇J of the cost function (J) with respect to each component $\gamma_k; k = \{1, \ldots, n^p\}$ of the uncertain parameters vector γ is given by:

$$\nabla J_k = \sum_i - [\nu(t_{i+1})]^T [\frac{\partial M_i[\mathbf{X}(t_i), \gamma]}{\partial \gamma_k}], k = \{1, \ldots, n^p\} \tag{5}$$

The main hurdle in the use of adjoint method is its implementation. Even with the use of adjoint compilers that have become available these days, this is a huge programming effort, that hampers new applications of the method. Secondly the adjoint equation needs to be integrated backward in time and therefore the states of the forward model have to be stored at each grid point for all time steps. The memory access will therefore be huge for large scale problems.

3 MODEL REDUCED VARIATIONAL DATA ASSIMILATION

The classical adjoint problem for a general model is a nonlinear constrained optimization problem which is difficult to solve. The problem can be simplified with the hypothesis

that the objective function J can be made quadratic by assuming that the nonlinear dynamics operator M_i can be linearized. The linearization of nonlinear high-order model (1) using the first order Taylor's formula around the background parameter γ_k^b gives

$$\triangle\overline{X}(t_{i+1}) = \frac{\partial M_i[\mathbf{X}^b(t_i), \gamma^b]}{\partial X^b(t_i)}\triangle\overline{X}(t_i) + \sum_k \frac{\partial M_i[\mathbf{X}^b(t_i), \gamma^b]}{\partial \gamma_k}\Delta\gamma_k \tag{6}$$

where $\overline{\mathbf{X}}$ is linearized state vector, $\mathbf{X}^b$ is the background state vector with the prior estimated parameters vector γ^b and $\triangle\overline{X}$ is a deviation of the model from background trajectory.

A model can be reduced if the incremental state $\triangle\overline{X}(t_{i+1})$ can be written as linear combination:

$$\triangle\overline{X}(t_i) = P\xi(t_{i+1}) \tag{7}$$

where $P = \{p_1, p_2, \cdots, p_r\}$ is a projection matrix such that $P^TP = I$ and ξ is a reduced state vector given by:

$$\xi(t_{i+1}) = \widetilde{M}_i\xi(t_i) + \sum_k \frac{\partial\widetilde{M}_i}{\partial\gamma_k}\Delta\gamma_k \tag{8}$$

or in matrix form

$$\begin{pmatrix} \xi(t_{i+1}) \\ \Delta\gamma \end{pmatrix} = \begin{pmatrix} \widetilde{M}_i & \widetilde{M}_i^\gamma \\ 0 & I \end{pmatrix} \begin{pmatrix} \xi(t_i) \\ \Delta\gamma \end{pmatrix} \tag{9}$$

Here $\Delta\gamma$ is the control parameter vector, $\widetilde{M}_i$ and $\widetilde{M}_i^\gamma$ are simplified dynamics operators which approximate the full Jacobians $\frac{\partial M_i}{\partial \mathbf{X}^b}$ and $\frac{\partial M_i}{\partial \gamma_k}$ respectively:

$$\widetilde{M}_i = P^T\frac{\partial M_i}{\partial \mathbf{X}^b(t_i)}P \tag{10}$$

$$\widetilde{M}_i^\gamma = P^T(\frac{\partial M_i}{\partial \gamma_1}, \cdots, \frac{\partial M_i}{\partial \gamma_u}) \tag{11}$$

The Jacobian $\frac{\partial M_i}{\partial \mathbf{X}^b}$, is obtained by approximating the nonlinear dynamics operator M_i by linearizing it with respect to background state $\mathbf{X}^b$. Instead of computing this huge Jacobian by approximating the partial differential with finite difference by perturbing the nonlinear operator M_i in the direction of each node, we perturb along the direction of p_h only:

$$\frac{\partial M_i}{\partial \mathbf{X}^b(t_i)}p_h = \frac{M_i[\mathbf{X}^b(t_i) + \varepsilon p_h, \gamma^b] - M_i[\mathbf{X}^b(t_i), \gamma^b]}{\varepsilon}, h = \{1, \ldots, r\} \tag{12}$$

with ε being the size of the perturbation. The reduced dynamics operator $\widetilde{M}_i$ can now be computed by pre multiplying the above formula by P^T:

$$\widetilde{M}_i = P^T(\frac{\partial M_i}{\partial \mathbf{X}^b(t_i)}p_1, \cdots, \frac{\partial M_i}{\partial \mathbf{X}^b(t_i)}p_r) \tag{13}$$

Notice also that only the original model simulations are needed here.The reduced model requires less computational time as it simulates a reduced state within the dimension r instead of the original dimension n where $r < n$. The dimension on which the reduced model operates is $(r + n^p) \times (r + n^p)$ with n^p being the number of estimated parameters.

