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A MESH REGULARIZATION APPROACH FOR SURFACE GRIDS

E. Stavropoulou*, M. Hojjat*, R. Wüchner*, K.-U.Bletzinger*

*Chair of Structural Analysis, TU München
Arcisstrasse 21, 80290 München, Germany
e-mail: {stavropoulou, hojjat, wuechner, kub}@bv.tum.de

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Abstract. *Within this contribution, a global method which regularizes a finite volume or a finite element mesh towards a desired condition is presented. In this method, an artificial stress state is applied on the surface or on the volume mesh which is going to be regularized and a global linear system of equations is solved. The applied fictitious stress adapts each element towards a desired predefined template geometry and at the end a globally smooth mesh is achieved. In this way, both the shape and the size of each element is controlled. The generality of the method with respect to the field of applications is discussed and examples from shape optimization, large structural deformation simulations and mesh quality improvement are shown as well.*

1 INTRODUCTION

The quality of the solution in finite element and finite volume methods strongly depends on the domain discretization. Hence, a good mesh is certainly desired as it is a prerequisite for meaningful results. However, many of the mesh generation techniques are not able to provide a discretization with good element shapes in the entire domain, especially in the case of complex geometries.

Moreover, even if the quality of the generated mesh is acceptable, in applications which deal with varying geometries or involve moving boundaries, during the evolution of the computation, the quality of elements could deteriorate and severely distorted elements might occur. In extreme cases, the elements become degenerate and further progress of analysis is restricted. For instance, in shape optimization problems and large deformation fluid-structure interaction simulations, the retaining of the initial discretization properties is not guaranteed. Within these iterative processes, large deformations, rigid body motions, big rotations and high changes in the curvature of the involved surfaces usually occur which leads to distortion of the mesh.

A significant amount of work has been done in the areas of mesh smoothing (mesh relaxation) and mesh deformation. Among the most common mesh smoothing techniques is the Laplacian smoothing,[5,7,9] an iterative method, which repairs the mesh by adapting the position of the single nodes. This method can result in distorted or degenerated elements in concave domains. Another class of smoothing algorithms which performs usually better but is more expensive is the optimization-based smoothing.[8,10] Here, the nodes are moved to satisfy the optimum of a cost function which describes a distortion metric. Furthermore, there are techniques which are based on physical observations and for instance, analogies with springs, materials and bubbles were introduced.[13] On the other hand, methods for deforming structured and unstructured meshes are of great interest due to the increasing demands of CFD simulations with moving boundaries. These methods, usually, add some physical properties on the mesh and the connectivity between the points is represented by springs or solid body elasticity.[2,6]

In this work, a regularization scheme is developed, which is generally applicable to all the aforementioned cases and offers an automatic control of the quality of the mesh, being parallely efficient for large scale computations. This global regularization method smoothens the mesh towards a desired target mesh by only solving a linear system of equations with number of unknowns as much as the degrees of freedom preserving the boundary of the mesh (two and three degrees of freedom for surface and volume meshes, respectively).

This method is closely related to form-finding,[3,14] which is used to determine numerically the shape of membrane structures. In regularization there are some additional aspects which should be considered. Firstly, a target mesh has to be defined such that the applied artificial prestress brings the faces to their desired shape. Secondly, the applied prestress has to control both, the shape and the size of each individual element and

