A novel multi-D finite-volume method for advection problems with embedded moving-boundaries

Yunus Hassen, Barry Koren

Centrum Wiskunde & Informatica (CWI)
Science Park 123, 1098 XG Amsterdam, The Netherlands
e-mails: yunus.hassen@cwi.nl barry.koren@cwi.nl

ABSTRACT

In this work, we present a multi-D immersed-boundary approach, for numerically solving advection problems. We employ the method of lines: a higher-order, cell-averaged, fixed-grid, finite-volume method for the spatial discretization, and an explicit time integrator.

Considering we have a solid body immersed inside a fluid domain, we obtain the discrete embedded boundaries (EBs) associated with individual control volumes, at any given time, by: (a) immersing the body into a fixed, Cartesian, finite-volume grid and identifying the finite volumes that contain (a part of) the boundary of the immersed body, and (b) detecting the points of intersection of the boundary of the immersed body with the faces of each cell and, using these points, degenerating the boundary, of the immersed body, into piece-wise linear components (discrete EBs in individual control volumes). Then, the discrete EBs are 'accurately' aligned, depending on the relative inclination (of the EB) with respect to the orthogonal axes and the area they subset in a cell. Now with these orthogonalized discrete-EBs in the respective cells, we achieve a sub-cell resolution of the immersed body in the fixed, Cartesian, finite-volume grid. The fluxes in the immediate neighborhood that are affected by these EBs are computed in such a way that they accurately and monotonously accommodate the boundary conditions valid on the moving body. The multi-D flux computation follows a local 1-D approach, described in [1].

Moreover, for the temporal discretization, we employ a special technique; the time integration is locally adaptive. Depending on the crossing of finite-volume faces by an orthogonalized discrete-EB, time steps are split in the vicinity of each EB to account for the abrupt flux-reversal. In effect, we achieve discrete exact fluxes, resulting in a gradual transition of the fluxes in time, at those faces.

To validate our method, we consider two discriminating test cases, with known exact solutions. The numerical results obtained are remarkably accurate, without requiring much computational overhead. They show a significant improvement in resolution over those computed using the standard methods. It is anticipated that the method can be easily extended to bodies immersed in the Euler flows, which we foresee to consider next.

REFERENCES

[1] Y. Hassen and B. Koren, Finite-volume discretizations and immersed boundaries, in: *Lect. Notes Comput. Sci. Eng.* **71**, Springer, Heidelberg (2010), pp. 229–268.