3.1 Collection of the snapshots and POD basis

The POD method is used here to obtain an approximate low-order formulation of the original tangent linear model. POD is an optimal technique of finding a basis which spans an ensemble of data (snapshots) collected from an experiment or a numerical simulation of a dynamical system. The POD modes are optimal at approximating a given dataset. Since the reduced model is used here to estimate uncertain parameters (depth D and manning coefficient c_m), the snapshots should be able to represent the behavior of the system for these parameters. Therefore the snapshot vectors $e_i \in \Re^s$ are obtained from the perturbations $\frac{\partial M_i}{\partial \gamma_k}$ along each estimated parameter γ_k to get a matrix:

$$E = \{e_1, \cdots, e_s\}; i = \{1, 2, \cdots, s\} \tag{14}$$

The dimension of this ensemble matrix E is $s = u \times n^s$, where n^s is the number of snapshot collected for each individual parameter γ_k. The covariance matrix Q can be constructed from the ensemble E of the snapshots by taking the outer product:

$$Q = EE^t \tag{15}$$

The projection matrix P used in the previous section is based on the dominant eigenvectors (POD modes) of this covariance matrix. This covariance matrix is usually huge as in the current application with state vector of dimension $\sim 3 \times 10^6$, so direct solution of eigenvalue problem is not feasible. To shorten the calculation time necessary for solving the eigenvalue problem for this high dimensional covariance matrix, we define a covariance matrix G as an inner product

$$G = E^t E \tag{16}$$

In the method of snapshots ([20]), one then solves the $s \times s$ eigenvalue problem

$$G\mathbf{z}_i = E^t E\mathbf{z}_i = \lambda_i \mathbf{z}_i, i \in \{1, 2, \cdots, s\} \tag{17}$$

where λ_i are the eigenvalues of the above eigenvalue problem. The eigenvectors $\mathbf{z}_i$ may be chosen to be orthonormal and the POD modes P are then given by:

$$\mathbf{p}_i = E\mathbf{z}_i/\sqrt{\lambda_i} \tag{18}$$

A physical explanation of POD modes is that they maximize the average energy in the projection of data onto subspace spanned by the modes. We define a measure ψ_i for the relative information to choose a low dimensional basis by neglecting modes corresponding to the small eigenvalues:

$$\psi_i = \frac{\lambda_i}{\sum_{l=1}^{s} \lambda_l} 100\%, i = \{1, 2, \cdots, s\} \tag{19}$$

We collect p_r $(r < s)$ modes such that $\psi_1 > \psi_2 > \ldots > \psi_r$ and they totally explain at least the required variance ψ^e,

$$\psi^e = \sum_{l=1}^{r} \psi_l \tag{20}$$

The total number of eigenmodes r in the POD basis P depends on the required accuracy of the reduced model.

3.2 Approximate objective function and its adjoint

In reduced model approach, we look for an optimal solution of the (1) to minimize the approximate objective function $(\hat{J})$ in an incremental way:

$$\hat{J}(\Delta\gamma) = \sum_i [\{Y(t_i) - H(X^b(t_i))\} - \hat{H}\xi(t_i, \Delta\gamma)]^T R_i^{-1} [\{Y(t_i) - H(X^b(t_i))\} - \hat{H}\xi(t_i, \Delta\gamma)] \tag{21}$$

The value of the approximate objective function $\hat{J}$ is obtained by correcting the observations $Y(t_i)$ for background state $X^b(t_i)$ which is mapped on the observational space through a mapping H and to the reduced model state $\xi(t_i, \Delta\gamma)$ which is mapped to the observational space through mapping $\hat{H}$, with $\hat{H} = HP$.