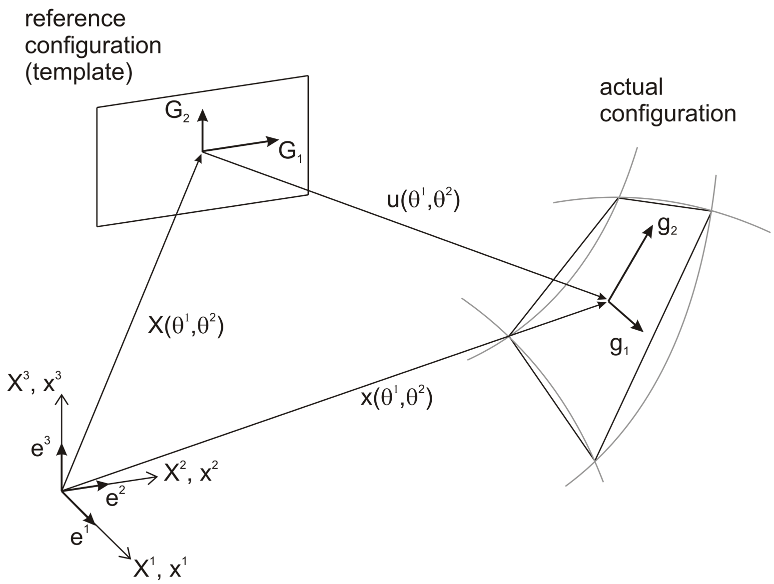


Figure 1: Deformation in the context of geometrical nonlinear analysis.

the resulting mesh has to preserve the geometry of the domain.

In section 2, the method is introduced. First, an introduction to form-finding is given and the governing equations of this procedure initiate the description of the method. In section 3, some test cases which may arise in various applications are examined and discussed. The strength of this method in the case of CFD simulations with moving boundaries as well as of shape optimization is presented.

2 THE REGULARIZATION METHOD

The mesh regularization method presented here is inspired by form-finding which is a method to determine the free-form shape of membrane and shell structures.[3,14]

Form-finding determines the structural shape from an inverse formulation of equilibrium in space due to a given pre-stress distribution acting on the deformed structure. In other words, assuming a stress field applied on the resulting structure, the displacement field which brings the system to equilibrium is found based on the principle of virtual work, as stated in the following:

$$\delta w = \int_a \boldsymbol{\sigma} : \delta \boldsymbol{\epsilon} da = \int_A \boldsymbol{S} : \delta \boldsymbol{E} dA = 0, \tag{1}$$

where $\boldsymbol{\sigma}$ is the prescribed Cauchy stress tensor acting on the resulting geometry with area α and $\boldsymbol{S}$ the 2nd Piola-Kirchhoff stress tensor acting on the reference geometry with area A. $\boldsymbol{E}$ and $\boldsymbol{\epsilon}$ are the Green-Lagrange and Euler-Almansi strain tensor, respectively.[1,11] When $\boldsymbol{\sigma} = \sigma \cdot \boldsymbol{G}$, with $\boldsymbol{G}$ being the unity tensor, the resulting surface is a minimal surface, which is the surface of minimal area content connecting given boundaries.

For the formulation of eq. (1), there is no material description needed, since the applied stress field is known and as a result, the problem is reduced to a geometrical one eventhough the formulation is initiated by a mechanical equilibrium. Eq. (1) is discretized by the finite element method considering large deformations, since the geometrical nonlinearity is not negligible.[4]

The resulting stiffness matrix is singular with respect to tangential shape variation. For this reason the updated reference strategy is applied,[3] in which eq. (1) is modified to the following:

$$\delta w = \lambda t \int_A det\boldsymbol{F}(\boldsymbol{\sigma} \cdot \boldsymbol{F}^T) : \delta\boldsymbol{F} dA + (1-\lambda)t \int_A (\boldsymbol{F} \cdot \boldsymbol{S}) : \delta\boldsymbol{F} dA = 0 \tag{2}$$

where $\boldsymbol{F}$ is the deformation gradient. The first term is equivalent to eq. (1). In the second term we assume the 2nd Piola-Kirchhoff stresses to be given instead of the Cauchy stresses σ. The solution of eq. (2) approaches the solution of the original problem by setting λ to a constant value small enough to stabilize eq. (1) and repeating the procedure, where S is adapted with respect to the actual geometry, assumed as the updated reference geometry. Taking $\lambda = 0$, the intermediate problem is linear. The procedure is robust and converges to the solution of eq. (1) rather quickly.