Since the reduced model has linear characteristics, it is easy to build an approximate adjoint model for the computation of gradient of the approximate objective function (21). The gradient of $\hat{J}$ with respect to $\Delta\gamma$ is given by:

$$\frac{\partial \hat{J}}{\partial(\Delta\gamma)} = \sum_i -[\hat{\nu}(t_{i+1})]^T \frac{\partial \xi(t_{i+1})}{\partial(\Delta\gamma)} \tag{22}$$

where $\hat{\nu}(t_{i+1})$ is the reduced adjoint state variable. Once the gradient has been computed, the process of minimizing the approximate objective function $\hat{J}$ is done along the direction of the gradient vector in the reduced space.

After the minimization process the initial parameters γ are updated and new set of updated parameters γ^{up} is obtained:

$$\gamma^{up} = \gamma + \Delta\gamma \tag{23}$$

This process of minimization is repeated several times by constructing new POD model with new set of updated parameters γ^{up} to get optimal parameters.

3.3 Convergence Criterion for Inner and Outer Iterations

We have defined two criterions, both for the inner and outer iterations of the optimization process. We stop the present inner iteration and switch to a new outer iteration with new set of parameters following the criterion μ, which is defined as:

$$\mu = \sum_{k=1} \parallel \bigtriangledown \hat{J}_k^{\eta} \parallel / \parallel \bigtriangledown \hat{J}_k^{b} \parallel \leq \delta \tag{24}$$

with $\bigtriangledown \hat{J}_k^{b}$ is value of gradient for $\Delta\gamma_k$ at start of inner iteration, $\bigtriangledown \hat{J}_k^{\eta}$ is value of gradient for $\Delta\gamma_k$ after each inner iteration. The value of δ is chosen considering that the gradient should decrease by at least three order of magnitude from the initial gradient ([14]) or the number of inner iterations are $(n^p + 1)$, where n^p is the number of uncertain parameters. The outer iteration cycle converges when the optimal value α of the objective function J is achieved.

$$\alpha = \parallel [J]^{\beta} - [J]^{\beta-1} \parallel \leq \kappa \tag{25}$$

where β is the number of outer iterations. We have chosen $\kappa = 25$ for the numerical experiment.

3.4 Computational Cost

The computational costs of the reduced model approach are dominated by the generation of the ensemble of forward model simulations. If the dynamics of the system does not change significantly during the coarse of simulation then a smaller simulation period can be chosen for the generation of ensemble. Using this ensemble the optimization problem can then be solved over whole period of model simulation. The efficiency of optimization process is also influenced by the ensemble size. A large ensemble size leads to a huge eigenvalue problem. It is possible to include only those snapshots in the ensemble where data is available.

The method needs to be updated in each outer iteration (β) by constructing a new POD model by generating a new ensemble of forward model simulations. The number of outer iterations β is influenced by the chosen abortion criterion κ. It should not be chosen too small as this cause jumping of objective function (J), since it is possible that reduced model overestimates γ_k due to the process of linearization.

4 DUTCH CONTINENTAL SHELF MODEL

The Dutch Continental Shelf Model (DCSM) is an operational storm surge model, used in the Netherlands for real-time storm surge prediction in North sea. Accurate predictions of the storm surges are of vital importance to the Netherlands since large areas of the land lie below sea level. Accurate forecasts at least six hours ahead are needed for proper closure of the movable storm surge barriers in Eastern Scheldt and the New Waterway. The governing equations used in DCSM are the non-linear 2-D shallow water equations. The shallow water equations, which describe large scale water motions,

are used to calculate the movements of the water in the area under consideration. These equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} - fv + \frac{\rho_0 g u\sqrt{u^2+v^2}}{(D+h)C^2} = \frac{\tau_x}{D+h} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial x} \tag{26}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} + fu + \frac{\rho_0 g v\sqrt{u^2+v^2}}{(D+h)C^2} = \frac{\tau_y}{D+h} - \frac{1}{\rho_w}\frac{\partial p_a}{\partial y} \tag{27}$$

$$\frac{\partial h}{\partial t} + \frac{\partial [u(D+h)]}{\partial x} + \frac{\partial [v(D+h)]}{\partial y} = 0 \tag{28}$$

with:

h = water-level
u, v = depth-averaged current in x and y direction respectively
D = water depth below the reference plane
f = Coriolis parameter
C = 2D Chezy coefficient
τ_x, τ_y = wind stress in x and y direction respectively
ρ_w = density of water
p_a = atmospheric pressure
ρ_0 = background density
g = acceleration of gravity

These equations are descretized using an Alternating Directions Implicit (ADI) method and the staggered grid that is based on the method by ([21]) and improved by ([1]). In the implementation, the spherical grid is used instead of rectangular (see e.g. [2]). Boundary conditions are applied at both closed and open boundaries. At closed boundaries, the velocity normal to the boundary is zero. So no inflow and outflow can occur through these boundaries. At the open boundaries the water level is described in terms of ten harmonic components ($M_2, S_2, N_2, K_2, O_1, K_1, Q_1, P_1, U_2, L_2$) as follows:

$$h(t) = h_0 + \sum_{j=1}^{10} f_j H_j \cos(\omega_j t - \theta_j) \tag{29}$$

where

h_0 mean water-level
H total water depth
$f_j H_j$ amplitude of harmonic constituent j
ω_j angular velocity of j
θ_j phase of j

The DCSM covers an area in the north-east European continental shelf, i.e. 12^oW to 13^oE and 48^oN to 62^oN, as shown in Figure 1. The resolution of the spherical grid is $1/8^o \times 1/12^o$, which is approximately 8×8 km. With this configuration there are 201×173

grid with 19809 computational grid points. The time step is $\triangle t$=10 minutes. All the open boundaries of the model are located in deep water (more than 200m). This is done in order to model explicitly the non-linearities of the surge tide interaction.

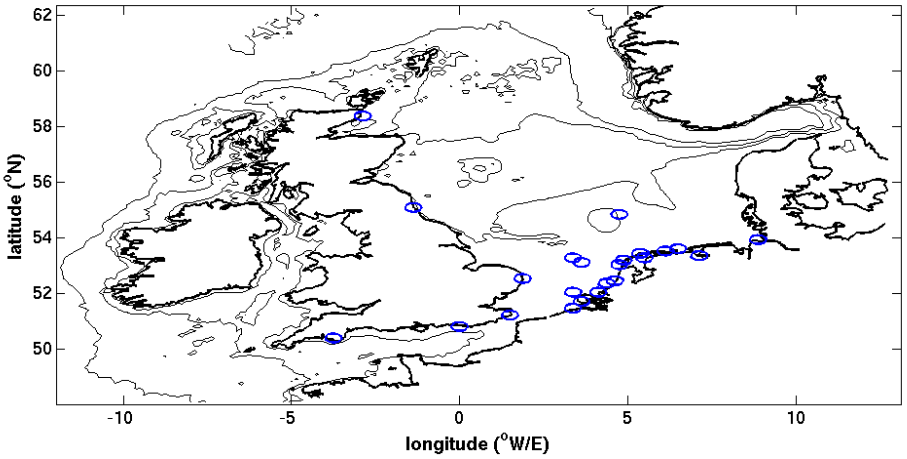
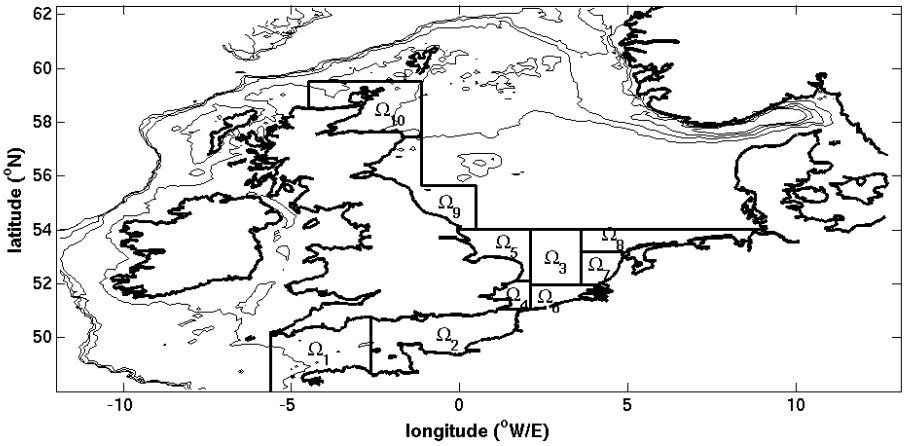


Figure 1: DCSM area with 50 m, 100 m and 200 m contour lines. The blue circles shows the locations of the tide gauge stations used for calibration.