The same principle applies to the mesh regularization method including additional constraints to the initial problem. The point of departure is again eq. (1). Now, the movement is restricted to the surface directions since the surface geometry should remain unchanged. Applying this constraint and $\lambda = 0$ to eq. (2) leads to a non-singular system of equations which is linear in the surface tangent space. As a consequence, the proposed method will generate proper meshes even for large distortions of plane or volumetric meshes after solving one linear system of equations. The kernel of the method is the proper choice of adequate element reference geometries or element templates. The can freely be chosen independently for each element representing the ideal element shape and/or size. Obviously, there will not be any modification if the actual element geometries are taken as templates.

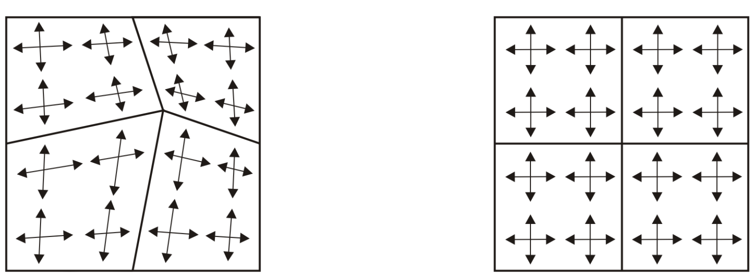


Figure 2: Regularization of a 4-element grid using a square template in the reference configuration with a schematic stress distribution before (left) and after (right) regularization.

For example, starting with the simple 4-element mesh of fig. 2 and applying the principle of virtual work with an isotropic stress field in the reference configuration, and without taking any caution for the reference geometry, the resulting displacemet field is going to be $\mathbf{0}$. Applying a square template in reference configuration, the strains and consequently the internal forces which are produced between the two configurations create a displacement field which is globally as close as possible to the defined template for every element since no external loading is applied.

The target (reference) element shape is chosen on element level without considering continuity with the neighbour elements and only depending on the needs of the mesh as it is going to be discussed in section 3. The standard notation of continuum mechanics is used to describe the displacement $\boldsymbol{u}$ as the difference between the actual position $\boldsymbol{x}$, and the reference position $\boldsymbol{X}$, with the reference configuration being derived based on an ideal target mesh (fig. 1).

3 RESULTS

The method presented in this work can have various applications due to its general formulation. It is not depending on the geometrical description of the elements of the mesh nor on the amount of distortion of the mesh.

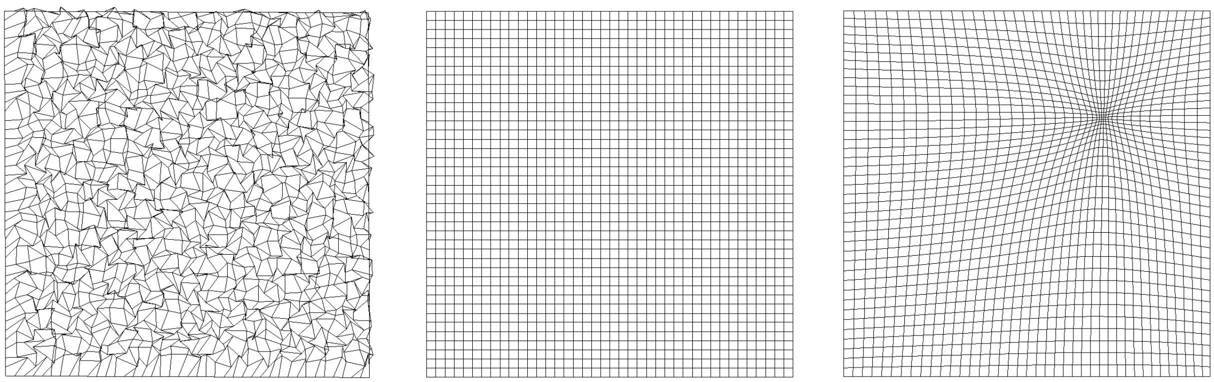
Additionally, there is a variety of choices for the element template which makes the method easily adjustable depending on the type of application. The template can preserve properties of the initial mesh, like size or shape, or it can be predefined with an ideal element shape.