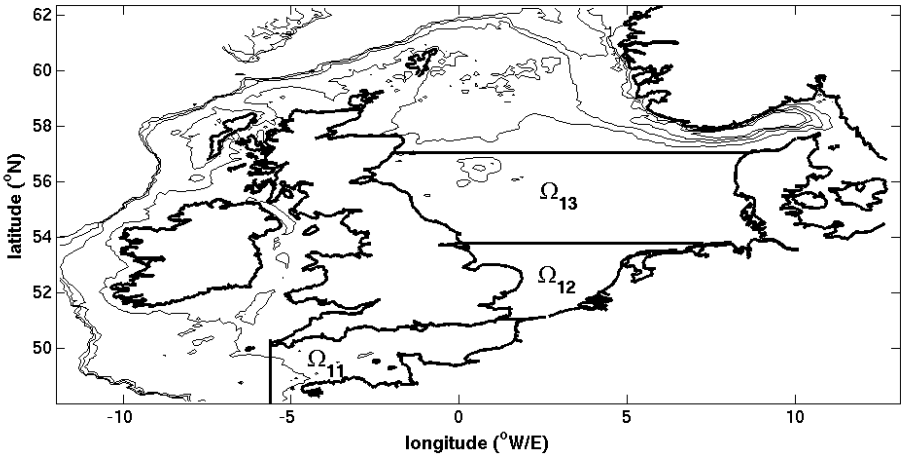
4.1 Estimation of Depth and Bottom Friction

The bathymetry for a model is usually derived from nautical maps. One of the purpose of these maps is to guide large ships safely through shallow waters. Therefore, these maps usually give details of shallow rather than deep-water areas. If we use these maps to prescribe the water depth, it is reasonable to assume that this prescription of the bathymetry is erroneous. So depth can be a parameter on which model can be calibrated. In the early years of the developments of the DCSM, the changes to these parameters were made manually. Later automated calibration procedures based on variational methods were developed starting from the work by ([6]; [22]). The complete description on the development of these calibrated procedures for DCSM can be found in ([23]). Both depth and bottom friction have to be prescribed at each grid cell of the model. Thus, theoretically, it is possible to consider depth and bottom friction at each grid cell as a parameter to adapt. Practically it is not possible to take the adaptation values of every grid point as a parameter since far too many parameters would then have to be estimated in proportion to the available amount of data. Including too many parameters, identifiability will become a problem ([24]). Here, the rectangular areas are chosen, which are considered as adaptation parameters. These rectangular areas are chosen based on the previous calibrations of the DCSM ([25]) and the spatial correlations within the rectangular regions.

The numerical domain is divided into 10 and 3 subdomains for the depth and bottom friction corrections respectively. (see Figure 2). For each subdomain Ω_k, a correction



(a)



(b)

Figure 2: The subdomains Ω_k of the DCSM used for the estimation of a) Depth values b) bottom friction coefficients

parameter γ_k^b is defined that is related to depth $D_{x,y}$ by:

$$D_{x,y}^{new} = D_{x,y}^{old} + \gamma_k^b, if(x,y) \in \Omega_k; k = \{1, \cdots, 10\} \tag{30}$$

and chezy coefficient C by:

$$C_{x,y}^{new} = C_{x,y}^{old} + \gamma_k^b, if(x,y) \in \Omega_k; k = \{11, \cdots, 13\} \tag{31}$$

The model performance can be assessed by comparing it to the measured (observed) dataset. The available data used in this research consists of water level measurements of the tide gauge stations from Dutch DONAR database. In the operational system the astronomical tide component of the observed data is replaced by the one predicted using the DCSM, obtained by running the DCSM without any wind input forcing ([26]). The target of the calibration of the parameters (i.e., depth and bottom friction coefficient)

is to optimize the model for its reproduction of the astronomical tide. The tide gauge data are therefore retrieved from the results of the harmonical analysis to exclude the meteorological influences.

For the calibration 24 water level locations are selected (see Figure 1). Observations obtained by the harmonic analysis from these 24 stations at every time step (10 minutes) are used for the calibration experiments. The calibration runs are performed for the period from 29 December 2006 to 30 January 2007 (33 days). The first 3 days are used to properly initialize the simulation. The measurement data are used for the remaining 30 days. This period is selected such that 2 spring-neap tide cycles are simulated. We have assumed that the observations Y of the computed water levels h have white noise process with standard deviation σ_m =0.10 meter.

As explained in section 3, the generation of the ensemble involves perturbations $\frac{\partial M_i}{\partial \gamma_k}$ with respect to each parameter for the whole simulation period. If the dynamics of the system does not change significantly then a smaller simulation period can be chosen to generate an ensemble of forward model simulations for an optimization problem over larger period ([19]). The ensemble E is generated using forward model simulations for a period from 29 December 2006 00:00 to 04 January 2007 24:00. The snapshot vectors in the ensemble are collected for the period where data is available, i.e., from 01 January 2007 00:00 to 04 January 2007 24:00. 30 snapshots are chosen with an equal interval for each γ_k.

Each snapshot vector consists of predicted water level h, velocities u and v. Before solving the eigenvalue problem as explained in the section 3.1 to find dominant eigenmodes, it is necessary to scale the snapshot vectors. The state vector should be scaled such that all state variables become equally observable. One approach here is based on the energy. The potential energy of a surface elevation h above the reference plane for one grid cell is

$$E_h = 1/2gh^2\rho_w\Delta x\Delta y \tag{32}$$

and the kinetic energy of the grid cell is

$$E_{u,v} = 1/2(u^2 + v^2)D\rho_w\Delta x\Delta y \tag{33}$$

where

g the gravitational acceleration

ρ_w the density of the air

Assume one measures surface elevations. Through propagation of the model, kinetic energy may become potential energy and because the model is dissipative, the sum of the two can only decrease or at most remain the same. This suggests that scaling the state variables according to the energy they represent creates approximately equal observability if the dissipation is small. In this case, the water levels should be scaled with $\sqrt{g}$ and the velocities u and v with $\sqrt{D}$ (see [27]).

Using this ensemble E of 390 snapshot vectors, we are able to form a basis consisting of only 24 dominant eigenmodes that captured 95% of the relative energy. Figure 3 shows

POD modes captured energy for 390 snapshot vectors. So a reduced model is built using these 24 POD modes and finally operates on dimension $\Re^{24+13}$. The low dimensional model is defined by assuming that the matrix $\widetilde{M}$ remains stationary throughout the experiment.

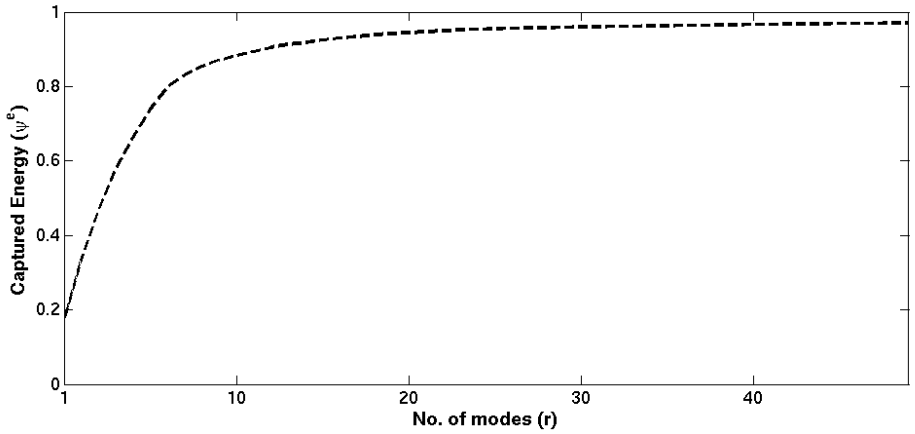


Figure 3: The POD modes capture energy for an ensemble of 390 snapshots of the water-level h(m), velocities u and v.