A first example for a template could be the prototype of an ideal element shape for all elements, for instance, a square or an equilateral triangle. For complicated domains, where mesh generators produce a mesh with specific properties, this template might be adapted to the special situation. For example, the local refinement around points, lines and surfaces have to be retained. In such cases, a template which preserves the properties of the generated mesh has to be used. For instance, the template can preserve the area, or the length of the edges of each element. Furthermore, in applications where an evolutionary process distorts the mesh, like in optimization or fluid structure interaction simulations, the ideal mesh can be assumed to be the mesh which was initially defined.

In the following, examples from all the aforementioned cases will be presented. In section 3.1, a noise, which was added as a distortion to a mesh, will be removed and local refinements will be produced by the use of an ideal square template. Furthermore, a structured mesh produced with a mesh generator is improved with respect to element distortion. In section 3.2, some cases of evolving element distortions due to deformation processes are presented.

3.1 MESH QUALITY IMPROVEMENT

In this section, the regularization method is applied to different meshes in order to improve their quality. Firstly, a square template is applied as reference geometry to remove the distortion (noise) from a 2 dimensional plane mesh consisting of quad elements. Figs. 3a and 3b show the distorted mesh and the mesh after regularization, respectively. The same square template was used for every element and the resulting mesh consists of elements which match perfectly with the predefined template.



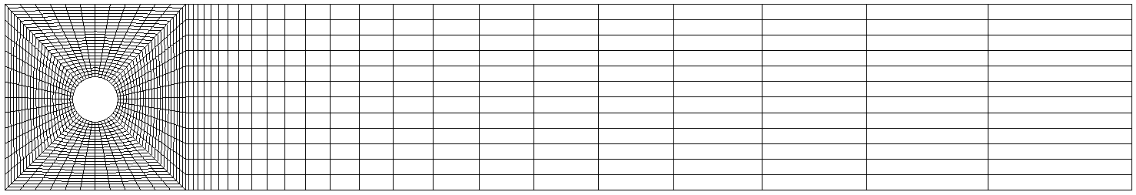
(a) (b) (c)

Figure 3: Noise removal and local refinement around a point.

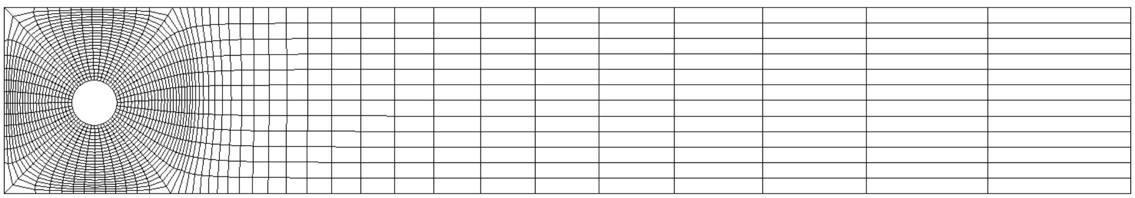
Changing the relative size of the template of each element, local refinements can be achieved. In fig. 3c, the size of the square template increases when the distance from the refinement point increases. In the same way, proper refinement needed for boundary layer resolution in fluid problems can be achieved.

In the next example, a mesh created by a mesh generator will be improved. Mesh generators usually create a mesh inside bounding boxes made out of straight lines and curves from of the boundary of the domain. But if the curvature of the domain boundaries is relatively high, the quality of the resulting mesh is affected and distorted elements occur in the region of the bounding boxes.