With this reduced model, the approximate objective function $\hat{J}$ is minimized in reduced space and the new values of the estimated variables γ^{up} are found. We have stopped the inner minimization process and switched to a new outer iteration with the new set of parameters following the criterion μ. The objective function J is reduced by more than 7% with the updated parameters γ^{up} after the inner minimization iteration (Figure 4).

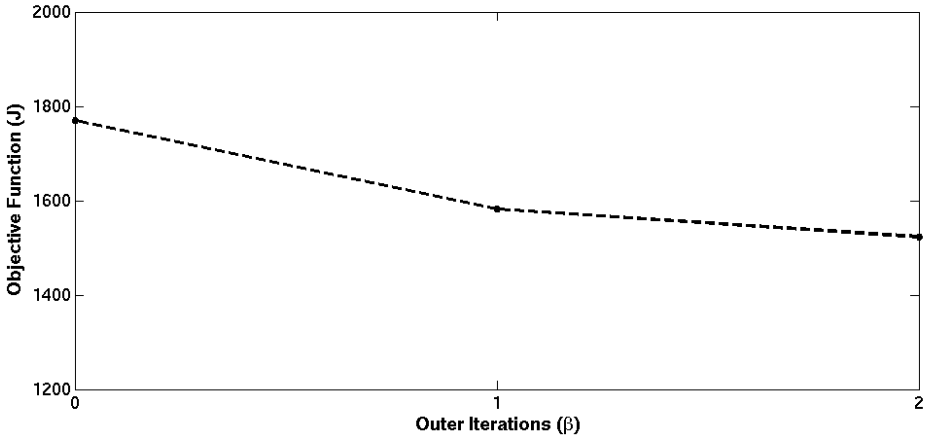


Figure 4: The value of the objective function J at successive outer iterations.

A new POD model is constructed in the 2^{nd} outer iteration. Again an improvement is observed in the objective function J after the inner minimization (see Figure 4) with the

new values of γ^{up} after the 2^{nd} outer iteration. Compared to the operational model the reduction of objective function J is more than 10%. It might seem that this improved performance is very minor. However, we must take into account that the operational DCSM is a well calibrated model of the tidal motion in the North Sea. Any improvement, achieved by adjusting the depth values and the bottom friction in the 13 subdomains, is therefore an encouraging result.

The validation runs were also carried out with 12 validation stations from the period 13 January 2007 to 14 February 2008 (33 days). The stations that were used for the calibration were not included in the dataset that was used for the validation (see Figure 5). Three validation runs were performed, 1^{st} with the initial values for the calibration parameters γ^b and two with the updated parameters γ^{up} after the 1^{st} and 2^{nd} outer iterations respectively. The validation experiments have to clarify whether these parameter adaptations really improve the model.

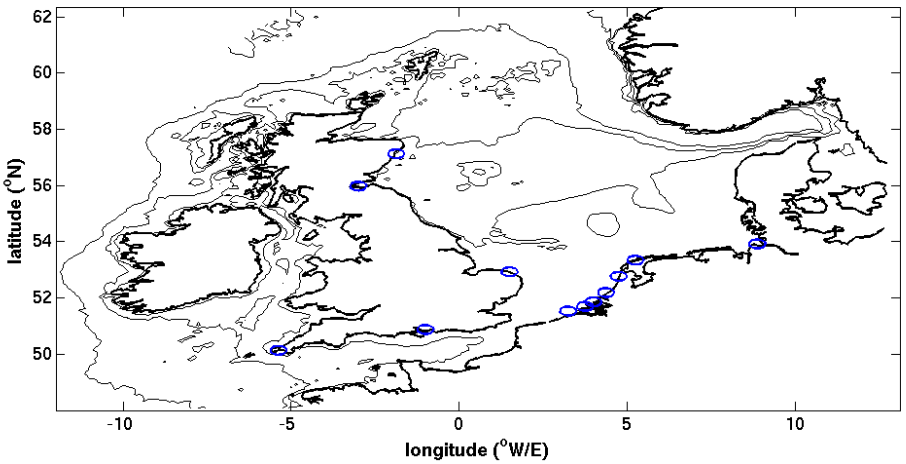


Figure 5: DCSM area with stations included in the validation experiment.