A mesh generated with such a technic is shown in fig. 4a. The elements are not severely distorted as in the previous section but still there is a group of elements around the boundary circle which are distorted. This type of non orthogonality is a remarkable source of error in numerical methods, especially in finite volume method. By removing the angle deformation from each element by assuming a quadrilateral template for each element with edges the midsegments of each element the mesh of fig. 4b is obtained. Now, the mesh is following the curved lines of the boundary and the mesh quality is significantly improved, as it can be seen in fig. 5. In this figure, the distribution of the angle of distortion is ploted and it can be observed that after regularization, more



(a)



(b)

Figure 4: Regularization of a structured mesh with a hole.

elements have small distortion angles and the elements with bigger distortion angles are less.

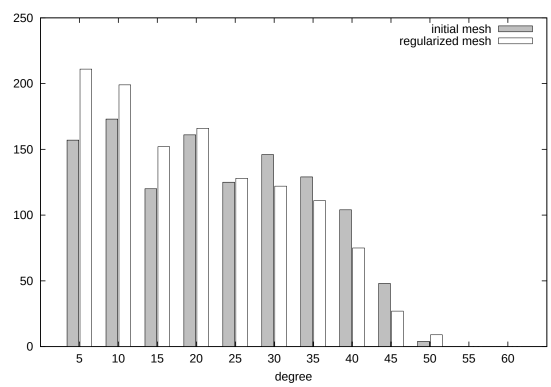
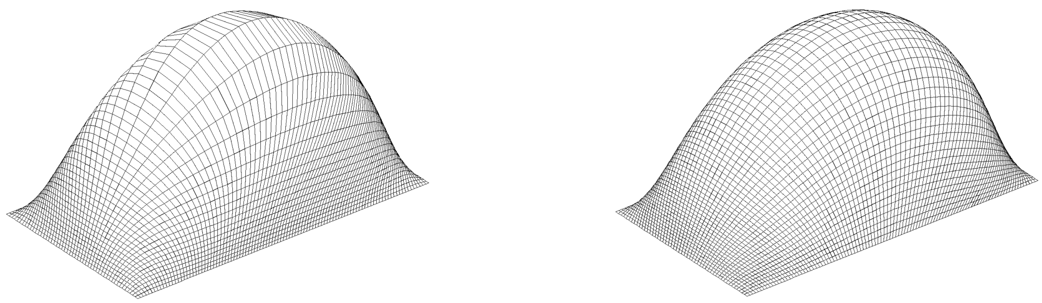


Figure 5: Distribution of angle of distortion in initial and regularized mesh.

3.2 APPLICATIONS IN EVOLUTIONARY PROCESSES

In this paragraph, the mesh regularization method is discussed in the context of evolutionary processes which begin with an initial mesh which gets distorted during the computation. In this type of applications, large variations of the surface curvature during the computation can affect the quality of the mesh or even restrict the whole process, since the elements might get severely elongated or overlaped.

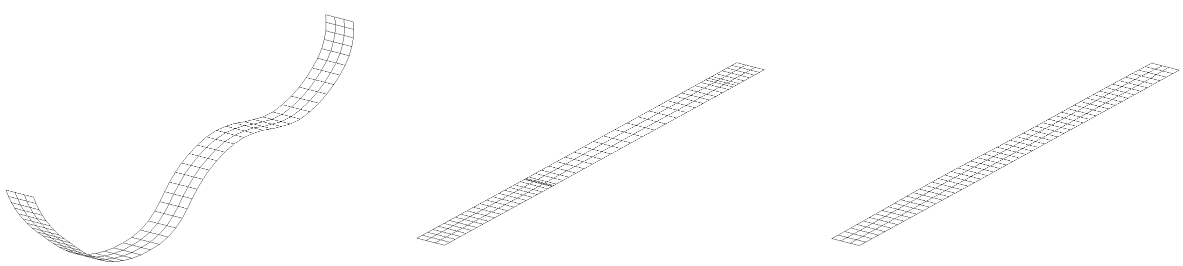
In fig. 6a, the resulting mesh of an initially plane surface, after applying a displacement field normal to the surface for 100 iterations, is shown. The resulting elements are elongated, even though, the initial mesh had a good quality consisting of square elements of the same size. Applying the regularization method, using the initial shape of each element as the reference template, the elements retain their initial shape (fig. 6b). The transmission from a flat or slightly curved surface to highly curved one is a common incident in parameter-free optimization.



(a) (b)

Figure 6: The final mesh of applying an evolutionary process without and with regularization, respectively.

The decrease of curvature in the surface mesh can have even more severe effects. The curved geometry of fig. 7a might reduce its curvature during simulation and this will lead some elements to overlap and become degenerate (fig. 7b). Applying the regularization method on the overall process, the problem can be dissolved and the computation is not restricted anymore because of failure of the mesh (fig. 7c).



(a) (b) (c)

Figure 7: Reduction of the curvature of a 2D mesh (a) to a plane mesh without (b) and with (c) regularization.

In the last example, the bulk motion of the circular boundary of the mesh of fig. 4a is tested. Both the shape and the position of the circular boundary is changed in three steps. The resulting mesh after applying regularization with slip boundaries in the upper and lower part of the domain is shown in fig. 8. Moving boundaries appear frequently in shape optimization problems and fluid structure interaction simulations[12] on non-fixed grids, as well as CFD calculations.

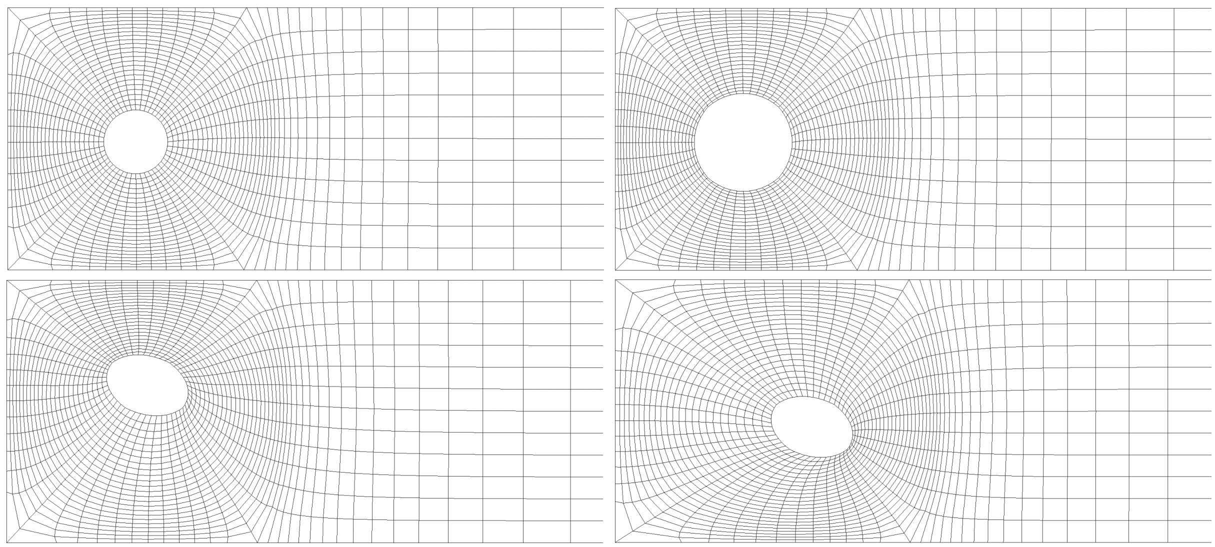


Figure 8: Adaptation of the mesh with a moving circular boundary.

4 CONCLUSION

In this contribution, a mesh regularization method for the smoothing of surface meshes is introduced. The method emanates from form-finding used to determine the shape of free-form membrane structures and because of its general formulation it is widely applicable. The method requires the solution of a global linear system of equations. The unknowns are the degrees of freedom preserving the boundary of the mesh. The method adapts the mesh towards a predefined target mesh. Some examples arising in several applications are presented and possible applications were addressed.

Further research includes an extension of the method for volume meshes and a comparison between the new method and existing methods with respect to accuracy and efficiency.

5 ACKNOWLEDGMENTS

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