Figure 6 summarizes the RMSE after each outer iteration β, separately for the all data observation points used for calibration (assimilation) and the data observation points used for the validation of results. The POD based calibration approach, reduces the RMS values of the water levels (h) of the tide gauge stations used for the assimilation and validations.

The computational cost of the calibration experiments is expressed in terms of the number of simulations with the original model. Thirteen parameters are estimated during 1^{st} and 2^{nd} outer iterations β of the calibration experiment. In both the outer iterations one forward model simulation is required for the calibration period, i.e., from 29 December 2006 to 30 January 2007, to obtain the value of the objective function J. As thirteen parameters are estimated , thirteen forward model simulations are performed from 29 December 2006 to 04 January 2007 to obtain an ensemble of the perturbations $\frac{\partial M_i}{\partial \gamma_k}$ along

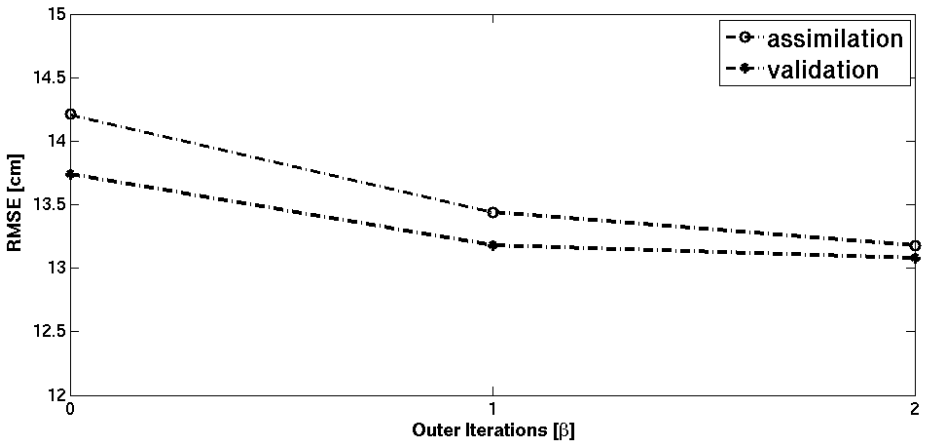


Figure 6: RMS errors with respect to water level observations at assimilation and validation stations.

γ_k. The snapshots are collected for last four days only, since observations are available for this period. As the number of snapshots chosen is 390, the computational time to solve eigenvalue problem and to construct the reduced model is negligible. Combined with thirteen estimated parameters, the reduced model simulates a reduced state within the dimension of a subspace $\Re^{37}$ instead of the original state space $\sim 6 \times \Re^{10^4}$. Similarly the cost of optimization in the reduced space is negligible and eventually the time required to estimate 10 depth values and 3 bottom friction coefficients is equal to approximately 10 forward model simulations.

5 CONCLUSIONS

- It is usually laborious to implement adjoint model for the computation of the gradient for large scale systems. The POD model-reduced approach is used here to simplify this problem, which shifts the minimization into lower dimensional space and avoids the implementation of the adjoint of the tangent linear approximation of the original nonlinear model. Compared to the classical adjoint method, the minimization in the reduced space converges faster due to better condition number of the reduced hessian.
- In this paper, the POD based calibration approach has been used to calibrate the two-dimensional shallow water flow model, the DCSM, defined over the entire European continental shelf. The method has been used to calibrate the DCSM with respect to the depth values and the space-varying bottom friction coefficient. The results show that the calibration method performs very efficiently. A POD reduced model of size $\Re^{37}$ is constructed instead of original model with state space $\sim 6 \times \Re^{10^4}$. The RMS errors for the tide gauge stations used for both calibration and validation periods have improved and an overall improvement of more than 10% is observed after the calibration in comparison with the operational model.
- The computational costs of the method is dominated by the generation of an ensemble

of forward model simulations. The simulation period of the ensemble is equivalent to the timescale of the original model. Here an accurate reduced model is obtained from an ensemble with a relatively short simulation period of first four days that is used for calibration over the whole calibration period of one month period.

- The results demonstrate that the time required to estimate 10 depth values and 3 bottom friction coefficients is equal to approximately 10 forward model simulations. Thus the POD calibration method offers an efficient minimization technique compared to the classical adjoint method without the burden of implementation of the adjoint.

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